Differential Evolution Algorithm-based Range Image Registration with a Novel Point Descriptor

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Abstract - Range image registration plays increasingly important role in a variety of research fields. In general, the registration results are sensitive to many practical factors, such as the sizes, geometries, and initial positions of the range images. This paper is such an attempt to register two range images with different sizes under complex initial situations. First, in coarse registration, a novel point descriptor is designed based on the electrostatic field theory. In term of the identified properties space, a good source of prior information is achieved. Then, combine with the coarse registration, an enhanced differential evolution algorithm is proposed for fine registration. Experiment results reveal that the proposed methods are able to provide competitive results to solve the challenging registration problems when compared with the other methods.

Keywords: Range image registration, Point descriptor, Differential evolution algorithm

1 Introduction

Range image registration is a critical technique in computer vision and pattern recognition areas. In the past years, it has been widely used in quality inspection [1], virtual museum [2], reverse engineering [3], and other fields. The goal of the registration is to find the optimal transformation matrix to align two range images as close as possible. The transformation matrix includes rotation and translation parameters, in some papers the scale parameters are taken into account also [4]. According to the matching accuracy, registration can be divided into coarse registration and fine registration [5]. In general, the coarse registration is an approximate matching to provide good initial position for fine registration.

In coarse registration, the property of each point is extracted to search the correspondence between two point clouds. To improve the efficiency, generally, only the points which can effectively contribute to finding the good corresponding point pairs are used. In literature, the normal and curvature are the popular information for point description [6, 7]. However, they are difficult to describe the distinctive points for the highly symmetric or planar models, and the process might result in selecting many points that essentially contain the same information. Rusu et al. [8] proposed a persistent feature histograms (PFH) to match point clouds from different views, and each point was estimated by a 16D features based on the normal. Guo et al. [9] presented a rotational projection statistics (RoPS) for local feature description of point set. Yang et al. [10] proposed a local feature statistics histogram (LFSH) for registration, in LFSH the local depth, point density, and normal were encoded to describe the local shape geometries. In fine registration, iterative closest point (ICP) [11] is the best known method. However, the ICP is sensitive to the initial position of two models, and it is easy trapped in local minima when two models are far away from each other. In practical application, the complex position and the geometric shape of the models make the registration problem more difficult. More recently, in the literature, range image registration is considered as an optimization problem, and some heuristic algorithms are employed to solve this problem. Such as the simulated annealing (SA) algorithm [12], genetic algorithm (GA) [13], scatter search (SS) algorithm [14], and artificial bee colony (ABC) algorithm [15].

In this paper, we propose a coarse-to-fine method for range image registration. In coarse registration, the point cloud is considered as a conductor with electrostatic equilibrium, and the points are seen as the free charges of the conductor. Then, a novel point descriptor is designed based on the electrostatic field theory (EFT). With EFT, the electric force, electric filed, and electric potential energy are encoded for point description. In fine registration, based on the initial solutions achieved in the coarse registration, we employed the improved differential evolution algorithm (DE) for searching the global optimal solution. The contributions of this paper are as follows.

- A novel point descriptor is designed based on the EFT for coarse registration.
- In terms of the coarse registration, an improved DE algorithm is proposed for fine registration.
- The proposed methods can achieve successful results for range image registration with complex initial situations.
The rest of this paper is structured as follows. Section 2 presents the EFT-based point descriptor. The improved DE algorithm is proposed in Section 3. Experiments and analyses are conducted in Section 4. Finally, Section 5 gives the conclusion and future work.

2 Electrostatic field theory-based point descriptor

In electrostatic field, the conductor contains a lot of free charges which move easily. Due to the interaction of each charge, finally a steady state (called electrostatic equilibrium) of the conductor is achieved. There are many similarities between point cloud and conductor. First, the location of the points and the free charges are fixed in the point cloud and conductor, respectively. Second, the property of the point and free charges can be approximated by its neighborhood. Furthermore, the curvature is an important information for point cloud, and the value of the curvature may be positive and negative for different points. Similarly, the free charges are the signed magnitudes, and the quantity of each charge is different. In this paper, the points are considered as the free charges with electrostatic equilibrium, and the property of each point is described by the electric force, electric filed, and electric potential energy.

2.1 Electric force

Given two static points \( c_1 \) and \( c_2 \), assuming that the electric quantity of \( c_1 \) and \( c_2 \) are \(+q_1\) and \(+q_2\), respectively. According to the Coulomb’s law, the vector form of the electric force between particles \( c_1 \) and \( c_2 \) is as follows:

\[
\vec{F}_{21} = \vec{r}_{21} \frac{q_1 q_2}{r_{21}^2}
\]

(1)

where \( k_e \) is Coulomb’s constant \( k_e=8.99\times10^9 \text{Nm}^2\text{C}^2\), and the \( \vec{r}_{21} \) denotes a unit vector pointing from \( c_2 \) to \( c_1 \). In this paper, we assign the quantity of electricity for each point based on the curvature value.

\[
q_i = \lambda \cdot C_{ci}
\]

(2)

where \( C_{ci} \) is the curvature value of point \( c_i \), and \( \lambda \) is the amplification coefficient, in this paper, \( \lambda \) is set as \( \lambda=200 \). As shown in Fig 1 (a), the points \( p_1, p_2, \ldots, p_5 \) represent the neighborhood of point \( p_0 \) with different colors, and the size of each point represents the quantity of electric charge. Because the electric force satisfies the superposition principle, the electric force at \( p_0 \) is \( \vec{F}_{p_0} = \vec{F}_{p_1} + \vec{F}_{p_2} + \vec{F}_{p_3} + \vec{F}_{p_4} + \vec{F}_{p_5} \).

2.2 Electric field

According to the Coulomb’s law, the distribution of charges \( c_1 \) and \( c_2 \) can create electric fields \( E_1 \) and \( E_2 \), respectively. The electric field is a vector field, and it satisfies the superposition principle also. This principle is useful to calculate the field created by multiple point charges. If charges \( c_1, c_2, \ldots, c_n \) are stationary in space at \( r_1, r_2, \ldots, r_n \), the resulting field is the sum of fields generated by each particle as described by:

\[
\vec{E}(r) = \sum_{i=1}^{N} \vec{E}_i(r) = \frac{1}{k_e} \sum_{i=1}^{N} \frac{q_i}{|r-r_i|}
\]

(3)

where \( k_e \) is Coulomb’s constant, and \( N \) is the number of neighbors of the point \( c \).

Fig 1. (a) Electric force computation of \( p_0 \). (b) Description of electric filed and the equipotential surfaces. (c) Point description of Bunny, and (d) is the zoom-in of the local region at the ear of Bunny.

2.3 Electric potential energy

In this paper, we use the electric potential energy \( U_e \) to illustrate the relationship between the center point and its neighborhood points, respectively. The electric potential energy of one neighborhood point charge \( c \) (with \(+q \) quantity) in the presence of a point charge \( C \) (with \(+Q \) quantity) is:

\[
U_e(d) = k_e \cdot \frac{qQ}{d}
\]

(4)
3 Differential evolution algorithm for registration

3.1 Rigid registration problem

The purpose of rigid registration is to find a transformation matrix to match the point clouds onto a common coordinate system. Given two point clouds \( P \) and \( Q \), where \( P = \{p_i | i=1,2,\ldots, N_p\} \) and \( Q = \{q_j | j=1,2,\ldots, N_q\} \), the \( N_p \) and \( N_q \) are the number of points of \( P \) and \( Q \), respectively. Assuming that \( Q \) is fixed and the \( P \) is moving. Based on the transformation matrix \( T=[T_r, T_t] \), \( P \) is updated as follows:

\[
p_{i}\text{-new} = T_r \cdot p_i + T_t, \tag{5}
\]

where \( T_r \) is the rotation matrix and \( T_t \) is the translation vector. Then, a corresponding table \( T_i \) is generated in terms of the temporary location of two models. According to the Euclidean distance [11], the current error is computed as follows:

\[
E_{r_i} = \frac{1}{N_p} \cdot \sum_{i=1}^{N_p} \left[ p_{i,\text{new}} - T_i(q_i) \right]^2 \tag{6}
\]

where \( T_i() \) means searching the correspondence between two models. The goal of the registration is to find a transformation matrix, which include three rotation parameters \( (r_x, r_y, \text{ and } r_z) \) and three translation parameters \( (t_x, t_y, \text{ and } t_z) \), the parameters with subscripts \( x, y, \text{ and } z \) mean they are updated along \( x, y, \text{ and } z \) axes, respectively. In this paper, we cast the registration as a six dimensional non-linear optimization problem, and the improved DE algorithm is employed to search the global optimal solution.

3.2 DE algorithm for fine registration

DE algorithm is proposed by Storn et al. [16], in the past several decades, various enhanced DE algorithms have been proposed for different applications. Because range image registration is a low-dimension optimization problem and the coarse registration is performed, in this paper, a simple and effective optimization structure of DE algorithm is preferred.

As mentioned previously, when use DE algorithm to solve the registration problem the transformation matrix \( \{t_x, t_y, t_z, r_x, r_y, r_z\} \) is set as the individual of population, and the real-code is employed. The objective function is to minimize the root mean square error (RMSE) with Euclidean distance between two point clouds. In practice, the mutation factor \( F \) plays a crucial role for the efficiency of the DE algorithm, and the setting of \( F \) should be problem dependent. In this paper, we introduce an adaptive mutation operator [17] into the DE algorithm as follows:

\[
F = F_0 \cdot 2^{\frac{G - G_{\text{max}}}{G_{\text{max}}}} \tag{7}
\]

where \( F_0 \) is the mutation constant, \( G \) is the current evolutional generation, and \( G_{\text{max}} \) is the maximum evolutional generation.

As for registration, because the units of the translation and the rotation are millimeter and degree, respectively, we assign different mutation constants for translation parameters and rotation parameters. Besides, to speed up the convergence of DE algorithm the centroids of two point clouds are translated to the origin of coordinates. Furthermore, we use the approximate solutions achieved by the coarse registration for population initialization of DE, and the mutation strategy “DE/best/2” is employed.

\[
v_{i,G} = x_{\text{best},G} + F \cdot (x_{r1,G} - x_{r2,G}) + F \cdot (x_{r3,G} - x_{r4,G}) \tag{8}
\]

where the \( r1, r2, r3, \text{ and } r4 \) are distinct integers randomly selected from the range \([1, \text{NP}]\) and any of them that are not equal to \( i \). \( \text{NP} \) is the population size, and \( x_{\text{best},G} \) represents the best individual among the current population based on the fitness value.

When use the DE for registration, the main computation time is searching the correspondence between two point clouds. In general, the corresponding point pairs are determined by the Euclidean distance of the 3D points in two models. Assuming that the number of points of \( P \) and \( Q \) are \( N_p \) and \( N_q \), respectively, the computation complexity is \( O(N_p N_q) \). In this paper, k-D tree method is employed to accelerate the computation, and the computation complexity is reduced to \( O(N_p \log(N_q)) \). In the procedure of evolution, the population
size of DE is set as $m$ and the number of iterations is $k$, and the whole computational complexity is $O(mkN_p^2(N_q))$.

4 Experiments and results

4.1 Experiment case

In this section, we compared the proposed registration algorithm, named FADE, with three deterministic methods (ICP [11], FICP [18] and ADF [19]) and two improved heuristic algorithms (IFFO [20] and IDE [21]). The test models are shown in Fig 2, from left to right they are Bunny, Dragon, Air Intake, and Fandisk models.

Fig 2. Test models.

The number of points of each model are 4026, 8261, 9001, and 11165, respectively. Because in registration the rotation matrix has higher effect than translation vector, we pay more attention to the rotation matrix. Following the work of Li et al. [19], we set the original point set as $Q$, and the moving point cloud $P$ is achieved by the presetting transformation matrices conducted on the $Q$. The presetting matrices are shown in Table 1, where $L$ is the largest size (width, height, and depth) of the model.

Table 1. Presetting transformation matrices.

<table>
<thead>
<tr>
<th>No. of Trans</th>
<th>$T_x$ (mm)</th>
<th>$T_y$ (mm)</th>
<th>$T_z$ (mm)</th>
<th>$R_x$ (°)</th>
<th>$R_y$ (°)</th>
<th>$R_z$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>0.1*L</td>
<td>0.1*L</td>
<td>0.1*L</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.5*L</td>
<td>0.5*L</td>
<td>0.5*L</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>$T_3$</td>
<td>1.0*L</td>
<td>1.0*L</td>
<td>1.0*L</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

4.2 Experiments

In the coarse registration experiments, we aim to find the corresponding points between $Q$ and $P_1=Q_{T_1}$, and $Q$ and $P_2=Q_{T_2}$, where $Q_T$ means the $Q$ is transformed by the matrix $T$, and $Q_S$ denotes the down-sampling of $Q$. In this section, we use the Bunny as the $Q$ to register $P_1=Q_{T_2}$ and $P_2=Q_{T_3}$, respectively. The number of the neighbors is set as $k=10$. Note that to keep the geometric structure of the model, we simplified the original point cloud based on the triangular meshes. Thus, although the number of points of $P_1$ is smaller than $Q$, the point cloud $P_2$ is not a real subset of $Q$. With 60% simplified triangular meshes of $Q$, the $P_2$ is achieved with 2446 points. The corresponding point pairs of $Q$ and $P_1$, and $Q$ and $P_2$ are shown in Fig 3 (a) and (b), respectively. For property bins comparison, we randomly selected one point pair from $Q$-$P_1$ and $Q$-$P_2$, respectively, and the results are shown in Fig 3 (c) and (d).

In fine registration, we use the simplified point sets to match the original point clouds. The simplified Bunny, Dragon, Air Intake, and Fandisk point sets have 2446, 4235, 4501, and 6699 points, respectively. ICP, FICP, and ADF are deterministic methods, the maximum iteration is set as $I_{ermax}=30$ for these methods. As for the IFFO, IDE, and FADE, the population size is set as $NP=30$, and the terminal condition in this experiment is the maximal number of function evaluations (FEs) $F_E=3000$. In IFFO, the maximum search radius of translation parameter is $\lambda_t=1.0*10^{-7}$, and the maximum search radius of rotation parameter is $\lambda_r=1.0*10^{-10}$. The parameters setting of IDE are follow with [21]. In FADE the initial mutation constants of translation and rotation are $F_t=0.2$ and $F_r=0.4$. In some papers, the initial transformation matrix is determined by only a few point pairs from the coarse registration, and if the bad point pairs are selected that will lead a false result in fine registration. Considering the effect of noise and occlusion, in this paper, we randomly select 3 different point pairs from two point clouds 2000 times to compute the transformation matrices, and the best 30 matrices are saved as the initial population of FADE. To enable a fair comparison, each heuristic algorithm is run 25 times independently and the median value is saved. The registration errors of each algorithm are shown in Table 2, where the bold values mean the best results in the rows.

![Fig 3. (a) Corresponding point pairs of Q and P1. (b) Corresponding point pairs of Q and P2. (c) Property bins of Q and P1. (d) Property bins of Q and P2.](image-url)
4.3 Discussion

In this section, we illustrated the performance of EFT-based point descriptor for point clouds coarse registration. From Fig 3 (a) we can see that all the points in $Q$ can find a right corresponding point in $P_i$, and the matched property bins of the corresponding points were shown in Fig 3 (c). This illustrated that the EFT-based point descriptor was invariant to 3D rigid transformations. Fig 3 (b) and (d) presented a snapshot of corresponding point pairs of two point clouds with different sizes, and some false correspondence were achieved on the low curvature regions of the model. That’s because the bins of the free charge with low electric quantity were sensitive to its neighbors.

In fine registration, we conducted experiments with complex initial positions. As shown in Table 2, the deterministic methods were sensitive to the initial position and the geometry of the models. ICP and ADF methods achieved successful results for Bunny and Dragon models with simple case $T_1$, however, they were stick in local minima with the Air Intake and Fandisk models. Relatively, the FICP performed better than ICP and ADF. As for heuristic algorithms, the results gained by IDE were more accurate than IFFO, however, the structure of IDE was more complicated than IFFO. It was found from our experiments that FADE performed best for the test models under current initial positions, benefit from the coarse registration it was more robust to the initial position and the geometry of the models.

5 Conclusion

In this paper, we aim to obtain the most possible accurate alignment of two models with complex initial position, and a coarse-to-fine method is proposed for range image registration. In coarse registration, a novel point descriptor is designed based on electrostatic field theory, and the initial correspondence is achieved to reduce the search space. In addition, combine with the coarse registration, an improved differential evolution algorithm is presented for fine registration. The extensive comparison studies reveal that the proposed point descriptor is invariant to 3D rigid transformations for coarse registration, and the FADE performs superior to many different existing algorithms used to solve the fine registration problem.

The limitations of this paper are as follows. First, although the proposed fine registration method is accurate and more robust, the heuristic algorithms take more computation time than the deterministic methods. Furthermore, in special situation, if the false corresponding point pairs take a greater proportion, they are different to be removed in terms of random selection.

In the future, other properties of the electrostatic field can be considered for point description, such as the electric flux and the charge density. Besides, in coarse registration, a novel point detection strategy can be developed to remove the false point pairs. Furthermore, to improve the efficiency of registration, deterministic methods can be employed to combine with the improved EFT-based point descriptor.

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6 References


