

Maximum Flow Routing in Multihop Wireless Networks

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Abstract—Path selection in multihop wireless networks is a hard problem. Given a performance goal, the best approach remains unclear. In hardwired networks, network flow may be maximized by using shortest path over link weights that are determined by a linear program to a multicommodity flow formulation. In this poster we generalize this linear program to include interference and mobility. We show that “In a static wireless multihop network If we route packets using any other algorithm than shortest path with links weights determined by the solution of the dual problem of maximizing the minimum spare capacity after adding the interference constraints, then the network flow saturates before reaching the maximum when we increase the demands; while we can always increase demands until we reach the maximum flow supported by the network, if we use shortest path with links weights determined as stated above. The same is true for mobile wireless multi-hop networks under protocol interference model if nodes are available within each other transmission, interference ranges within constant probabilities”.

Keywords - maximum flow, conflict graph, linear programming, shortest path, optimum routing.

I. INTRODUCTION

In [2] the authors proved shortest path is optimum in hardwired networks by maximizing the minimum spare capacity for elastic traffic and accordingly more data can be accommodated. The minimum spare capacity z may be maximized by solving the LP in Eq. 1,

$$\begin{bmatrix} \mathbb{A} & \mathbf{0} \\ \mathbb{I} & \mathbf{1} \end{bmatrix} \begin{bmatrix} X \\ z \end{bmatrix} = \begin{bmatrix} V \\ C \end{bmatrix} \quad (1)$$

$$\mathbb{A} = \begin{bmatrix} A & 0 & 0 & \dots & 0 \\ 0 & A & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & A \end{bmatrix} \quad (2)$$

$$\mathbb{I} = [II\dots I] \quad (3)$$

$$X(k) \geq 0, 1 \leq k \leq K, z \geq 0 \quad (4)$$

A is the node-link incidence matrix, of dimension $N \times L$. X is links flow vector, V is the demands vector and C is the links capacity vector. There are K demands.

A solution to the above linear program exists when routes are selected according to shortest path where the link weights are the optimal values for the dual variables y^T of the dual problem. The dual problem is a minimization subject to:

$$[u^T y^T] \begin{bmatrix} \mathbb{A} & \mathbf{0} \\ \mathbb{I} & \mathbf{1} \end{bmatrix} \geq [0^T \mathbf{1}] \quad y^T \geq 0 \quad (5)$$

where u^T is a vector of size $1 \times KN$ of the dual variables corresponding to the equality constraints. y^T is a vector of size L corresponding to the inequality constraints.

II. INTERPRETATION OF MAXIMIZING THE MINIMUM SPARE CAPACITY OPTIMALITY RESULT

We will prove the following proposition which we first proved in our paper [1] although stated differently:

“In a hard-wired network if we route packets using any other algorithm than shortest path with links weights determined by the solution of the dual problem of maximizing the minimum spare capacity, then the network flow saturates before reaching the maximum when we increase the demands; while we can always increase demands until we reach the maximum flow supported by the network, if we use shortest path with links weights determined as stated above.”

Let us call a routing strategy (algorithm) that solves maximizing the minimum spare capacity z by γ and all other strategies that don't solve this optimization problem by ω . we will prove:

In any network: (i) when γ is used, which means the minimum spare capacity is maximum, then we can inject more demand (traffic) in the network, i.e. we can increase the demand, if the flow is not maximum. (ii) On the other hand when we have the flow is maximum then the routing strategy (algorithm) used must also be a solution to maximizing the minimum spare capacity, i.e. γ , and the maximum minimum spare capacity is zero. Accordingly, when the flow is not maximum and we are using ω , we cannot reach the maximum flow by increasing the demand, because if we keep increasing the demand until we reach the maximum then that means we are using γ which is a contradiction.

Proof:

The first part of the proposition (i) is clear because spare capacities of all links are at least $z = \min_{l \in L} z_l$, z_l is the spare capacity of link l , $1 \leq l \leq L$ and that minimum z is at its maximum value when γ strategy (algorithm) is used. Hence we can increase the load of each link by that minimum which means we can increase the load (demand). We prove the 2nd part of the proposition (ii) as follows:

Let the flow is maximum whatever the routing strategy used ω or γ , i.e. for the collection of all routing strategies (algorithms), then z_l should be zero for at least one value of l , $1 \leq l \leq L$. Let this is not the case, i.e. z_l is not zero

for all values of l . Take now any path from a source node to a destination node. Increase the traffic (flow) on the links of this path by $\min_{l \in \text{links of the path}} z_l$ and thus we were able to increase the total flow. This is a contradiction because the flow is maximum and accordingly z_l should be zero for at least one value of l , $1 \leq l \leq L$. We have proved for any routing strategy γ or ω that at least one spare capacity is zero. Thus $z = \min_{l \in \mathbb{L}} z_l = 0$ is independent of the routing strategy. Thus, the maximum of minimum spare capacities, i.e. $\max z$, equals to 0 for the collection of all routing strategies (algorithms) ω or γ . Now when we use γ , we get z maximized and when we use any ω strategy (algorithm) we have z less than its maximum value but this means $z < 0$ which is a contradiction since $z \geq 0$. Hence only strategy γ can be used when the flow is maximum.

Q.E.D

III. CONSTRAINTS IMPOSED BY INTERFERENCE

Jain et al [3] developed a model to calculate the maximum capacity of a wireless multi-hop network under interference using a conflict graph. In addition to the well-known formal formulation of the maximum flow problem in hard-wired networks. They added two constraints imposed by interference.

$$\sum_{i=1}^{K'} \lambda_i \leq 1 \quad (6)$$

$$f_{ij} \leq \sum_{l_{ij} \in I_i} \lambda_i \text{Cap}_{ij} \quad (7)$$

K' is the number of maximal independent sets in the conflict graph, I_i is a maximal independent set, λ_i is the fraction of time maximal independent set I_i is active (carrying transmitted data), f_{ij} is the flow carried by link ij and Cap_{ij} is the capacity of link ij . An Independent set in the conflict graph is simply a set of non-interfering links.

IV. OPTIMUM ROUTING IN STATIC WIRELESS MULTI-HOP NETWORKS

By adding the interference constraints in Eqs. 6 and 7 to the model in the section I and letting λ be the vector $[\lambda_1, \lambda_2, \dots, \lambda_i, \dots, \lambda_{K'}]^T$. It can be easily shown that Eq. 1 in section I becomes:

$$\begin{bmatrix} \mathbb{A} & 0 & \mathbf{0} \\ 0 & \bar{\mathbb{I}} & 0 \\ \mathbb{I} & 0 & \mathbf{1} \end{bmatrix} \begin{bmatrix} X \\ \lambda \\ z \end{bmatrix} = \begin{bmatrix} V \\ 1 \\ \sum \lambda_i C \end{bmatrix} \quad (8)$$

Where \mathbb{A} , $\mathbf{0}$, \mathbb{I} , and $\mathbf{1}$ are the same as in Eq. 1. The first 0 in the left matrix of Eq. 8 is a matrix of dimensions $KN \times K'$ of all zeros. The first 0 in the second row is all zeros vector of dimensions $1 \times KL$; $\bar{\mathbb{I}}$ is a vector of all 1s of dimensions $1 \times K'$; and the last 0 in the second row is just a zero. The middle 0 in the last row of the matrix is all zeros matrix of dimensions $L \times K'$. The last matrix in the right column is $L \times 1$ vector such that the j th element is $\sum_{j \in I_i} \lambda_i C_j$. This can be further written as :

$$\begin{bmatrix} \mathbb{A} & 0 & \mathbf{0} \\ 0 & \bar{\mathbb{I}} & 0 \\ \mathbb{I} & M & \mathbf{1} \end{bmatrix} \begin{bmatrix} X \\ \lambda \\ z \end{bmatrix} = \begin{bmatrix} V \\ 1 \\ 0 \end{bmatrix} \quad (9)$$

M is $L \times K'$ matrix where $M_{ij} = -C_i$ if link $i \in I_j$ and 0 otherwise. The dual problem is a minimization subject to:

$$[u^T \quad v \quad y^T] \begin{bmatrix} \mathbb{A} & 0 & \mathbf{0} \\ 0 & \bar{\mathbb{I}} & 0 \\ \mathbb{I} & M & \mathbf{1} \end{bmatrix} = [0^T 1] \quad y^T \geq 0 \quad (10)$$

v is the dual variable corresponding to the constraint $[0 \quad \bar{\mathbb{I}} \quad 0]$. $[X \quad \lambda \quad z]^T = 1$. 0^T is a vector of dimensions $1 \times (KL + K')$ of all 0s. It can be readily seen that the solution is also shortest path with link weights are optimum values assumed by the dual variables of the vector y^T . We can use the algorithm in [3] or any other algorithm to find (maximal) independent sets and solve the optimization problem in this section, approximating links weights, but may be in non-polynomial time.

V. OPTIMUM ROUTING IN MOBILE WIRELESS MULTI-HOP NETWORKS

Assuming nodes mobility where each node is available within transmission, interference, range of each other node with constant probabilities p_i , q_i respectively. These probabilities may be different for each pair of nodes. Then by following the same approach in [1], we can arrive at a similar formulation in static wireless multi-hop networks when we set the the objective function as maximizing the minimum mean spare capacity to accommodate for more mean flow after adding interference constraints. However, due to an astronomical number of possible different networks topologies, the number of possible independent sets in such topologies is so huge that calculating links weights which depends on the two stated probabilities becomes infeasible unless we have a very limited mobility. One possible scenario is VANETs that vehicles move on specific routes with minor network topology changes. This is left for future work to figure out potentially successful algorithms that compute such weights. In spite of this shortest path is proved to be also optimum in some setting that allows the network to carry more mean flow contrary to what is generally believed by the research community. As future work also a simulation will be carried out to validate our findings.

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