On the Achievable Diversity-Complexity Tradeoffs of Joint Network-Channel Coded Cooperative Communication

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Abstract—In this paper, a novel mathematical framework for the analysis and optimization of Joint-Network–Channel Coded Diversity (JNCCD) protocol is presented. The analysis is applicable to relay–aided protocols based on the error propagation model, which rely on appropriately designed diversity combining demodulators at the destination. Wireless networks with an arbitrary number of sources and relays are considered. Arbitrary multilevel modulation schemes and network codes constructed over Galois Field (GF) are analyzed and JNCCD protocols are investigated. Our results show that if the modulation order is smaller than the Galois field; we are still able to achieve the higher diversity order with lower decoding complexity.

I. INTRODUCTION

Network coding (NC) is a natural choice for wireless networks due to their broadcast nature, i.e., a signal transmitted by a node is overheard by all neighbor nodes. The extra information at neighboring nodes can help to reduce the overall number of transmissions and therefore increase the overall throughput [1]-[8]. Network coding has also been extensively used in corporative relay channel to obtain improved diversity gains [9]-[11]. Where limited power budget and throughput rates are driving forces for the design of the next-generation wireless ad hoc, sensor, and cellular networks [12]. Basically channel fading is one of the major underlying causes of performance degradation and energy consumption in wireless networks.

The rationale of this paper is to exploit NC at the relays in order to combine the data received from the sources, hence reducing the number of channel uses needed by half– duplex relays and increasing the achievable throughput. The fundamental design issue that has been addressed in this paper is consists of the appropriate choice of both the encoding vectors at the relays and the development of decoder at the destination for gaining the best achievable diversity. Against this background,

Mathematical framework, analysis and design of JNCCD protocols for wireless networks are challenging due to the exploitation of both relay-aided transmission and NC operation. Indeed, mathematical frameworks for error performance and diversity analysis as well as for network code design are still unavailable for arbitrary network topologies, modulation schemes and size of the Galois Field (GF) used for NC.

A comprehensive literature can be found in [1]–[6] In the present paper, we are interested in the analysis and design of JNCCD protocols using digital NC and based on the so–called error propagation model [2], [5]. The reader interested in recent advances on CRC–based NCCD protocols is invited to

consult [5], [7]–[10]. In addition to the mathematical performance analysis, the paper provides important guidelines to the design of diversity–achieving network codes. The remainder of this paper describes the proposed algorithm, the system model, notations, Galois field size, modulation order and channel models used throughout this paper and the simulation results.

A. System Model and Terminologies

We study the canonical two-source two-relay cooperative network. Fig.1 presents the basic flow of studied scheme.

The following notations is used throughout the paper. $x \in GF(p)$ denotes a symbol of a GF of size $p.n \sim \mathscr{CN}(\mu, \sigma^2)$ denotes a circular symmetric complex Gaussian Random Variable (RV) with mean equal to μ and variance equal to σ^2 . \boxplus , \square and \boxtimes denotes addition, subtraction and multiplication in GF, respectively. $\Sigma \boxplus$ denotes a summation performed in GF. $\arg\{.\}$ denotes the phase angle operator. $\mathscr{H}(x,y)$ denotes the Hamming distance of x and y, i.e., $\mathscr{H}(x,y) = 0$ if x = y and $\mathscr{H}(x,y) = 1$ if $x \neq y$. arg max $\{.\}$ stands for argument of the maximum, means the set of points of the given argument for which the given function attains its maximum value. $diag_N(X)$ denotes a block diagonal matrix given by the matrix direct sum of \mathscr{N} matrices equal to \mathscr{X} .

B. Signal Model and transmission Protocol

Assume a multi-source-multi-relay network with N_S sources (S_t) for $t = 1, 2, ..., N_s$, N_R relays (R_q) for q =



Fig. 1: Proposed Scheme.



1,2,..., N_R), and a single destination (*D*). The transmission of sources and relays occur in orthogonal time-slots. In timesot T_t , the source S_t broadcasts its data symbol to *D* and to the N_R relays. This transmission phase lasts N_S time-slots. Let μ_{S_t} be the symbol transmitted by S_t ì, which is assumed to be an element of GF of size $M = 2^m$ with *m* being a positive integer i.e., $\mu_{S_t} \in GF(M)$. Then the signals received at R_q and *D* in time-slot T_t are as follows:

$$\begin{cases} y_{SR_q} = \sqrt{E_s} h_{SR_q} \mathscr{M}_S(\mathbf{x}_S) + n_{SR_q} \\ y_{SD_d} = \sqrt{E_s} h_{SD_d} \mathscr{M}_S(\mathbf{x}_S) + n_{SD_d} \end{cases}$$
(1)

where $\mathcal{M}_S(\mathbf{x}_S) \in \boldsymbol{\chi}$ is the complex modulated symbol symbol transmitted by the source, and $\hat{\boldsymbol{z}}_S^{(R_q)}$ is the source's estimated bits at R_q , and $\tilde{\mathbf{x}}_S^{(R_q)}$ is the trial bits used in the hypothesis– detection problem at R_q , for q = 1, 2, ..., M. The signal y_{SD_d} , for d = 1, 2, ..., N, is processed by the destination during the next time–slot. The AWGN is independent and identically distributed (i.i.d.) with zero mean and variance. per real dimension equal to N0/2, i.e., $n_X Y \sim \mathcal{CN}(0, N0)$. Upon reception of yStRq, the relay R_q , demodulates the signal y_{SR_q} using the Maximum–Likelihood (ML) criterion as follows:

$$\sum_{S}^{a(R_q)} = \arg\min_{\begin{pmatrix} a(R_q) \\ S \end{pmatrix} \in GF(M)} \left\{ \left| y_{SR_q} - \sqrt{E_S} h_{SR_q} \varphi M \begin{pmatrix} a(R_q) \\ S \end{pmatrix} \right|^2 \right\}$$
(2)

where $\mu_{S_t^{Rq}}^{Rq}$ is the estimate of $\mu_{S\sqcup}^{l}$ at R_q . In this paper;a Channel Coding-based NCCD protocol is considered, where all the available NR relays forward a symbol to *D*. At the end of the $N_{S_{th}}$ time-slot, Rq takes turn transmitting, in time-slot $T_{N_{S+q}}$, a network- coded symbol to *D*. This transmission (relaying) phase lasts NR time-slots. Let $g_{Rq} = [gS_1Rq, gS_2Rq, ..., gS_{N_s}Rq]$ be the $(1 \times NS)$ -element encoding vector used at R_q , where $gs_tR_q \in GF(p)$ with $p = 2^l$ and lbeing a positive integer. Major assumptions are made: 1)the *NR* relays are assumed not to encode their own data with the data received from the sources.2) the setup M > p with M/p being an integer is studied. Under these assumptions, the network-coded symbol transmitted by Rq can be formulated as follows:

$$\hat{\mu}R_q = \sum_{t=1}^{N_S} \boxplus \left(gs_t R_q \boxtimes \hat{\mu}_{S_t}^{(R_q)} \right)$$
(3)

where $\hat{\mu}_{R_q} \in GF(M)$.

Then, the signal received at D can be formulated as follows:

$$\hat{y}_{R_{qD}} = \sqrt{E_{R_q} h_{R_q} D^{\hat{x}} R_q + n_{R_q} D}$$
(4)

where $\hat{x}_{R_q} = \varphi_M(\hat{\mu}_{R_q})$ and E_{R_q} is the average symbol energy of R_q . Finally, the source node *St* broadcasts the modulated and coded symbol during the first time slot for (*S*1) and the second time slot for (*S*2). The signal model considered in this section is based on the so–called symbol–by–symbol transmission assumption, i.e., the atomic information unit emitted by the sources is a symbol. Current communication protocols, however, are based on the transmission of packets that consist of several symbols.

II. DETECTION AT THE DESTINATION

The Joint-Network-Channel Coded Cooperative-Maximum Ratio Combining (JNCCC–MRC) demodulator is a low–complexity alternative to ML–optimum demodulation. As an example, this performance vs. complexity trade–off is studied in [11].

For binary NC and binary modulation, by considering three different demodulation schemes at the destination: i) the Minimum Distance Demodulator (MDD), which is the simplest alternative that does not take into account demodulation errors at the destination; ii) the Hard–decision Maximum–Likelihood Demodulator (H–MLD); and iii) the Soft–decision Maximum– Likelihood Demodulator (S–MLD), which is the optimal but most computationally–intensive alternative. The H–MLD is an intermediate solution that is further investigated in [4]. Unlike the MDD option that neglects demodulation errors providing a low–complexity implementation at an unacceptable performance degradation and the ML–optimum option that provides the best performance at an unfeasible computational complexity,

A. Mathematical Derivation of C-MRC Decoder

Following the same line of thought as [3][8] and [5], the JNCCC–MRC demodulator in (5) and (6) originates from the concept of equivalent channel between the NS sources and the generic relay Rq. For its derivation, the probability, P_{Rq} , that Rq forwards an incorrect symbol to D has to be computed.

This probability can be formulated as shown in (30), and i) is valid for high–SNR, where it is plausible to assume that the forwarding error is dominated by the situations when a single source is incorrectly demodulated and all the other sources are correctly demodulated, [3] and ii) takes into account that $1 - \text{sign}(g_{S_lR_q}) P_r\{\hat{\mu}_{S_l} \neq \mu_{S_l}\} \approx 1$ for high–SNR. $P_r\{\hat{\mu}_{S_l} \neq \mu_{S_l}\}$ is the probability that the symbol that are from *St* are wrongly demodulated at *Rq*. For high–SNR, it can be approximated for many modulations [12]. Based on above derivation the JNCCC–MRC demodulator can be written as below in (6); where: i) $\lambda(,)$ is basically the decision unit of the decoder;

ii) $\mu = [\mu_{S_1}, ..., \mu_{S_{N_S}}, \mu_{R_1}, ..., \mu_{R_{N_R}}]$ is the $(1 \times (NS + NR))$ element that contains the symbols broadcasted by sources and relays between the non.existance of the decoding errors at the relays, where $\mu_{R_q} = \sum_{t=1}^{N_S} \boxplus (g_{S_t R_q} \boxtimes \mu_{S_t})$ iii) $\hat{\mu} =$ $[\mu_{S_1}^2, ..., \mu_{S_{N_S}}, \mu_{R_1}, ..., \mu_{R_{N_R}}]$ denotes the hypothesis of μ at the demodulator; iv) $\hat{\mu}_S = [\mu_{S_1}^2, ..., \mu_{S_{N_S}}]$ is the $(1 \times N_S)$ are the elements that contains the information symbols of the sources and are approximated at *D*. It is important to note that $y_{R_q D}$ in (6) depend on $\hat{\mu}_{R_q}$ and in general $\hat{\mu}_{R_q} \neq \mu_{R_q}$ due to the some decoding errors at $\hat{\mu}_{R_q}$ v) $\gamma_{XY} = (EX/N0)|hXY|^2$ for the basic pair of nodes *X* and *Y*; vi) γ_{eq}, R_q defined in (7) generates from the idea of equal channel between the sources *NS* and *Rq*, as discussed above. It gives an idea about the reliability of the network–channel-coded symbol that are broadcasted by *Rq* is correct, i.e., $\hat{\mu}_{R_q} = \mu_{R_q}$. In particular, the larger ($\gamma_{eq}; R_q$) is the greater the probability of right processing and forwarding is; and vii) $0 < \lambda_{R_q} \le 1$ defined in (7) is a reliability measuring factor applied to the signal received from Rq in order to take possible demodulation errors. Hence we can call this the function of relay to destination link (γ_{R_q}) and equal channel (γ_{eq}, R_q) links quality. In particular, the better the reliability of the equivalent channel is the closer to one γ_{R_q} is. For the special case that all links between the *NS* sources and *Rq* are ideal, i.e., $\gamma_{S_t} \longrightarrow \infty$ for $t = 1, 2, ..., N_S$ we have $\lambda_{R_q} = 1$ and the JNCCC–MRC demodulator splits into the traditional MRC demodulator [6]–[8], [10]. From (7), it means that the JNCCC–MRC demodulator is totally independent from the real coefficients of the selected encoding vectors. In fact, it depends only on these coefficients being either zero or non–zero. [13]

III. ERROR PROBABILITY CALCULATION AND DIVERSITY ORDER ANALYSIS

A. Error Probability Calculation

This section is about the end-to-end error probability of the JNCCC-MRC demodulator is computed by taking into account the potential demodulation errors on the relays.

The mathematical framework is obtained for calculation of the Average Symbol Error Probability (ASEP) which is calculated with the help of the union bound scheme. From (6), In Section 4.4, it is mentioned that the decision metric in (6) depends on $\hat{\mu}_{R_q}$, which are the network channel coded symbols that are really transmitted by the relays. For estimating the ASEP, all possible transmitted information symbols must be taken into consideration. With the use of the total probability theorem [21], the ASEP can be formulated as shown in(8), where: i) E_{m_q} denotes the event that $\hat{\mu}_{R_q} = \mu_{m_q}$ assume that μ_{R_q} is the symbol that would have been transmitted by R_q in the non-existence of the demodulation errors and ii) $P_r \wr \mathscr{E}_{m_q} |h\wr$ can be formulated as follows

$$P_{r}\{\mathscr{E}_{m_{q}}|h\} = P_{r}\{\hat{\mu}_{R_{q}} = \mu_{m_{q}}|\mu_{R_{q}};h\}$$

$$= P_{m_{q}}^{(CF)}(h) = P_{r}\{\hat{\mu}_{R_{q}} = \mu_{m_{q}} = \mu_{R_{q}}|\mu_{R_{q}};h\} \quad (8)$$

$$= P_{m_{q}}^{(EF)}(h) = P_{r}\{\hat{\mu}_{R_{q}} = \mu_{m_{q}} \neq \mu_{R_{q}}|\mu_{R_{q}};h\}$$

where $P_{m_q}^{(CF)}()$ and $P_{m_q}^{(EF)}()$ denote the probability of Correct (right) Forwarding (CF) and Erroneous (wrong) Forwarding (EF) at R_q , respectively. It is very important to note that the events \mathscr{E}_{m_q} for $q = 1, 2, ..., N_R$ are independent [13].

Proposition: A closed-form expression for $P_{m_q}^{(CF)}()$ and $P_{m_q}^{(EF)}()$ is given. Let a common M-ary modulation. For high-SNR, $P_{m_q}^{(CF)}(\cdot)$ and $P_{m_q}^{(EF)}(\cdot)$ can be computed as follows:

$$P_{m_q}^{CF}(h) \approx 1 - \sum_{t=1}^{N_S} \xi_{s_t R_q}^{(EF)} P_r \left\{ \hat{\mu}_{S_t}^{(R_q)} \neq \mu_{S_t} \right\} \\ P_{m_q}^{EF}(h) \approx \sum_{t=1}^{N_S} \xi_{s_t R_q}^{(EF)} P_r \left\{ \hat{\mu}_{S_t}^{(R_q)} \neq \mu_{S_t} \right\}$$

where: i) $P_r\{\hat{\mu}^{(R_q)} \neq \mu_{S_t}\}$ is probability that the information symbol transmitted from *St* is wrongly demodulated at R_q and ii) $\xi_{S_tR_q}^{CF} = (g_{S_tR_q} \text{ and iii})\xi_{S_tR_q}^{EF}$ depends on the specific approximation being used if a demodulation error appears. Let us consider the two major cases: 1) the probability of decoding any other information symbol is considered to be equally distributed. This appropriation can be called as Uniform Approximation (UA) and secondly 2) the decoding information symbol is considered to be, with uniform probability, as a closest neighbor.

Comments: The computation of (8) needs a expression $P_r \left\{ \hat{\mu}_{S_t}^{(R_q)} \neq \mu_{S_t} \right\}$ which is the symbol error probability at Rq. More specifically, it can be formulated as the linear combination of integrals.

In (8), only the symbol error probabilities $P_r\left\{\hat{\mu}_{S_t}^{(R_q)} \neq \mu_{S_t}\right\}$ indeed depends on the channel gains *h*. On the other hand, the weighting factors $\xi_{S_tR_q}^{CF}$ and $\xi_{s_tR_q}^{(EF)}$ are independent of *h* for example see equation. (III-A). Let us now turn our attention to $(\mu, \hat{\mu}); E_{m_1}, E_{m_2}, \dots E_{m_{m_R}}$ in (10).

*Lemma*1: Let us assume the per–link symbol error probability. Let $EX/N0 = \mathscr{K}_{\mathscr{X}}(E0/N0)$ and $\Omega_{XY} = \mathscr{K}_{\mathscr{X}}\sigma_{XY}^2$. The ASEP of the NCC–MRC demodulator in (10) can be formulated, for high–SNR, as follows:

ASEP
$$(\mu \to \hat{\mu}) = \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} \dots \sum_{m_{N_R}=1}^{M} ASEP_m(\mu \to \hat{\mu})$$
 (10)

where *m* is a integrated collecting the indexes m_q for $q = 1, 2..., N_R$.

$$\mathscr{M}_{R_q}^{(mq)}(S) = \begin{cases} \mathscr{M}_{R_q}^{(CF)}(S) & \text{if } \mu_{m_q} = \mu_{R_q} \\ \mathscr{M}_{R_q}^{(EF)}(S) & \text{if } \mu_{m_q} \neq \mu_{R_q} \end{cases}$$
(11)

with $\mathscr{M}_{R_q}^{(CF)}(\cdot)$ and $\mathscr{M}_{R_q}^{(EF)}(\cdot)$ referring to the cases of correct and incorrect forwarding at the relay [14].

B. Diversity Order Analysis

From Lemma 1, the diversity order provided by the JNCCC- MRC demodulator is given in Proposition 2 below.

Proposition2: Let the JNCCC–MRC demodulator in (III-A). Let

$$w^{(\mathscr{H})}(\mu,\hat{\mu}) = \sum_{q=1}^{N_R} d_{S_t}^{(\mathscr{H})} + \sum_{t=1}^{N_S} d_{R_q}^{(\mathscr{H})}$$

be the Hamming weight of μ and $\hat{\mu} \neq \mu$. Let $S_t = \min_{\mu,\hat{\mu}} \{w^{(\mathscr{H})}(\mu,\hat{\mu}) | d_{S_t}^{(\mathscr{H})} \neq 0\}$ be the *t*th entry of the Separation Vector (SV) of the network code [24, Definition 1], i.e., the minimum Hamming weight of the network code. The diversity order of the generic source S_t is $D_{S_t} = S_{V_t}$, i.e., the diversity order of the sources is the SV of the network code.

From *Proposition* 2, a high–SNR expression of the ASEP of S_t is obtained as follows.

Proposition 3: Let the JNCCC–MRC demodulator in(12). For high–SNR, the ASEP of S_t can be formulated as follows:

It follows by applying the union-bound [21] and by noting that: i) for high–SNR, the ASEP of S_t is determined with the lowest Hamming weight, which is SV_t .

IV. NETWORK CODE DESIGNING

Proposition 2, referred us many significant guidelines for the construction of the network codes for under discussion system.

Comments: Proposition 2 implies that the diversity order is determined only by the network code, means by the encoding vectors used at the relays. In particular, the diversity order is independent of the decoding errors and at the destination. This originates from using the NCC– MRC demodulator, which takes into account the SNRs of the

$$\begin{cases} \begin{bmatrix} \alpha(D) \\ S \end{bmatrix} = \underset{\substack{\hat{\mu}_{S} \in OF \text{ for } = 1, 2, \dots, N_{S} \\ R_{q}^{-} = \left\{ \Omega_{R} | \mathbf{D}_{R_{q}} = \overset{\pi(D)}{S} \right\} \\ R_{q}^{-} \left\{ \sum_{t=1}^{N_{S}} \frac{\left| y_{SD_{d}}(\mu_{S}) - \sqrt{E_{s}} h_{St_{D}} \varphi_{M}\left(\tilde{z}_{S} \right) \right|^{2}}{N_{0}} + \sum_{q=1}^{N_{R}} \lambda_{R} \frac{\left| y_{RqD}(\hat{\mu}_{R_{q}}) - \sqrt{E_{R}} h_{R} \varphi_{M}(\hat{\mu}_{R_{q}}) \right|^{2}}{N_{0}} \end{cases}$$
(6)

$$\begin{cases} \gamma_{eq,R_q} = \min\left\{\operatorname{sign}^{-1}(g_{S_1R_q})\gamma_{S_1R_q}, \operatorname{sign}^{-1}(g_{S_2R_q})\gamma_{S_2R_q}, \dots, \operatorname{sign}^{-1}\left(g_{S_{N_S}R_q}\right)\gamma_{S_{N_S}R_q} \right\} \\ \lambda_{R_q} = \left(1/\gamma_{R_qD}\right)\min\left\{\gamma_{eq,R_q}, \gamma_{R_qD}\right\} \end{cases}$$
(7)

$$ASEP_{S_t} \approx \frac{1}{M^{N^S}} \sum_{\mu_S} \sum_{\hat{\mu}_S} ASEP(\mu \to \hat{\mu}) \mathscr{H}(\mu_{S_t}, \hat{\mu}_{S_t}) \mathscr{H}(w^{\mathscr{H}}(\mu, \hat{\mu}), SV_t)$$
(12)

source–to–relay links. On the other hand, the coding gain [15]-[25] depends on these error probabilities, as shown in Lemma 1. As for the achievable diversity, this implies that, even though incorrect demodulation and forwarding are allowed by the protocol, the encoding vectors can be designed by assuming perfect demodulation at the relays. Hence, assuming a network topology with N_S sources and N_R relays, the network code may be chosen to be a non–binary ($N_S + N_R, N_S$) linear block code [26] having N_S systematic and NR parity symbols, respectively. If so, the diversity order would be the SV of the ($N_S + N_R, N_S$) block code.

Comments: If, according to Remark 4, a non-binary $(N_S + N_R, N_S)$ linear block code is used as a network code, the achievable diversity is determined by the network topology, i.e., N_S a (N_S, N_R, p) , Proposition 2 implies that the network code may provide either the same or a different diversity order to the sources. These two classes of codes are referred to as Equal Error Protection (EEP) [16], [17] and Unequal Error Protection (UEP) [18], [19]–[21] codes, respectively. Any EEP and UEP linear block code available in the literature may be used as a network code and its SV determines the achievable diversity. Useful code constructions are available in [16] and [17], e.g., EEP codes based on Reed–Solomon (RS) and Extended RS (ERS) methods.From [16], [17] and [19], [22], [23].

Reduced Complexity of NC operations in GF: Consider a network topology with NS sources and NR relays. The outcome network code is called as a full diversity achieving. This problem can be handled by using a special class of $(N_S + N_R, N_S)$ linear block codes, which is called the Maximum Distance Separable (MDS) [16][Ch. 11], [17]. In fact, MDS codes obtains the singleton bound [27], which implies that $SV_t = N_R + 1$ fort $= 1, 2, ..., N_S$, only couple of them, as shown in [16] [Fig. 11.2]. If a MDS code occurs, the size of the GF should meet the inequalities $p \ge N_R + 1$ and $p \ge N_S + 1$ for $N_S \ge 2$ and $N_R \ge 2$ [16], [17]. This proof's that, for achieving the best diversity, the size of the GF has to be of the order of max NS, NR. This may lead to a non-negligible encoding and decoding complexity, even for network topologies with a moderate number of nodes. In fact, both encoding and decoding complexity increase with the size p of the GF. As a result, there is a trade-off between achievable diversity order and encoding/decoding complexity. If $N_S = 1$ or $N_R = 1$, the

MDS codes are called "trivial" and they exist for any p [16]. This also implies that, in general, binary NC is optimal only for single–relay network topologies. It is worth noting that MDS codes find application to the design of CRC–based NCCD protocols too [7], [8], [9], [10].

In above comment, it is shown that full-diversity achieving network codes may not always be a practical option, because of complexity issues that may constrain the size of the GF to a given upper-bound. In these circumstances, two network code design problems may be of interest: 1) given N_S , N_R and p, the determination of the best achievable SV of the sources and 2) given N_S , p and SV, the determination of the minimum number of relays N_R such that SV is achieved.

Mathematical analysis and design are valid under two assumptions: i) the size of the GF coincides with the modulation order, i.e., M = p and ii) the N_R relays are full-cooperative. The transmission of M - ary symbols, either from the sources to the relays or from the relays to the destination, is assumed. Thus, $\mu_{S_t} \in GF(M)$ and $\hat{\mu}_{S_t}^{R_q} \in GF(M)$ The assumption $M \neq p$ affects only the operations performed at the relays after demodulating the data received from the sources.

The following operations are performed at the relays: i) $\mu_{S_t} \in GF(M = 2^m)$ is converted to its binary representation of m bits; ii) the resulting m bits are grouped in l-tuples and converted to a symbol of GF, thus obtaining m/l sub- symbols for each source; iii) NC is applied to the sub-symbols of different sources by performing operations in $GF(p=2\hat{l})$, thus obtaining $\frac{m}{T}$ network–coded sub–symbols. NC is not applied to the sub-symbols of the same source. Also, the same network code is applied to different sub-symbols; iv) the resulting m/l network-coded sub-symbols are converted to their binary representations of l bits each, leading to a concatenated binary string of m bits; and v) this string of bits is eventually converted to a network-coded symbol of GF(M), which is transmitted to D. By applying this encoding at the relays, Mand p may be chosen independently from each other hence the encoding/decoding complexity of NC operations is reduced since the operations are performed in a GF of smaller size p < M.

The mathematical expressions in (10) and (III-A) are still applicable if $M \neq p$. This originates from two observations: 1) the probability of correct/incorrect data forwarding of Proposition 1 is not affected by the NC operations performed at the relays. Under the high–SNR assumption that the forwarding errors are dominated by the events that the data of only a single source is not demodulated correctly, the probability of forwarding an incorrect network–coded symbol to the destination depends only on the demodulation error probability of the individual sources.

This latter probability, $P_r\{\hat{\mu}^{(R_q)} \neq \mu_{S_l}\}$, is independent of how NC is performed inside the relays and it only depends on M and 2) the APEP and ASEP in (10) and (III-A) are computed and formulated as a function of Euclidean $(d_{S_l}$ and $d_{R_q})$ and Hamming $(d_{S_l}^{\mathcal{H}} \text{ and } d_{R_q}^{\mathcal{H}})$ distances of the symbols of GF(M). The mathematical derivation leading to (10) and (III-A) is independent of how these symbols of GF(M) are computed at the relays. Thus, (10) and (III-A) are applicable to arbitrary *M* and *p*. However, Euclidean and Hamming distances do depend on how NC is performed at the relays.

Hence for the achievable diversity, the modulation order M and the size p of the GF may be chosen independently from each other. In particular, M may be chosen by taking into account only the desired rate and the end–to–end performance (coding gain). On the other hand, p may be chosen by taking into account only the desired diversity order and the constraints on the encoding/decoding complexity

V. SIMULATION RESULTS AND DIVERSITY ANALYSIS

In this section, we analyze the complexity and diversity trade-off of the proposed splitting scheme for various symbols, estimated sub-symbols for three channel conditions including Block fading Channel (BFC), Quasi- Static fading channel(QSFC) and Fully Interleaved fading channel (FIFC) [12] for C-MRC decoder.

In order to verify our claims about the performance of the proposed scheme, and to compare its performance with two most famous and currently in-use state of the art protocols relay-aided Cooperative Diversity (CD) [1], Network Coding (NC) and Channel Coding [2] the Monte-Carlo simulation results have been presented. In proposed approach we forward the estimated bits with or without decoding errors (error channel model). The receiver will make use of the knowledge of the error probability at the relay. A network encoder which encodes the interleaved estimated bits to get the network coded information messages. Then, a channel encoder encodes the messages to get the codeword which are then mapped into the modulated signal. In our experimental setup, a codeword consists of F blocks, and the relay estimates the error probability of the codeword based on the knowledge of the channel [12].

CD vs. NCCD vs. proposed JNCCD: Performance Comparison: In Figs. 2 and 3, the ASEP of CD, NCCD and proposed JNCCD protocols is compared for various network topologies. For analysis purpose, Monte Carlo simulations are shown. The comparison is performed under two major assumptions: 1) The network rate is the same. The network rate is defined as Rate = $(N_S log2(M))/(N_S + N_R)$ and Rate = $(N_S log2(M))/(N_S + N_A)$ bits per channel use (bpcu) for JNCDD based protocol.

ii) The total transmit energy is the same. Let E0 be the average symbol energy of each network node. The total transmit energy is defined as $E_T = E_0(N_S + N_R)$ and $E_T = E_0(N_S + N_A)$ for JNCCD based protocol.

where M and E_0 are chosen appropriately for achieving the same network rate and for consuming the same total transmit energy.

Simulation Setup : The results are obtained under the following general assumptions: The major considerations are : i) $\sigma_{XY}^2 = \sigma^2 = 1/2$ for every wireless link; and ii) $E_S = E_m \log_2(M)$, $E_{R_r} = E_m \log_2(M) + E_m \log_2(N)$ for r = 1, 2, ..., M, where E_m denotes the average energy per transmitted symbol. iii) the operations in GF(p) are performed by using the primitive polynomials PM(x) = x if Mnc = 2, PM(x) = x2 + x + 1ifMnc = 4 and PM(x) = x3 + x2 + 1ifMnc = 8 and so on. We show the Average Symbol Error Probability (ASEP) performance as a function of the average transmit-energy per coded symbol (E_m) .

Let us use the same (NS,NR,NA) for CD, NCCD and JNCCD protocols, M and E0 are chosen appropriately for achieving the same network rate and for consuming the same total transmit energy. The results in Fig. 2 illustrate the achievable benefits of NCCD against CD based protocols. It clearly allows the reduced number of channel uses, looking at achieved diversity a lower– order modulation scheme can be used with higher order field size for NCCD protocols, which results in a better ASEP. The network code considered in this simulation is obtained from [28].

In Fig. 3, i) the proposed splitting scheme (JNCCD) is compared with CD, NCCD schemes.

Broadcasted symbols are M = 4, 8, 16 and sub-symbols (Splitted) are Mnc = 2, 4, 8, 16 respectively. Three different types of fading channels have been analysed and a comparison with CD and NCCD schemes has also been presented. The network code considered in this simulation is obtained from [28]. It can be seen that even lower-order modulation scheme can be used for JNCCD protocols, which results in a better ASEP. Also, increasing Mnc beyond the minimum required for achieving full-diversity has a negligible impact on the achievable performance. The potential benefits of JNCCD protocols against CD and NCCD protocols based on proposed scheme are very clear, as shown in Fig. 3. Due to the fundamental limitations of the achievable diversity of CD and NCCD protocols discussed in Section V-D, JNCCD protocols may outperform CD and NCCD protocols in the low/medium-SNR region, but a crossing point is expected for high-SNR.

Fig. 3 shows that the ASEP of NCCD protocols degrades, due to the need of increasing M in order to achieve the same network rate. Furthermore, the ASEP of NCCD protocols is worse than the ASEP of CD protocols for low–SNR as well, and it can be seen that JNCCD protocol is capable of outperforming the both traditional protocols.

Hence a lower- order modulation scheme can be used for JNCCD protocols, which results in a better ASEP. More specifically, the achieved diversity orders when M = 32 and Mnc = 8,16 for JNCCD are 4 and higher, respectively, while that of CD and NCCD is only 2 and respectively. The simulation results confirm the diversity order of N + 1 for JNCCD protocol.

VI. CONCLUSION

In this work, a novel mathematical frameworks for the analysis of CD, NCCCD and JNCCD protocols have been proposed and have been investigated using the Monte Carlo



Fig. 2: Performance comparison of traditional cooperative (CD) protocol and state-of-art network-channel coding protocol (NCCD).



Fig. 3: Performance comparison of proposed joint networkchannel coding cooperative protocol (JNCCD) with CD and NCCD.

simulation methods. The frameworks provide a deep insight about the guidelines and method to the detailed design of higher diversity obtaining codes, by considering the practical implementation constraints. In particular, two takeaway messages emerge from our analysis: 1) spitting–based JNCCD protocols are able to obtain higher diversity even if the size of the field is greater than the number of sources and relays and 2) JNCCD protocols based are able to achieve the full–diversity under more restrictive assumptions compared to other state-ofart protocols. Finally, it is proved that joint network-channel coding with the aid of splitting approach has the ability of improving the performance of relay–aided communications even if the used modulation order is very small then the Galois field size.

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