**Volumetric Segmentation**

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**Abstract** - Image segmentation plays a crucial role in effective understanding of digital images, planar or volumetric images. The current research in graph based methods is oriented towards producing approximate solution (or sub-optimal solution) for such graph matching problem to reduce processing time. We are introducing an algorithm for volumetric segmentation based on virtual tree-hexagonal structure (prisms) constructed on the image voxels to improve the speed of segmentation. Here, a graph-based theoretic framework is considered by modeling image segmentation as a graph partitioning problem using input spatial graph. Then we can use the graph facilities and their related algorithms and computational complexity can be viewed as slow as the fundamental graph algorithms. The key to the whole algorithms of volumetric segmentation method is the prism cells as vertices. Volumetric segmentation algorithm contains many other algorithms but only segmentation algorithm is presented based on the limited space of paper.

**Keywords:** Graph Based Segmentation, Color Based Segmentation, Syntactic-based Segmentation, Spatial Graph Algorithms, Dissimilarity

1 Introduction and related work

Higher-level problems such as object recognition and image indexing can also make use of segmentation results in matching to address problems such as figure-ground separation and recognition by parts.

In our paper, we develop a visual feature-based method which uses a spatial graph constructed on cells of prism with tree-hexagonal structure containing less than half of the image voxels. Thus, the volumetric image segmentation is treated as a spatial graph partitioning problem.

It was determined the normalized weight of an edge by using the smallest weight incident on the vertices touching that edge [1]. Other methods for planar images [2], [3] use adaptive criterion that depends on local properties rather than global ones. In contrast with the simple graph-based methods, cut criterion methods capture the non-local cuts in a graph, being designed to minimize the similarity between pixels that are being split [4], [5]. The normalized cut criterion [5] takes into consideration self similarity of regions. An alternative to the graph cut approach is to look for cycles in a graph embedded in the image plane. In [6] and [7] the quality of each cycle is normalized in a way that is closely related to the normalized cut approaches. Other approaches to digital planar image segmentation consist in splitting and merging regions according to how well each region fulfills some uniformity criterion. Such methods [8] use a measure of uniformity of a region. In contrast [2] and [3] use a pair-wise region comparison rather than applying a uniformity criterion to each individual region. Complex organizing phenomena can emerge from simple computation on these local cues [9]. A number of approaches to segmentation are based on finding compact regions in some feature space [10]. Our previous works for planar segmentation algorithms [11], [12], [13] and [14] are related to the works in [2] and [3] in the sense of pair-wise comparison of region similarity. In this paper, we are using new algorithms for planar segmentation based on graphs. Our work for planar segmentation consists in adding new steps for volumetric segmentation algorithms that allow us to determine regions closer to it [15], [16], [17] and [18].

2 Virtual tree-hexagonal structure

Volumetric segmentation module creates virtual cells of prisms with tree-hexagonal structure defined on the set of the digital image voxels of the input RGB volumetric image and a spatial grid graph having tree-hexagons (prism) as cells of vertices. In addition for each component, the dominant color of the region is extracted. The surface extraction module determines for each segment of the image its boundaries. The boundaries of the determined visual objects are closed surfaces represented by a sequence of adjacent tree-hexagons. At this level, a linked list of voxels representing the surface is added to each determined component. This implies that there will be less ambiguity in defining volumetric surface and volumes [16], [17], [18].

Let I be an initial volumetric image having the three dimensions $h \times w \times z$ (e.g. a matrix having ‘h’ rows, ‘w’ columns and ‘z’ deep of matrix voxels). To construct a tree-hexagonal grid (prism cells) on these voxels we retain an eventually smaller image with:

$$
\begin{align*}
    h &= h - (h - 1) \mod 2; \\
    w &= w - w \mod 4; \\
    z &= z 
\end{align*}
$$

(1)
We associate to each tree-hexagon H from V two important attributes representing its dominant color and the coordinates of its pseudo-gravity center, denoted by c(h) and g(h). The dominant color of a tree-hexagon is denoted by c(h) and it represents the color of the voxel of the tree-hexagon which has the minimum sum of colors distance to the other twenty voxels. Each tree-hexagon H in the tree-hexagonal grid is thus represented by a single point, g(h), having the color c(h). By using the values g(h) and c(h) for each tree-hexagon, information related to all voxels from the RGB initial image is taken into consideration by the spatial segmentation algorithm.

3 Volumetric segmentation algorithm

Let \( V = \{h_1, \ldots, h_{|V|}\} \) be the set of tree-hexagons (prisms) constructed on the volumetric image voxels as presented in previous section and \( G = (V; E) \) be the undirected spatial grid-graph, with E containing pairs of prism cells (tree-hexagons) that are neighbors in a 16-connected sense. Components of an input digital image represent compact volumes containing voxels with similar properties. The set V of vertices of the graph G is partitioned into disjoint sets, each subset representing a distinct visual object of the initial image.

**Definition 1.** Let \( G = (V; E) \) be the undirected spatial graph constructed on the tree-hexagonal structure of an input digital image, with \( V = \{h_1, \ldots, h_{|V|}\} \). A proper segmentation of V, is a partition S of V such that there exists a sequence \( (S_i; S_{i+1}; \ldots; S_{f-1}; S_f) \) of segmentations of V for which:
- \( S_f \) is the final segmentation and \( S_i \) is the initial segmentation,
- \( S_j \) is a proper refinement of \( S_{j+1} \) (i.e., \( S_j \subset S_{j+1} \)) for each \( j = i, \ldots, f-1 \),
- segmentation \( S_j \) is too fine, for each \( j = i, \ldots, f-1 \),
- any segmentation \( S_j \) such that \( S_j \subset S_i \) is too coarse,
- segmentation \( S_i \) is neither too coarse nor too fine.

Our volumetric segmentation algorithm starts with the most refined segmentation, \( S_0 = \{\{h_i\}\} \ldots \{h_{|V|}\} \) and it constructs a sequence of segmentations until a proper segmentation is achieved. Each segmentation \( S_j \) is obtained from the segmentation \( S_{j-1} \) by merging two or more connected components for which there is no evidence of a boundary between them. For each component of a volumetric segmentation, a spanning tree is constructed and thus for each segmentation algorithm we use an associated spanning forest. The evidence of a boundary between two components is determined by taking into consideration some features in some model of the digital image. When starting, for a certain number of segmentation components the only considered feature is the color of the volumes associated to the components and in this case we use a color based region model. When the components become complex and contain too much tree-hexagons, the color model is not sufficient and geometric features together with color information are considered. In this case we use a syntactic-based with a color-based region model for volumes. In addition, syntactic features bring supplementary information for merging similar volumes in order to determine objects. In the color of graph-based model, the volumes are modeled by a vector in the RGB color space. This vector is the mean color value of the dominant color of tree-hexagons belonging to the regions. The evidence for a volumetric surface between two volumes is based on the difference between the internal contrast of volumes and the external contrast between them [2], [16] and [18]. Both notions of internal contrast and external contrast between two volumes are based on the dissimilarity between two colors.

**Definition 2.** Let \( G = (V; E) \) be the undirected spatial graph constructed on the tree-hexagonal structure of a
volumetric input image and $S$ a color-based segmentation of $V$. The segmentation $S$ is too fine in the color-based region model if there is a pair of components $(C', C'') \in S$ for which $\text{adjacent}(C', C'') = true$ and $\text{ExtVar}(C'; C'') = \text{IntVar}(C'; C'') + \text{thresh}(C'; C'')$, where the adaptive threshold $\text{thresh}(C'; C'')$ is given by

$$\text{thresh}(C'; C'') = \text{thresh}/(\min(|C'|, |C''|).$$  \tag{2}

The threshold ‘thresh’ is a global adaptive value defined by using a statistical model.

The maximum internal contrast between two components, $(C', C'') \in S$ is defined as follows:

$$\text{IntVar}(C'; C'') = \max\{\text{IntVar}(C'); \text{IntVar}(C'')\}.$$  

Algorithm 1 Volumetric Segmentation Algorithm
1: procedure SEGMENTATION(l, c, d, P, H, Comp)
2: Input l, c, d, P
3: Output H, Comp
4: $H \leftarrow$ CREATEHEXAGONALSTRUCTURE(l, c, d, P)
5: $G \leftarrow$ CREATEINITIALGRAPH(l, c, d, P, H)
6: CREATECOLORPARTITION(G, H, Bound)
7: $G' \leftarrow$ EXTRACTGRAPH(G, Bound, thresh)
8: Createsyntacticpartition(G, $G'$, thresh)
9: Comp $\leftarrow$ EXTRACTFINALCOMPONENTS($G'$)
10: end procedure

The input parameters represent the digital image resulted after the pre-processing operation: the array $P$ of the volumetric image voxels structured in ‘l’ lines, ‘c’ columns and ‘d’ depths. The output parameters of the segmentation procedure will be used by the surface extraction procedure: the tree-hexagonal grid stored in the array of tree-hexagons $H$, and the array $\text{Comp}$ representing the set of determined components associated to the objects in the input digital volumetric image.

The color-based segmentation and the syntactic-based segmentation are determined by the procedures CREATECOLORPARTITION and CREATESYNTACTICPARTITION respectively.

The color-based and syntactic-based segmentation algorithms use the tree-hexagonal structure $H$ created by the function CREATEHEXAGONALSTRUCTURE over the voxels of the initial volumetric image, and the initial triangular grid graph $G$ created by the function CREATEINITIALGRAPH. Because the syntactic-based segmentation algorithm uses a graph contraction procedure, createsyntacticpartition uses a different graph, $G$, extracted by the procedure EXTRACTGRAPH after the color-based segmentation finishes.

4 Segmentation results and quantitative evaluation

A true volumetric segmentation remains a difficult problem to tackle due to the complex nature of the topology of volumetric objects, the huge amount of data to be processed and the complexity of the algorithms that scale with the new added dimension [19]. Martin thesis [20] states that human segmentation can be used as the ground-truth reference in benchmarking segmentations produced by different methods.

Fig.3. Experimental results.

The segmentation method used for the experimental results is based on simple hysteresis threshold. All voxels with the density within a specified threshold ‘thresh’ will be treated as boundary voxels while the others as empty spaces. The over-segmented volume has high recall and low precision (Figure 3), while the under-segmented image has low recall because it fails to find salient features for the volume and also low precision [18], [21].

5 Conclusions

In this paper, a graph-based theoretic framework is taken into consideration by modeling digital image segmentation as a graph partitioning and optimization problem using input spatial graph. We are introducing an algorithm for volumetric segmentation based on virtual tree-hexagonal structure (prisms) constructed on the image voxels. The key to the whole algorithms of volumetric segmentation method is the prism cells.

6 References


