Uncertainty Aspects in Designing Transportation Networks for Extractive Industries

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***Abstract*—In transportation networks for extractive industries there are typically several uncertain factors including the volume of deposits, the duration of logistics operations, and the methods of delivery. All transportation networks have a unique outlet, usually a single cargo station for shipping all extracted products to other regions. In this paper, the authors demonstrate how methods of interval analysis and fuzzy sets theory are used for designing structures of such networks.**

***Keywords*—transportation systems; uncertainty; computation**

I. Introduction

In this paper, methods are developed for designing structures of transportation networks under the conditions of uncertainty of the initial information. As a rule, the initial information is imprecise for any type of transportation network, but the degree of uncertainty and the number of undetermined factors are especially noticeable when designing networks for extractive industries. Networks for extractive industries are meant to be transportation networks that are created for development and operation of deposits of minerals in regions without infrastructure. For these networks, the following undetermined factors are typical: the volume of deposit reserves, the duration of network operations, information about engineering-geological construction conditions, the directions of network development, the methods of construction, etc. A special feature of the construction of networks for extractive industries is the existence of a unique outlet (since there is no consumption inside the local region, and all the extracted product is transported to other regions from a single cargo station which is the network outlet).

Two types of factors cause indeterminacy of parameters:

1) Factors that are externally relative to the models and directly affect model's parameters equally. These are: duration of network utilization, capacity of deposits, construction methods and so on.

2) Factors that are not completely determined due to incompleteness of the engineering-geological information about construction conditions and of network exploitation, which affect the internal parameters of the model. The first group of factors determines the external indeterminacy of the model, while the second determines the internal.

For describing external as well as internal indeterminacy, methods of interval analysis [1] and the theory of fuzzy sets [2] are used in this paper. The procedure of choosing optimal versions of structures of networks is based on introducing a preference relation and using a decomposition approach [3].

In the second section, two types of indeterminacy in networks for extractive industries are presented. In the third section, preference relations are introduced on the set of versions of structures of the network, and a model of decomposition of the problem is described. In the fourth section, algorithms for designing transportation networks based on the decomposition approach are described. In conclusion, practical utilization of the results obtained are outlined.

II. Statement of the Problem

Usually, the initial information for the design of the transportation network is presented in the form of a connected graph, $G = (I \cup \{i_0\}, E)$, where I is the set of sources and i_0 is the sink. E is the set of edges corresponding to the admissible communications; the following are specified $\forall$ (i, j) $\in$ E: the cost of building an edge $w_{ij} \geq 0$, the cost of transporting a unit of product across an edge $v_{ij} \geq 0$.

For each source, $i \in I$ its capacity $p_i \geq 0$ is provided.

The problem of designing the transportation network is to minimize the objective function.

$$F(T) = \sum_{(i,j)\in T} (w_{ij} + v_{ij} y_{ij}(T)) \to \min, T \in \Omega, \qquad (2.1)$$

where T is the spanning tree of the graph G, y_{ij} is the flow over the edge (i, j) $\in$ T which is uniquely determined for a given sink i_0, T is the set of spanning trees of the graph G.

In the case of external indeterminacy, the problem of designing transportation networks can be posed in the form of ‘‘minimizing’’ the following objective function:

$$F(\tilde{\alpha}, \tilde{\beta}, T) = \sum_{(i,j)\in T} (\tilde{\alpha} w_{ij} + \tilde{\beta} v_{ij} y_{ij}(T)) \to \min, T \in \Omega, \qquad (2.2)$$

where $\tilde{\alpha}, \tilde{\beta}$ are indeterminate parameters

Under internal indeterminacy, it is required "to minimize" the

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objective function

$$F(T) = \sum_{(i,j)\in T} (\tilde{w}_{ij} + \tilde{v}_{ij}\tilde{y}_{ij}(T)) \to \min , T \in \Omega , \qquad (2.3)$$

where $\tilde{w}_{ij}, \tilde{v}_{ij}, \tilde{y}_{ij}$ are indeterminate parameters defined on the edges of the graph G.

The indeterminate parameters are modeled by interval numbers or fuzzy sets.

III. Preference Relations and the Decomposition Approach to the Problem with Interval and Fuzzy Estimates

To solve the problem (2.1) in [4] a decomposition model is proposed based on the approach [3]. The basic idea of the decomposition approach is as follows: instead of the initial, complex optimization criterion in the problem, several particular simpler criteria are introduced. If the partial criteria are monotonically coordinated with the global one (i.e., the fact that an alternative is superior with respect to the partial criteria implies its superiority with respect to the global one), then the solution of the initial problem is contained in the Pareto set of the coordinated multi-criterion problem. An application of the decomposition approach is most advisable when the global problem and the problems with partial criteria have different classes of complexity [4]. This model allows us to reduce the solution of the initial NP-complex problem (2.1) to a solution of the sequence of partial problems of polynomial complexity. In this paper, we propose application of the model [4] for solving problems (2.2) and (2.3).

Since an estimate of the alternative in problems (2.2) and (2.3) is either an interval or a fuzzy set, it is necessary for solving these problems to introduce scalar and vector preference relations (PR) on the set of alternatives, and to generalize the decomposition model to the case of these PR- For the problem (2.2), the partial criteria which are introduced are deterministic. A decomposition scheme for problem (2.2) with fuzzy estimates is described in [4]. Evidently, this method is applicable for solving the problem (2.2) with interval estimates. Unlike the problem (2.2), in problem (2.3) partial criteria are also undetermined and are defined by a certain PR. Hence, for application of the decomposition approach, it is necessary to formulate a condition of compatibility of the global PR and partial PRs. The compatibility condition is a generalization of the condition of monotonicity of the criteria in the deterministic case [3] namely PRη is compatible with relations $\nu_1,\dots,\nu_m$ if and only if the fact that $\forall$ i, x' dominates x" with respect to the PR implies that x' dominates x" with respect to the PR η. It is easy to verify that, in the case of compatible PRs, the set of non-dominated solutions with respect to the PR η. It is contained in the set of non-dominated solution with respect to vector PR ($\nu_1,\dots,\nu_m$).

A. Preference relation for the case of interval estimates

In the case of a single criterion, PR is introduced naturally: two intervals $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ are incomparable if and only if $(b_1 - a_2)(a_1 - b_2) \le 0$, I_1 dominates I_2 ($I_1 < I_2$) if and only if $a_2 > b_1$. A solution of the problem constitutes the set of non- dominating intervals (which are incomparable with each other). For multi-criteria problems with indeterminacy, several PR are introduced in [5], but their application in problems with interval indeterminacy is not advisable, since the problem becomes completely deterministic which results in a partial loss of information contained in the model and an unjustified narrowing down of the selection region.

We introduce a vector PR for the multi-criteria problem with interval parameters

$(U_1 (x), U_2 (x), \dots, U_m (x)) \to \min, x\in X$

In the following manner: to each alternative x there corresponds an m-dimensional parallelepipeds in the space of criteria $[a_i, b_i]$, where $D(x) = [a_1, b_1]\times [a_2, b_2]\times\dots \times [a_m, b_m]$, and $[a_i, b_i]$ is the interval of the values of the criterion $U_i(x)$. Parallelepipeds $D(x')$ dominates parallelepipeds $D(x'')$ if and only if $\forall i\ b_i'' < a_i''$.

B. PR for the case of fuzzy estimates

A scalar PR for fuzzy sets is determined in [2] as a PR induced by the natural order (<) on the real axis. The vector PR in this case is described in [3]. However, the given PR do not allow us to take into account the form of the curves described by membership functions and depend only on the mutual disposition of regions with maximal values of membership functions. In this paper, it is proposed to express these PR in terms of PR for interval estimators.

Let a fuzzy set be given by the membership function $\nu:R^1 \to [0, 1]$. Following [2] we shall call the set $X_\alpha^\nu = \{x \mid x\in U\ R^1 , \nu (x) \ge\alpha\}$, $0< \alpha < 1$ an α-level set of the fuzzy set ν. Evidently, for convex fuzzy sets, X_α^ν is an interval. A fuzzy set ν dominates a fuzzy set μ at level α if and only if X_α^ν dominates X_α^μ. The sets ν and μ are incompatible if and only if the corresponding intervals are incompatible. A vector PR at level α is introduced analogously. Note that for the normal convex fuzzy subsets, the PR introduced herein for the case $\alpha = 1$ coincides with the PR in [2], while the set of non-dominated alternatives with respect to a given PR monotonically increases as a decreases.

When the estimates are nonconvex (X_α^ν is not continuous), we take instead of X_α^ν an approximating interval whose lower bound coincides with the minimal lower bound, while the upper bound coincides with the maximal upper bound of intervals comprising X_α^ν . In this case, the set of non-dominated solutions can only increase.

IV. An Algorithm for Synthesizing Transportaion Networks under Indeterminacy Conditions

An algorithm for solving problem (2.2) for the case of fuzzy estimators was described in [5]. Since an interval estimate is a particular case of a fuzzy set, this algorithm can be applied also for solving problem (2.2) with interval estimates.

To solve the problem (2.3) with interval estimates, partial problems of polynomial complexity are distinguished:

$$U_1(T) = \sum_{(i,j)\in T} \tilde{w}_{ij} \to \min , T \in \Omega , \qquad (4.1)$$

$$U_2(T) = \sum_{(i,j)\in T} \tilde{v}_{ij}\tilde{y}_{ij}(T) \to \min , T \in \Omega , \qquad (4.2)$$

A solution of the problem can be obtained in the course of constructing a set of efficient solutions P, of the problem $(U_1(T), U_2(T)) \rightarrow \min, T \in \Omega$.

To each solution T_i of problem (2.3) in the space of the criteria there corresponds a region $D(T_i) = [a_1^i, b_1^i] \times [a_2^i, b_2^i]$ where $U_1(T_i) = [a_1^i, b_1^i]$, $U_2(T_i) = [a_2^i, b_2^i]$.

By definition, $T_i \in P_u$ if there exists no T_j such that $U_1(T_j) \leq U_1(T_i)$ and $U_2(T_j) \leq U_2(T_i)$ and one of the inequalities is strict.

Solution of problem (2.3) is reduced to the following.

1. A set of trees of shortest paths $\{T_i'\}$ possessing non-dominated estimates is constructed (i.e., the interval problem (4.2) is solved). To construct the set $\{T_i'\}$ it is proposed to use Dykstra's algorithm modified for the case of interval lengths of edges:

$$b_k' = \max_{\{T_i\}} \{b_i'\}, \text{ where } U_1(T_i') = [a_i', b_i'].$$

2. The spanning trees $T_i'' \in \Omega$ are generated in order of increase of a_i'', where $U_1(T_i'') = [a_i'', b_i'']$, as long as $a_i'' < b_k'$. The algorithm given in [6] is used for generation.

All the remaining spanning trees are certainly not efficient solutions. In the course of generating spanning trees, a set of solutions is formed which are not dominant with respect to cost of construction and operation of the transportation networks.

As follows from Section B, problem (2.3) with fuzzy estimates of alternatives can be reduced to the problem (2.3) with interval estimates. As far as the choices of the level α at which a comparison of fuzzy estimates is carried out is concerned, it is difficult to provide specific recommendations in the general case. It should be noted that as α decreases, the number of non-dominating alternatives increases monotonically. In a specific problem, the choice of (x must be made empirically originating from the required capacity of the set of non-dominated alternatives.

V. CONCLUSION

Based on the proposed approach, a package of programs has been developed which allows generation of a set of non-dominating versions of networks in the case of internal and external indeterminacy.

REFERENCES

[1] R. Moore, R. Kearfott, and M. Cloud, "Introduction to Interval Analysis," Society for Industrial and Applied Mathematics Philadelphia, PA, 2009.

[2] S.A. Orlovskiy, "Problems of Decision Making under Fuzzy Initial Information," Nauka, Moscow, 1981.

[3] P.S Krasnoshchekov, V.V. Morozov, and V.V. Fedorov, "Decomposition in Planning Problems," Tekhnicheskaya Kibernetika, No. 2, 1979.

[4] S. Peltsverger, B. Peltsverger. "Classification of Types of Indeterminacy in Designing Structures of Transportation Networks," Optimization Days "JOPT-07," Montréal, Canada, May 2007.

[5] V.I. Zhukovskiy, and V.S. Molostvov, "Multi Criterion Decision Making under Uncertainty Conditions," MNIIPU, Moscow, 1988.

[6] H.N . Gabow, Two algorithms for generating weighted spanning trees in order. SLAM J . Compute.,6, No.1, pp .139 - 150, 1977.