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Quantum Cost Reduction of Reversible Circuits Using New Toffoli Decomposition Techniques

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***Abstract*—Quantum cost is the most important criteria to evaluate reversible and quantum circuits. Also the fundamental building blocks of reversible and quantum circuits are Multiple-Control Toffoli (MCT) gates. The synthesis of MCT based reversible circuits are usually conducted into two steps. First, MCT circuits are decomposed into quantum circuits and then they are optimized using various techniques such as template matching, moving rules to reduce the quantum cost of reversible circuits. In this paper, we propose new techniques to decompose the Toffoli gates, in which MCT based circuits are mapped into a corresponding quantum realization. The main improvement is that the resulting quantum realization of MCT based circuits makes significantly better realization than those achieved in the earlier approaches and further reduction is possible using some other optimization techniques. Experimental results show that our new techniques enable to get sub-optimal realization of the MCT based reversible circuits in decomposition stage and quantum cost reduction of the reversible circuits is achieved by using that sub-optimal realization.**

***Keywords*-Reversible circuits; Quantum Circuits; Quantum Cost; Toffoli Decomposition;**

I. Introduction

Synthesis of reversible logic has been an active research area since power dissipation due to the information loss can be avoided, according to Landauer's principle. More recently, synthesis of quantum circuits has taken the great attentions, motivated by the promise of exponential speed-up in quantum computation. Now the question is how to construct the quantum circuit and what savings can be achieved if one compare such circuit with the circuit having the minimal number of elementary quantum gates. Such research have been already performed in [1]. However, its authors apply quantum decomposition directly to 3-bit Toffoli gates, while larger Toffoli gates are first decomposed into an equivalent circuits built from small Toffoli gates. In [2], a whole 4-bit reversible circuit is constructed directly from the quantum gates. In this way they have found a new quantum decompositions for some pairs of MCT gates. Those pairs lead to significant savings in the number of elementary quantum gates required for their realizations.

Many methods of reversible circuit synthesis have been developed [3], [4]. Most of them build circuits from MCT gates, which are then decomposed into cascades of the elementary quantum gates. A number of papers have been published on constructing such decompositions for any size of the reversible gates [5], [6], [7], [8], [9], [10]. There are also papers that focus on reducing the number of elementary quantum gates in the given reversible or quantum circuit [1], [10], [11], [12]. The number of elementary quantum gates required to build the circuit is a common metric called quantum cost (QC). One of the well known examples of savings in quantum cost of the reversible circuit is the Peres gate [13], [14]. Peres gate can be considered as a pair of Toffoli and CNOT gates. The sum of quantum costs of those two gates is 6, however the reversible function of Peres gate can be implemented with only 4 elementary quantum gates. Maslov [15] used a mixture of different techniques (including MMD algorithm, Reed-Muller spectra based algorithm, template application and resynthesis) to improve either gate count or quantum cost which led to improving results for some benchmarks from [16]. However, exact minimization of QC was not the aim of this approach. Donald and Jha [17] added a new option for optimizing QC to their earlier algorithm. They performed similar experiments for an extended library of gates including also SWAP, Fredkin and Peres gates. Wille [18] formulated a synthesis problem as a quantified Boolean formula and then solved it by applying Binary Decision Diagrams. This enabled to find the minimal as well as the maximal QCs for the specified number of gates up to seven gates. Grosse [19] considered synthesis for networks made of multiple control Toffoli gates using SAT-like engines. Their approach to reducing quantum cost was the same, i.e. minimization of the number of gates as the first step and only then trying to reduce quantum cost with the fixed gate count. Some efforts have been recently made to reduce QCs of designs. One of the recent approaches consists in looking for circuit realizations using quantum elementary gates like NOT, controlled NOT, two square roots of NOT [1], [6], [7], [20] or Hadamard gates [21]. The paper [11] shows that significant reduction of QC can also

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be obtained without considering elementary quantum gate library.

In this paper, we propose a new techniques to decompose the Toffoli, gate in which MCT based circuits are mapped into a corresponding quantum realization. This new techniques give the sub-optimal realization of the MCT based circuits compared to other existing techniques and we show that after applying merge rules into the newly formed quantum circuits further reduction is possible.

The paper is organized as follows. Section II recalls basic concepts of reversible logic and quantum logic. Section III presents the techniques to decompose the Toffoli gate with an algorithm. Sections IV presents an algorithm to optimize the reversible circuits in terms of quantum cost or elementary gate count using the proposed techniques with some examples. In Section V our experimental results are collected and compared to known circuits from benchmark pages and from the literature. Section VI summarizes the paper with conclusions and suggestion for further research.

II. Preliminaries

We present the basic concepts of reversible circuits and logic operations in quantum circuits in this section.

In a binary boolean context, a reversible gate is an elementary circuit component that realizes a bijection. To satisfy this requirement, the function must have the same number of inputs and outputs. A reversible function can be realized by cascading reversible gates with fanout-free and feedback-free realization. Many reversible gates have been proposed and Toffoli, Peres and Fredkin are conventionally used to synthesize reversible circuits.

In particular cascade of the generalized Multiple Control Toffoli (MCT) gates is called an MCT based reversible circuit. An MCT gate with no control line is called a NOT gate, with a single control line is called a controlled-NOT(CNOT) gate and with two control lines is the original Toffoli gate as shown in Fig. 1.

On the other hand, the logic representation in quantum computation is quite different from the logic representation in classical computation. The basic unit of information in quantum computation is a qubit represented by a state vector. The states $|0\rangle$ or $|1\rangle$ are known as the computational basic states. The state of an arbitrary qubit is described by the following vector

$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}, \tag{1}$$

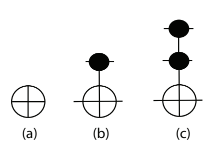

Figure 1. (a) NOT, (b) CNOT, (c) Toffoli

where α and β are complex numbers which satisfy the constraint $\left|\alpha^2\right|+\left|\beta^2\right| = 1$. The measurement of qubit results in either 0 with probability $\left|\alpha^2\right|$, that is, the state $|0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}$ or 1 with probability $\left|\beta^2\right|$, that is, the state $|1\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$. Contrary, a classical bit has a state either 0 or 1, which is analogous to the measurement of a qubit state either $|0\rangle$ or $|1\rangle$ respectively. The main difference between bits and qubits is that a bit can be either state 0 or 1 whereas a qubit can be a state other than $|0\rangle$ or $|1\rangle$ according to (1).

Similarly a generalized two qubit state can be described as

$$|\Psi\rangle = \lambda_1\,|00\rangle + \lambda_2\,|01\rangle + \lambda_3\,|10\rangle + \lambda_4\,|11\rangle = \begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\end{pmatrix}, \tag{2}$$

where $\lambda_1\lambda_4 = \lambda_2\lambda_3$. If $\lambda_1\lambda_4 \neq \lambda_2\lambda_3$ then the state $|\Psi\rangle$ is referred to as an entangled state that is not separable as the tensor product of two single qubit.

Many quantum gates have been defined and studied but we concentrate on the elementary quantum gates NOT, CNOT, Controlled-V, and Controlled-$V^\dagger$, also known as quantum primitives. These gates have been widely used to synthesize binary reversible functions.

A single-qubit NOT and CNOT (and generally every MCT gate) are self-inverse gates. The 2-line controlled-V gate changes the target line using the transformation defined by the matrix $V = \frac{1+i}{2}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}$ if the single control line has the value 1. The 2-line controlled-$V^\dagger$ gate changes the target line using the transformation defined by the matrix $V^\dagger = V^{-1} = \frac{1+i}{2}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}$ if the single control line has the value 1. Gates V and $V^\dagger$ are referred to as square-root-of-NOT gates since $V^2 = (V^\dagger)^2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ and therefore, Controlled-V and Controlled-$V^\dagger$ are inverse of each other. Therefore, two adjacent identical MCT gates can be removed, an adjacent V , $V^\dagger$ pair (any order) with the same target and control can be removed, two adjacent V (or $V^\dagger$) gates with the same target and control can be replaced by a CNOT. Any one primitive among these three can be formed by cascading the other two primitives, referred to as splitting rules that are shown in Fig. 2 (a) and (b) respectively. Moreover, Controlled-V and Controlled-$V^\dagger$ can be replaced with each other resulting in two more splitting rules shown in Fig. 2 (c) and (d) respectively. The inverse of splitting rules is referred to as merge rules. However, in quantum computation, the splitting of a quantum primitives does not increase the number of two-qubit operations.

Definition 1: The size of a circuit c is defined as the number of its gates and denoted by $|c|$. The size of an NCV circuit is also known as quantum cost.

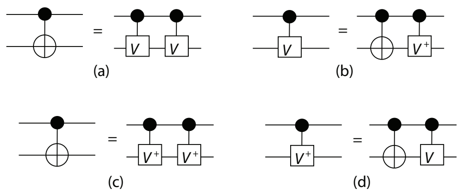

Figure 2. Splitting and Merging rules in Quantum Primitives

The mobility of gates is determined by the moving rule that relies on the following property [22]:

Property 1: Two adjacent gates g_1 and g_2 with controls c_1 and c_2 and targets t_1 and t_2 can be interchanged if $c_1 \cap t_2 = \emptyset$ and $c_2 \cap t_1 = \emptyset$.

III. Proposed Decomposition Techniques of Toffoli Gate

Quantum cost (QC) of a quantum circuit is usually defined by the number of quantum primitives in the quantum circuit. There are three possible Toffoli-3 gates for 3-bit reversible circuits and the circuit representation is shown in Fig. 3. For each case, exactly five elementary quantum gates are required and the circuit representation for the decomposition of the Toffoli-3 gate varies depending on the order of selection of the control bits. According to Lemma 6.1 in [23], the classical reversible Toffoli-3 gate has a quantum implementation of five quantum primitives. The realization of a Toffoli-3 gate can be used in four distinct ways: as given, reversed, and in both those cases with the V and $V^\dagger$ gates interchanged. We note further that, the right most Controlled-V gate can be move anywhere into the circuit. Therefore, different arrangements of quantum implementations of Toffoli-3 are shown in Fig. 4, 5 and 6 respectively, and any one of these quantum implementations of Toffoli-3 can be used in decomposing reversible circuits into quantum circuits without changing the functionality of reversible circuits.

Decomposition of Toffoli gate in MCT based reversible circuit has a significant role to optimize the reversible circuit. The decomposition of Toffoli-3 and n-bit Toffoli network has been studied for decades. But there is no clear direction and specification for the better realization of decomposition of Toffoli gate when the MCT based circuit is decomposed into quantum circuit. Interestingly, it has significant impact into the MCT based circuit to reduce the quantum cost

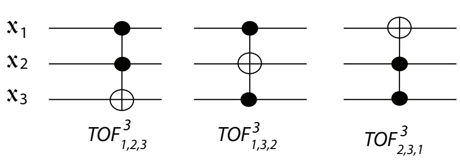

Figure 3. Three possible Toffoli-3 gate representation for 3-bit reversible circuit

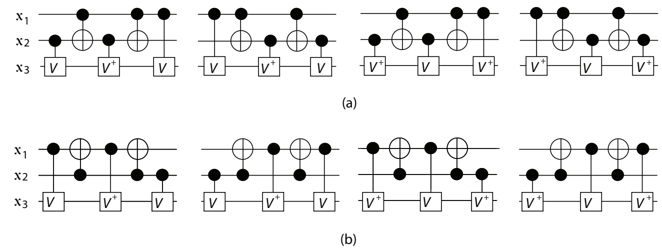

Figure 4. Decomposition of $TOF^3_{1,2,3}$ when (a) first controller is x_1 and second controller is x_2, and (b) first controller is x_2 and second controller is x_1.

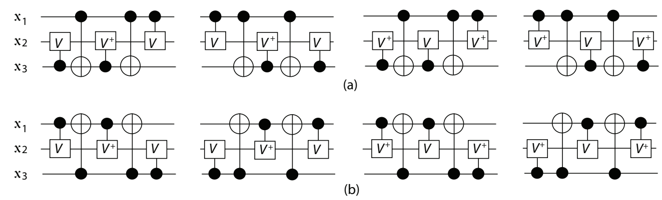

Figure 5. Decomposition of $TOF^3_{1,3,2}$ when (a) first controller is x_1 and second controller is x_3, and (b) first controller is x_3 and second controller is x_1.

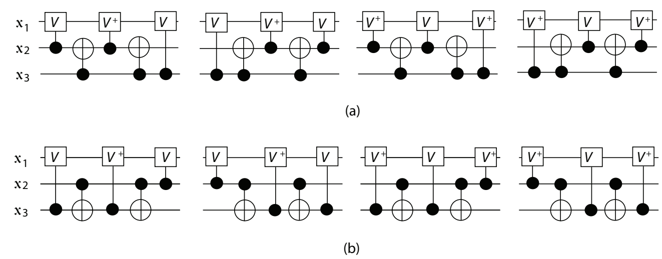

Figure 6. Decomposition of $TOF^3_{3,2,1}$ when (a) first controller is x_2 and second controller is x_3, and (b) first controller is x_3 and second controller is x_2.

of the reversible circuit. Here we propose an algorithm to decompose the Toffoli gate which shows the reduction of quantum cost of the reversible circuit. Fig. 7 shows the six possible CNOT gates for 3-bit reversible circuits.

As we shown that three possible Toffoli gates for 3-bit reversible circuits can be realized into two different ways. Choosing the optimal representation for each Toffoli gate in the MCT based circuit depends on its adjacent gates. Table I shows the best ways for decomposing the Toffoli gate with minimal quantum cost depending on the adjacent gates into MCT based circuits. If we apply this circuit decomposition depending on its adjacent gates in the MCT based circuit, it will provide sub-optimal representation for the circuit. The

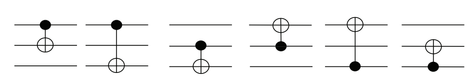

Figure 7. Six possible CNOT gate representation for 3-bit reversible circuit

algorithm to decompose the Toffoli gate is as follows.

procedure TOFFOLI GATE DECOMPOSITION ALGORITHM: ▷ Input: TR is the set of all possible realization of Toffoli gate and CNT is the set of all possible realization of CNOT gate. ▷ Output: $OptReal$ is the set of all optimized realization of Toffoli gates depending on its adjacent gates and $OptCost$ is corresponding new quantum cost.
  var $OptReal \leftarrow \emptyset, OptCost \leftarrow 0$; ▷ Initialize all the possible realization of toffoli gate
  var $M \leftarrow length(TR), N \leftarrow length(CNT)$;
  for $i \leftarrow 1$**to**M **do**
    for $j \leftarrow 1$**to**N **do**
      for $k \leftarrow 1$**to** $all\ possible\ realization\ of\ TR[i]$ **do**
        if $Is\ Optimized\ Realization?$ **then** ▷ We consider minimum number elementary gates required to the quantum representation as optimized circuit
          $Update\ OptReal$ and $OptCost$
        end if
      end for
    end for
    for $x \leftarrow 1$**to**M **do**
      if $TR[i] \neq TR[x]$ **then**
        $Do\ nothing$
      else if $Is\ Optimized\ Realization?$ **then**
        $Update\ OptReal$ and $OptCost$
      end if
    end for
  end for
end procedure

$Remark$: In Table I CNOT is denoted by CN for convenience.

IV. QUANTUM COST REDUCTION USING PROPOSED TOFFOLI DECOMPOSITION

This section describes how to decompose the Toffoli-3 gates to reduce the quantum cost of reversible circuits using the new techniques. For circuit optimization we apply our new gate library and new techniques to decompose the Toffoli gate in the MCT based circuit depending on its adjacent gate. For some cases, we need to realize the Toffoli gate with interchanging V and V+ gates.

Consider the function $4mod5\_v_0\_18$ (taken from the RevLib [24]). Its MCT realization with quantum cost 25 are shown in Fig. 8(a) and the decomposition of Toffoli-3 depending on its adjacent gates is shown in Fig. 8(b). We can apply moving rules and reorder the obtained circuit in Fig. 8(c). Gates 1 and 4 form a new two-qubit gate and gates 2 and 13 form another new two-qubit gate. Gates 7 and 8, 11 and 12, and 15 and 16 will be deleted according to deletion rule. The cost of the circuit is reduced by 14. That is, the total quantum cost of the circuit after applying gate library and the new techniques of decomposition is 11, whereas paper [25] found quantum cost of 17 for the same circuit.

The following example shows what is the impact to decompose the Toffoli gates depending on the order of selection of the control bits and the new techniques to decompose the Toffoli gate in the MCT based circuit depending on its adjacent gate. First we decomposed each toffoli gate to obtained the optimized quantum circuit. After that we will apply new techniques to show the efficiency of our techniques. Consider the function $ham3\_102$ (taken from the RevLib [24]). The MCT realization of the function $ham3\_102$ is shown in Fig. 9(a).Fig. 9(b) shows the elementary quantum gate realization of the function after decompose the $TOF^3_{3,2,1}$, in which first controller is x_3 and

Table I
NEW CIRCUIT DECOMPOSITION OF TOFFOLI GATE DEPENDING ON ADJACENT GATES

Gate	Adjacent Gate	New Circuit Decomposition	New QC
$TOF^3_{a,b,c}$	$CN_{a,b}$	$V_{b,c}CN_{a,b}V^{\dagger}{}_{b,c}V_{a,c}$	4
$TOF^3_{a,b,c}$	$CN_{a,c}$	$V_{b,c}CN_{a,b}V^{\dagger}{}_{b,c}CN_{a,b}V^{\dagger}{}_{a,c}$	5
$TOF^3_{a,b,c}$	$CN_{b,c}$	$V_{a,c}CN_{a,b}V^{\dagger}{}_{b,c}CN_{a,b}V^{\dagger}{}_{b,c}$	5
$TOF^3_{a,b,c}$	$CN_{b,a}$	$V_{a,c}CN_{b,a}V^{\dagger}{}_{a,c}V_{b,c}$	4
$TOF^3_{a,b,c}$	$CN_{c,a}$	$V_{b,c}CN_{a,b}V^{\dagger}{}_{b,c}CN_{a,b}[V_{a,c}CN_{c,a}]$	5
$TOF^3_{a,b,c}$	$CN_{c,b}$	$V_{a,c}CN_{a,b}V^{\dagger}{}_{b,c}CN_{a,b}[V_{b,c}CN_{c,b}]$	5
$TOF^3_{a,c,b}$	$CN_{a,b}$	$V_{b,c}CN_{a,c}V^{\dagger}{}_{b,c}CN_{a,c}V^{\dagger}{}_{a,b}$	5
$TOF^3_{a,c,b}$	$CN_{a,c}$	$V_{b,c}CN_{a,c}V^{\dagger}{}_{b,c}V_{a,b}$	4
$TOF^3_{a,c,b}$	$CN_{b,c}$	$V_{a,b}CN_{a,c}V^{\dagger}{}_{c,b}CN_{a,c}[V_{c,b}CN_{b,c}]$	5
$TOF^3_{a,c,b}$	$CN_{b,a}$	$V_{b,c}CN_{a,c}V^{\dagger}{}_{b,c}CN_{a,c}[V_{a,b}CN_{b,a}]$	5
$TOF^3_{a,c,b}$	$CN_{c,a}$	$V_{a,b}CN_{c,a}V^{\dagger}{}_{a,b}V_{c,b}$	4
$TOF^3_{a,c,b}$	$CN_{c,b}$	$V_{a,b}CN_{a,c}V^{\dagger}{}_{b,a}CN_{a,c}V^{\dagger}{}_{c,b}$	5
$TOF^3_{b,c,a}$	$CN_{a,b}$	$V_{c,a}CN_{c,b}V^{\dagger}{}_{b,a}CN_{c,b}[V_{b,a}CN_{a,b}]$	5
$TOF^3_{b,c,a}$	$CN_{a,c}$	$V_{b,a}CN_{c,b}V^{\dagger}{}_{b,a}CN_{c,b}[V_{c,a}CN_{a,c}]$	5
$TOF^3_{b,c,a}$	$CN_{b,c}$	$V_{c,a}CN_{c,b}V^{\dagger}{}_{c,a}V_{b,a}$	4
$TOF^3_{b,c,a}$	$CN_{b,a}$	$V_{c,a}CN_{c,b}V^{\dagger}{}_{b,a}CN_{c,b}V^{\dagger}{}_{b,a}$	5
$TOF^3_{b,c,a}$	$CN_{c,a}$	$V_{b,a}CN_{c,b}V^{\dagger}{}_{b,a}CN_{c,b}V^{\dagger}{}_{c,a}$	5
$TOF^3_{b,c,a}$	$CN_{c,b}$	$V_{b,a}CN_{c,b}V^{\dagger}{}_{b,a}V_{c,a}$	4
$TOF^3_{a,b,c}$	$TOF^3_{a,c,b}$	$V_{b,c}V_{a,c}CN_{a,b}[V^{\dagger}{}_{b,c}V_{c,b}]V^{\dagger}{}_{a,b}CN_{a,c}V^{\dagger}{}_{c,b}CN_{a,c}$	8
$TOF^3_{a,b,c}$	$TOF^3_{b,c,a}$	$V_{a,c}V_{b,c}CN_{b,a}[V^{\dagger}{}_{a,c}V_{c,a}]V^{\dagger}{}_{b,a}CN_{c,b}V^{\dagger}{}_{b,a}CN_{c,b}$	8
$TOF^3_{a,c,b}$	$TOF^3_{b,c,a}$	$V_{c,b}V_{a,b}CN_{c,a}[V^{\dagger}{}_{a,b}V_{b,a}]V^{\dagger}{}_{c,a}CN_{b,c}V^{\dagger}{}_{c,a}CN_{b,c}$	8
$TOF^3_{a,c,b}$	$TOF^3_{a,b,c}$	$V^{\dagger}{}_{c,b}CN_{c,a}V_{a,b}[CN_{c,a}V_{a,c}]V_{a,b}V^{\dagger}{}_{b,c}CN_{a,b}V_{b,c}$	8
$TOF^3_{b,c,a}$	$TOF^3_{a,b,c}$	$V_{c,a}V_{b,a}CN_{b,c}[V^{\dagger}{}_{c,a}V_{a,c}]V^{\dagger}{}_{b,c}CN_{a,b}V^{\dagger}{}_{b,c}CN_{a,b}$	8
$TOF^3_{b,c,a}$	$TOF^3_{a,c,b}$	$V_{b,a}V_{c,a}CN_{c,b}[V^{\dagger}{}_{b,a}V_{a,b}]V^{\dagger}{}_{c,b}CN_{c,a}V^{\dagger}{}_{a,b}CN_{c,a}$	8

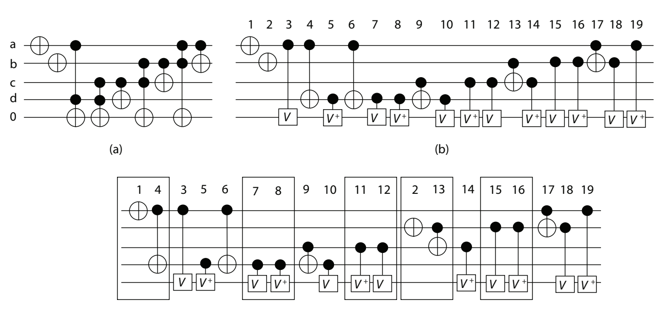

Figure 8. (a) MCT representation of $4mod5\_v_0\_18$ circuit, (b) decompose Toffoli gates depending on its adjacent gates, (c) deletion and merge rules applied two gates after applying moving rules.

second controller is x_2. The implementation shown in Fig. 9(b) has quantum cost 9. We reorder the gates shown in Fig. 9(c) so that they can be realized into two-qubit gates in our new quantum gate library. Gate pairs 4, 6 and 7,9 form new two qubit gates and the cost of the circuit shown in Fig. 9(b) is reduced by 2. Using straight forward methods the total quantum cost of the circuit in Fig. 9(c) is 7.

Now, consider the decomposition of $TOF_{3,2,1}^{3}$ according to our new techniques as shown in Table III and the elementary quantum gate realization of the function $ham3\_102$ shown in Fig. 9(d). The implementation shown in Fig. 9(d) has quantum cost 7, which is already lower than Fig. 9(b). After that we reorder the gates of Fig. 9(d) for further reduction. We reorder in that way, so that they can be also realized into two qubit gates in our new quantum gate library. The new obtained quantum circuit after reordering gates is shown in Fig. 9(e). Gates 4 and 6 form another new two qubit gate as well as gates 5 and 7 form a further one. The cost of the circuit is reduced by 4. That is, the total quantum cost of the circuit in Fig. 9(e) is 5, which is lower than shown in Fig. 9(c).

From the above two examples, we can say that the decomposition of the Toffoli gate in MCT based reversible circuit can play a very important role such as reducing the quantum cost of the reversible circuit. The algorithm for circuit optimization is as follows.

procedure CIRCUIT-OPTIMIZATION ALGORITHM:
    Decompose the Toffoli gates.
    Apply moving rules and find out the following rules to reduce the quantum cost.
    Apply Deletion Rule: If two adjacent gates are identity then delete two gates.
    Apply Merge Rule: If two adjacent gates can merged then replace into a single one.
    Apply Gate Library: Create the sequences of gates which match with the new two-qubit gate library.
end procedure

After decomposing the Toffoli gate according to new techniques, we use the moving rule with the adjacent gate to create sequences of two qubit gates that operate on the same two qubit lines. Two adjacent gates g_1 and g_2 with controls c_1 and c_2 and targets t_1 and t_2 can be interchanged if $c_1 \cap t_2 = \emptyset$ and $c_2 \cap t_1 = \emptyset$. These sequences of two qubit gates can be deleted if they are identity, and can be merged into a single one according to the new two-qubit gate library.

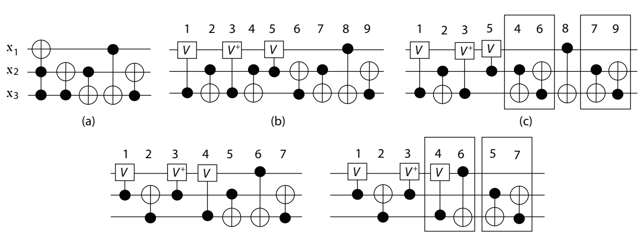

Figure 9. (a) $ham3\_102$ circuit, (b) decompose Toffoli gate when first controller is x_3 and second controller is x_2, (c) merge two gates into one after applying moving rules, (d) decompose Toffoli gate depending on its adjacent gate, (e) merge gates after applying moving rules.

V. Experimental Results

We used Lemma 6.1 in [23], decomposition method to decompose the reversible circuit into quantum circuits using only quantum primitives. Using MATLAB, we wrote a program to get new decomposition circuit for the decomposition of Toffoli gate in the MCT based circuits depending on its adjacent gates. We used reversible circuits with MCT gates reported in RevLib [24] to verify the power of our new techniques. After the decomposition of Toffoli gate considering both two ways, the moving rule was used to reduce the quantum costs of reversible circuits. We showed in the previous section with example that how the new techniques of decomposing the Toffoli gates can further reduce the quantum cost of the reversible circuits. The results are shown in Table II. Columns 2 and 3 represent the number of MCT gates and lines in the original circuits needed to decompose the circuit into quantum circuits respectively. Columns 4, 5 and 6 show the number of elementary quantum gates required in the original MCT circuits before optimization, in paper [25] and after optimization for the quantum implementation of reversible circuits respectively for benchmark functions reported in RevLib [24].

VI. Conclusions

New decomposition techniques of Toffoli gate in MCT based circuit have been proposed that play a significant role in reducing quantum costs of reversible circuits. Proposed techniques for decomposing the Toffoli gate depend on its adjacent gate and the order of selection of the control

Table II
QUANTUM COST REDUCTION OF REVLIB BENCHMARK CIRCUITS

Benchmark	Lines	Gates	QC [1]	QC [12]	QC [This Work]
3_17_1	3	6	14	9	8
4mod5-V_0_18	5	9	25	17	11
3_17_14	3	6	14	10	9
mod5d2_70	5	8	16	12	11
4gt11_83	5	8	12	9	8
rd32-V_1_68	4	5	13	10	9
alu-V_3_34	5	7	19	15	14
alu-V_1_28	5	7	15	12	11
4mod5-V_1_23	5	8	24	20	19
rd32-V_0_66	4	4	12	10	8
decod24-enable_125	6	9	21	18	17
4gt11-V_1_85	5	4	8	7	6
4mod5-V_0_20	5	5	9	8	6
ham3_102	3	5	9	8	5
toffoli_double_4	4	2	10	9	8
one-two-three-V_2_101	5	8	24	22	21
mod5mils_65	5	5	13	12	10
mod5mils_71	5	5	13	12	10
hwb4_52	4	11	23	23	22
peres_9	3	2	6	6	4

bits. The main improvement of this new techniques is that the resulting quantum realization of MCT based circuits is significantly better than those achieved in the earlier approaches when MCT based reversible circuits is decomposed into quantum realization. The experimental results show the efficiency of the new decomposition techniques. Still there is enough room to improve this techniques for further development in the area of reversible logic and quantum logic. Limited interaction distance between gate qubits is one of the most common limitations of the current technologies. For example, trapped ions, liquid nuclear magnetic resonance (NMR), and the original Kane model have been designed based on the interactions between linear nearest neighbor (LNN) qubits. Our next aim is to develop an algorithm for n*n-input reversible circuits and transform the optimized quantum circuits into LNN architecture for physical developments of this emerging technology.

REFERENCES

[1] N. Scott and G. Dueck, "Pairwise Decomposition of Toffoli Gates in a Quantum Circuit," *Great Lakes Symposium on VLSI*, pp. 231–235, 2008.

[2] P. Kerntopf and M. Szyprowski, "Reducing Quantum Cost of Pairs of Multi-Control Toffoli Gates," *International workshop on Boolean Problems*, pp. 263–268, 2012.

[3] A. D. Vos, "Reversible Computing," *Wiley-VCH Verlag*, 2010.

[4] M. Saeedi and I. L. Markov, "Synthesis and Optimization of Reversible Circuits A Survey," *ACM Computing Surveys*, vol. 45, pp. 21–34, 2013.

[5] R. Wille, D. M. Miller, and Z. Sasanian, "Elementary Quantum Gate Realizations for Multiple-Control Toffoli Gates," *Proc. 41st IEEE International Symposium on Multiple-Valued Logic*, pp. 288–293, 2011.

[6] D. M. Miller, "Lower Cost Quantum Gate Realizations of Multiple-Control Toffoli Gates," *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, pp. 308–313, 2009.

[7] D. M. Miller and Z. Sasanian, "Lowering the Quantum Gate Cost of Reversible Circuits," *53rd IEEE International Midwest Symposium on Circuits and Systems*, pp. 260–263, 2010.

[8] Z. Sasanian, "Technology Mapping and Optimization for Reversible and Quantum Circuits," *Ph.D. dissertation, Department of Computer Science, University of Victoria, Victoria, BC, Canada*, 2012.

[9] Z. Sasanian and D. M. Miller, "Transforming MCT Circuits to NCVW Circuits, in: A. De. Vos and R. Wille (Eds)," *Reversible Computation, RC 2011, LNCS, Springer-Verlag, 2012*, vol. 7165, pp. 77–88, 2011.

[10] ——, "Reversible and Quantum Circuit Optimization: A Functional Approach, in: R. Gl ¨ uck and T. Yokoyama (Eds)," *Reversible Computation, RC 2012, LNCS, Springer-Verlag, 2013*, vol. 7581, pp. 112–124, 2013.

[11] M. Szyprowski and P. Kerntopf, "Reducing Quantum Cost in Reversible Toffoli Circuits," *Proc. 10th Reed- Muller Workshop*, pp. 127–136, 2011.

[12] ——, "A Study of Optimal 4-bit Reversible Circuit Synthesis from Mixed-Polarity Toffoli Gates," *Proc. 12th IEEE Conference on Nanotechnology*, pp. 1–6, 2012.

[13] D. Maslov and G. W. Dueck, "Improved Quantum Cost for n-bit Toffoli Gates," *IEEE Electronic Letters*, vol. 39, pp. 1790–1791, 2003.

[14] A. Peres, "Reversible logic and quantum computers," *Physical Review A*, vol. 32, pp. 3266–3276, 1985.

[15] G. Dueck, D. Maslov, and D. Miller, "Techniques for the Synthesis of Reversible Toffoli Networks," *ACM Trans. on Design Automation of Electronic Systems*, vol. 12, pp. 1–28, 2007.

[16] D. Maslov, "Reversible Logic Synthesis Benchmarks Page," *http://www.cs.uvic.ca/˜dmaslov*.

[17] J. Donald and N. K. Jha, "Reversible Logic Synthesis with Fredkin and Peres Gates," *ACM Journal on Emerging Technologies in Computing Systems*, vol. 4, pp. 1–19, 2008.

[18] G. W. Dueck, R. Wille, H. M. Le, and D. Große, "Quantified Synthesis of Reversible Logic," *Proc. DATE*, pp. 1015–1020, 2008.

[19] G. W. Dueck, D. Grosse, X. Chen, and R. Drechsler, "Exact SATbased Toffoli Network Synthesis," *Proc. 17th Great Lakes Symposium on VLSI, Italy*, pp. 96–101, 2007.

[20] Z. Sasanian and D. M. Miller, "Mapping a Multiple-Control Toffoli Gate Cascade to an Elementary Quantum Gate Circuit," *Proc. 2nd Workshop on Reversible Computation*, pp. 83–90, 2010.

[21] D. Maslov and M. Saeedi, "Reversible circuit optimization via leaving the Boolean domain," *IEEE Trans. on CAD*, pp. 806–816, 2011.

[22] R. Wille, D. Miller, and R. Drechsler, "Reducing reversible circuit cost by adding lines," *IEEE International Symposium on Multiple-Valued Logic*, pp. 217–222, 2010.

[23] A. Barenco, C. H. Bennett, R. Cleve, D. P. Divincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, "Elementary gates for quantum computation," *Physical Review A*, vol. 52, no. 5, pp. 3457–3467, 1995.

[24] "RevLib: An online resource for reversible functions and circuits, http://revlib.org/."

[25] R. M. Mazder, A. Banerjee, G. W. Dueck, and A. Pathak, "Two-Qubit Quantum Gates to Reduce the Quantum Cost of Reversible Circuit," *41st IEEE International Symposium on Multiple-Valued Logic*, pp. 86–92, 2011.