An optimal Biorthogonal M-channel Signal Matched FIR Filter Bank

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***Abstract*—This paper deals with the problem of finding an optimum M-channel biorthogonal signal adapted filter bank. The concept of Principal Component Filter Bank (PCFB) has been proposed, independently by many researchers, as the optimal solution for signal adapted filter banks. PCFB, however, exist only for three special cases of orthonormal filter banks: for the class of transform coders, for 2-channel subband coders and for sub-band coders with unconstrained filters. Although various rigorous approximations for practically realizable PCFBs have also been proposed, but they are only sub-optimal. In this work, we present a biorthogonal M-channel signal matched FIR filter bank which satisfies the properties of spectral majorization and total decorrelation, thus yields optimum coding gain, for any class of sub-band coders. The design strategy does not assume the input statistics to be WSS. To validate the theory, the performance of the proposed algorithm has been compared with the existing literature, using simulations.**

***Index Terms*—Biorthogonal signal adapted FIR filter bank, coding gain optimization, data compression, spectral mojorization, total decorrelation.**

I. Introduction

Signal adapted filter banks offer high coding gain and high energy compaction, which makes them useful in a variety of applications involving compression, representation, denoising, signal decomposition etc. [1]–[7]. Researchers have proposed different design strategies for the same [3], [6], [8]–[11]. Amongst these, Principal component analysis is one of the most popular technique [3], [8], [9], [11]–[15]. In [16], Vaidyanathan et al. stated two necessary and sufficient conditions to ensure optimum coding gain, which are:

1) for an M-channel filter bank, variances of the decimated sub-band signals should satisfy the condition of *Spectral majorization* i.e. $\sigma_0^2 \geq \sigma_1^2 \geq \cdots \geq \sigma_{M-1}^2$, where σ_i^2 is the variance of i-th channel, and
2) the set of sub-band signals should have *total decorrelation*, that implies decorrelation across channels.

Since PCFB satisfies both these conditions, it has been suggested as the optimum solution. PCFB, however, exist only for three special cases of orthonormal filter banks [14]: for the class of transform coders, for 2-channel subband coders and for sub-band coders with unconstrained filters. Various rigorous approximations for practically realizable PCFBs have also been proposed which are only sub-optimal [15], [17], [18]. A signal adapted M-channel biorthogonal filter bank of finite length, was proposed by Lu et al. in [9], obtained by minimizing a coding gain related objective function. However, their coding gain results are not better than PCFB and the initial values of parameters affect the performance of the algorithm. Using the concept of PCFB, Jhawar et al. [14] proposed an FIR PU (Para Unitary) filter bank, for a uniformly decimated 3-channel sub-band coder. Although the filters are of finite length, the results and the design presented were only for a 3-channel filter bank, also the complexity increases many folds as the filter order increases. Recently, Weng and Vaidyanathan [3] proposed that a restricted class of biorthogonal GTD (generalized triangular decomposition) filter banks can give optimum coding gain when the input is wide sense stationary.

In this paper, we propose a biorthogonal M-channel signal adapted FIR filter bank, which satisfies the conditions of *spectral majorization* as well as *total decorrelation* and, thus, yields an optimum coding gain. The proposed filter bank exist for any class of sub-band coders and is not restricted to only WSS signals. The filter bank is designed based on the following conditions:

1) Output of first channel is a M-step ahead prediction error, second channel output is (M-1)-step ahead prediction error and so on and output of (M-1)-th channel provides 1-step ahead prediction error. This structure ensures that the sub-band signals follow spectral majorization.
2) Set of sub-band signals are orthogonalized using a lower triangular matrix, with the diagonal elements as 1.

These two steps results in orthogonalization in time as well as across channels respectively, and, thus, yields an optimum coding gain. The algorithm proposed in this paper, gives M-channel N-order FIR filter bank matched to a single realization of the input signal.

This paper is organized as follows: section II presents design of the proposed signal matched filter bank. In section III, we discuss the implementation of the algorithm for the given data case, by developing the required geometrical framework. In section IV, performance of the proposed algorithm is compared with the results of GTD sub-band coder [3], DCT, with the biorthogonal filter bank [9] and with the signal matched filter bank [6]. Conclusions are presented in section V.

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A. Notations Used

Given data vectors are represented using bold letters and corresponding matrices by capital letters. The notations V^{-1} and V^T denotes inverse and transpose of the matrix V, respectively. The projection (in this work projection would always mean orthogonal projection) of a vector ν, on the space $S \equiv Span\{\nu_i \mid 1 \leq i \leq n\}$, is denoted by $\nu \mid S$ and the orthogonal complement space of S is denoted by $S^{\perp}$. The notation $\mathbb{R}^M$ stands for the real vector space of dimension M. The norm of a vector x is denoted by $|| \ x \ ||$, where x belongs to a Hilbert space.

II. Optimal signal matched filter bank

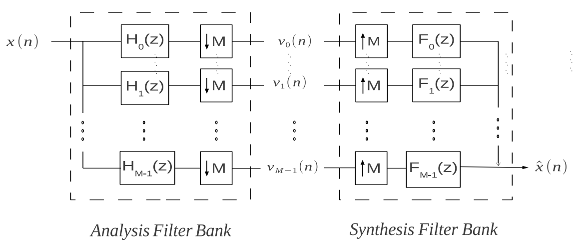

Fig. 1: M-channel multirate filter bank

From the multirate filter bank theory [19], i-th sub-band signal i.e. decimated version of output of i-th analysis filter, $\mathrm{H_i(z)} = \sum_{\mathrm{k=0}}^{\mathrm{N-1}} \mathrm{h_i(k)z^{-k}}$, Figure 1, can be written as:

$$v_i(n) = \sum_{\mathrm{p=0}}^{\mathrm{N-1}} \mathrm{h_i(p)}x(\mathrm{Mn-p}), \qquad 0 \leq \mathrm{i} \leq \mathrm{M-1}. \quad (1)$$

If we substitute $\mathrm{h_i(p)} = 0$ for $0 \leq p \leq M-1$, $p \neq i$ and $\mathrm{h_i(i)} = 1$, equation(1) reduces to the following form:

$$v_i(n) = x(\mathrm{Mn-i}) + \sum_{\mathrm{p=M}}^{\mathrm{(N-1)}} \mathrm{h_i(p)}x(\mathrm{Mn-p}), \qquad 0 \leq i \leq M-1. \quad (2)$$

We attach geometric significance to this expression by regarding each one of them, $0 \leq i \leq M-1$, as prediction error, therefore we can write the above equation as:

$$v_i(n) \equiv e_i(Mn-i) \triangleq x(\mathrm{Mn-i}) + \sum_{\mathrm{p=M}}^{\mathrm{(N-1)}} \mathrm{h_i(p)}x(\mathrm{Mn-p}), \qquad 0 \leq i \leq M-1. \quad (3)$$

The channel outputs, so defined, represent different step ahead predictors starting from 1-step(when i=0) to M-step (when i=M-1), as shown in Figure 2.

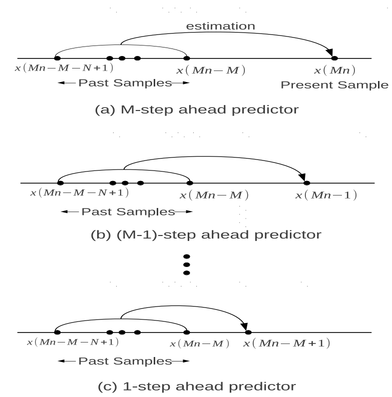

Fig. 2: M-forward linear predictors

It is pertinent to mention here that, although a similar filter bank was proposed in [6], coding gain was not optimized. As stated in the introduction, our objective is to obtain optimized coding gain, thus we want the sub-band signals, $e_i(Mn-i)$ for $0 \leq i \leq M-1$, to achieve *spectral majorization* and *total decorrelation*. The structure, proposed above, will ensure that the sub-band signals follow spectral majorization. As the output variance of the 1-step ahead prediction error filter should be minimum and that of M-step ahead prediction error should be the maximum, i.e. $\sigma_0^2 \geq \sigma_1^2 \geq \cdots \geq \sigma_{M-1}^2$, where σ_i^2 is the variance of (M-i)-step ahead prediction error.

Spectral majorization is necessary but not sufficient condition for optimum coding gain, we must satisfy the condition of total decorrelation also. To achieve this, a_i's are selected in the following equation, such that $e_i(Mn-i)$ for $0 \leq i \leq M_1$ are orthogonalized:

$$\epsilon_i(Mn-i) = e_{\mathrm{i}}(\mathrm{Mn-i}) + \sum_{\mathrm{p=1}}^{\mathrm{M-1}} \mathrm{a_i(p)}e_{\mathrm{i-p}}(\mathrm{Mn-i+p}), \qquad 0 \leq i \leq M-1. \quad (4)$$

The proposed M-channel signal matched filter bank is, therefore, obtained by selecting filter parameters, Figure 3, which satisfy the following constraints:

1) a_i's are chosen such that $e_i(Mn-i)$ are orthogonal to space $S_i \equiv \mathrm{Span}\{e_j(Mn-j) \mid i+1 \leq j \leq M-1\}$, for $0 \leq i \leq M-1$, to ensure orthogonalization across channels, and
2) h_i's should be chosen such that $|| \ e_i(Mn-i) \ ||^2$ is minimized, satisfying constraint 1.

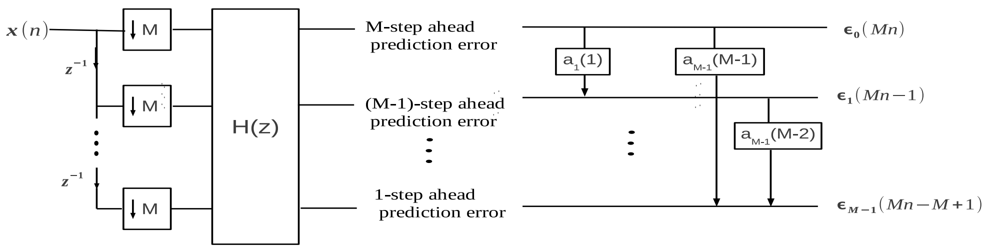

Fig. 3: Signal Matched Filter Bank

III. Implementation of the algorithm

Equations (3) and (4) gives the mathematical representation of the proposed filter bank. In this paper, we present the design of the filter bank matched to a single realization of the given signal such that it satisfies constraints (1) and (2) as stated in the previous section. In this section, we first propose the geometrical framework required for the development and implementation of the given data algorithm.

A. Notations

We call a signal v(n) to be in a pre-windowed form if $\mathrm{v(n)} = 0$ for $n < 0$. For a given discrete time signal/sequence, $\mathbf{v}(n)$, the data vector at time 'n', is defined as $1 \times L$ (L is a fixed number) vector :

$$\mathbf{v}(n) \equiv \left[\begin{array}{cccccc} 0 & \cdots & 0 & \mathrm{v}(0) & \mathrm{v}(1) & \cdots & \mathrm{v(n)} \end{array}\right],$$

with $L \gg n$. Note that we have inserted enough number of zeros (the condition on L i.e. $L >> n$ ensures that the dimension, of this vector, does not change with time). This is basically a collection of all present and past values of v(n), upto time n, in its natural chronological order.

The corresponding $1 \times L$ vector, for the i-th delayed and M-down-sampled version of the signal v(n), denoted as $\mathbf{v}(Mn - i) \in \mathbb{R}^{\mathbb{L}}$, is given as follows:

$$\begin{aligned}\mathbf{v}(Mn-i) \equiv [\ 0 \quad \cdots \quad 0 \quad \mathrm{v(M-i)} \\ \mathrm{v(2M-i)} \quad \cdots \quad \mathrm{v(Mn-i)}\ ],\end{aligned} \tag{5}$$

and the set of p vectors,$\{\mathrm{v(Mn-k)} | \mathrm{i} \le \mathrm{k} \le \mathrm{i+p-1}\}$, forms a $p \times L$ matrix, denoted as $\mathrm{V_p^{Mn-i}}$, and is given as follows:

$$\mathrm{V_p^{Mn-i}} \equiv \left[\begin{array}{c} \mathrm{v}(Mn-i) \\ \mathrm{v}(Mn-i-1) \\ \vdots \\ \mathrm{v}(Mn-i-p+1) \end{array}\right]. \tag{6}$$

Here the superscript denotes the top row vector used and the subscript "p" denotes number of rows. The projection operator is denoted by P and $\mathrm{P}^{\perp} = (\mathrm{I - P})$, denotes the projection operator corresponding to the orthogonal complement space. π is the pining vector defined as $\left[\begin{array}{cccc} 0 & \cdots & 0 & 1 \end{array}\right] \in \mathbb{R}^{L}$.

B. Geometrical framework for the given data case

Using equations (3), (4) and the notations defined above, we now present the geometric setting required for the development of the algorithm for signal matched filter bank. Using equation(3), we write all the outputs for time upto Mn, in matrix form as follows:

$$\begin{aligned}\left[\begin{array}{cccccc} 0 & \cdots & 0 & e_i(2M-i) & \cdots & e_i(Mn-i) \end{array}\right] = \\ \left[\begin{array}{cccccc} 0 & \cdots & 0 & x(2M-i) & \cdots & x(Mn-i) \end{array}\right] \\ + \left[\begin{array}{cccc} h_i(M) & h_i(M+1) & \cdots & h_i(M+N-1) \end{array}\right] \\ \left[\begin{array}{cccccc} 0 & \cdots & 0 & x(M) & \cdots & x(Mn-M) \\ 0 & \cdots & 0 & x(M-1) & \cdots & x(Mn-M-1) \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & x(M-N+1) & \cdots & x(Mn-M-N+1) \end{array}\right], \\ 0 \le i \le M-1.\end{aligned} \tag{7}$$

Using the notations defined in the above section, equation(7) can be written, in vector form, as follows:

$$\mathbf{e_i}(Mn-i) = \mathbf{x}(Mn-i) + \mathbf{h}_{\mathrm{i}} \mathrm{X_N^{Mn-M}}, \quad 0 \le \mathrm{i} \le \mathrm{M-1}. \tag{8}$$

Equation (8) is the " given data case" counterpart of (3). The filter parameters, $\mathbf{h}_i$'s, can be obtained using the least squares criteria, i.e. minimizing $||\ e_i\ ||^2$, and , thus, can be written as:

$$\mathbf{h_i} = \mathbf{x}(Mn-i) X_N^{Mn-M^T} \left[X_N^{Mn-M} X_N^{Mn-M^T} \right]^{-1}, \quad 0 \le i \le M-1. \tag{9}$$

In a similar manner, the given data case counterpart of equation (4) can be written as follows:

$$\epsilon_{\mathbf{i}}(Mn-i) = \mathbf{e_i}(Mn-i) + \mathbf{a}_{\mathrm{i}} \mathrm{E_i^{Mn-i+1}}, \quad 0 \le \mathrm{i} \le \mathrm{M-1}. \tag{10}$$

where, $\mathrm{E_i^{Mn-i+1}} \equiv \left[\begin{array}{c} \mathbf{e_{i-1}(Mn-i+1)} \\ \mathbf{e_{i-2}(Mn-i+2)} \\ \vdots \\ \mathbf{e_0(Mn)} \end{array}\right]$.

The filter parameters, $\mathbf{a}_i$'s, can be obtained using the least squares criteria:

$$\mathbf{a_i} = \mathbf{e_i}(Mn-i)E_i^{Mn-i+1^T}\left[E_i^{Mn-i+1}E_i^{Mn-i+1^T}\right]^{-1}, \quad 0 \le i \le M-1. \tag{11}$$

The errors $\epsilon_i(Mn-i)$'s, so obtained, satisfy the conditions of *spectral majorization* and *total decorrelation* and, thus, gives an optimum coding. It should also be noted here that, for equations similar to (9) and (11), we already have fast order as well as time recursive algorithms existing in the literature [20]. These algorithms can be used to obtain computationally efficient solution of the proposed methodology.

IV. Simulation Results

We now present simulation results to validate the proposed algorithm. Since we have not come across any other work dealing with given data case algorithm in the context of signal adapted multirate filter banks, we compare our work with most recent block processing algorithms in the context. We consider three cases, considering different input signals and compare our results with different signal adapted sub-band coders.

Case 1: Simulations are performed for a four channel SMFB, with filters of order 8. We consider two AR(2) inputs, with poles at $0.975e^{\pm j\theta}$ and compare the coding gain of proposed filter bank with the results presented by Lu et al. [9] and with Nalbalwar [6]. Coding gain of any transform coder is defined as the ratio of the mean square error in pulse coded modulation over that in the transform coder [21]. The results have been appended in Table I.

TABLE I: Coding Gain Comparison

θ	[9]	[6]	Proposed algorithm
$\pi/2.8$	6.6411	7.5498	12.8996
$\pi/1.75$	4.9174	7.1294	10.5934

Case 2: In [3], design of a GTD-biorthogonal sub-band coder is presented, it has been shown that as number of channels increases the coding gain comes closer to the theoretically maximum coding gain. Here, for the same input signal, an AR(2) process with poles at $0.975e^{\pm j\pi/3}$, we show that the coding gain of the proposed filter bank converges even when the number of channel is as small as 2, this result is embedded in Table II. The filter bank is designed with each analysis filter of order 5.

Case 3: Here a comparison, between the coding gain performance of DCT(Discrete Cosine Transform) algorithm and the proposed algorithm, is presented for a WSS AR(1) input signal with pole at 0.95.

TABLE II: Behavior of Coding gain (in dB) as number of channels change

Number of channels	G_{SBC} using the proposed algorithm	Approximate G_{SBC} from [3]
2	11.4773	10.4
3	11.5202	9.9
4	11.4947	11.2
5	11.5233	11.3
6	11.8081	11.1

TABLE III: Coding Gain Comparison between DCT and the proposed algorithm

	Discrete Cosine Transform	Proposed Filter bank
4×4 point	7.57	10.0421
8×8 point	8.83	10.075

Coding gain is a measure to compare performance of different transforms [21], and it is evident from tables I,II and III that the proposed filter bank provides better coding gain than other sub-band coders. Also, the proposed analysis filter bank is essentially a single input multi output whitening filter. The filter bank can, thus, be used as a classifier for pattern recognition or machine learning, where coding gain can be the decision criteria and the filter coefficients provides the required features. Since the algorithm can be implemented in an order as well as time recursive manner, the proposed method is also computationally efficient and can be used in real time applications.

V. Conclusions

In this paper, a biorthogonal signal matched filter bank, which yields an optimum coding gain, has been proposed. Unlike PCFB, the proposed filter bank exist for any class of sub-band coders, filters are of finite length and the input is not assumed to be WSS only. It has also been shown, in Tables I-III, that the proposed algorithm has better coding gain performance than other sub-band coders. Since the filter bank parameters can also be obtained by fast algorithms, the proposed method is computationally efficient. In this work, we discussed given data case, however, for given statistics also the algorithm can be easily developed.

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