Application of Kalman Filter to Estimate Position of a Mobile Node in Indoor Environments

Mounika S. K. Gudipati and Shivakumar Sastry
Department of Electrical and Computer Engineering
The University of Akron, Akron OH 44325-3904, USA

Abstract—Indoor location estimation is an important problem for many emerging applications that involve networked embedded systems. We present an approach to estimate the location of a mobile node using a Kalman Filter. The system model comprises four anchor nodes and one or more mobile nodes. Each mobile node uses the average value of the received signal strength to each anchor node to estimate its position using an extended Kalman Filter. These data are fused with data from a local accelerometer to improve the estimate. We present experimental results to demonstrate the precision of our location estimates. In the future, this work can be extended to reduce the error in the location estimates.

Index Terms—Extended Kalman Filter, Position Estimation, MultiSensor Fusion.

1 INTRODUCTION

Estimating the position of mobile entities is an important problem for several emerging applications in areas such as advanced manufacturing, Internet of Things, and healthcare systems. The problem is especially challenging when it must be addressed indoors and with high precision. To address this problem, we designed and carried out experiments to understand how much precision could be achieved for position estimation in indoor environments using commercial, low-power WSN devices.

The problem of localization is very well studied in the literature [1], [2], [3]. For example, the GPS system and route navigation is widely used across the world. This problem, however, remains challenging when the resolution of the localization must be high. The localization methods in the literature are based on measuring the distance between a mobile node and a fixed set of anchor nodes at known positions. The distance between the nodes can be calculated using ToA (Time of Arrival), AoA (Angle of Arrival) or RSSI (Received Signal Strength Indicator). While the ToA and AoA can provide more accurate estimates, these approaches rely on additional hardware that are expensive. It is also not clear how much resolution these approaches can offer in laboratory and manufacturing environments that are indoor.

Several reports in the literature address this problem by using the received signal strength and triangulation based on a fixed set of anchor nodes. The option to use RSSI to estimate distance, while attractive, is complicated because of the irregularity of RF propagation in the low-power regime and because the RSSI measurements are noisy.

To address the issue of noise, we utilized an Extended Kalman Filter. A Kalman Filter is a widely used recursive prediction-update based state estimator algorithm that can minimize error variance [4]. Thus, by viewing the position of the mobile node as its state, our aim was to minimize the error in the position estimates. The prediction and update steps are combined via the Kalman gain which is calculated in each step to minimize the mean-square error. Such approaches are widely-reported in the literature in several areas including robot localization, guidance and navigation and tracking[5], [6], [7], [8].

The Kalman Filter is, however, known to provide an optimal estimate of the unknown state for a linear dynamic system with Gaussian distribution. The measurements required to assess the state of the mobile node were non-linear. To cope with this challenge, we used an Extended Kalman Filter. This framework allowed us to improve the accuracy of the position estimates, further, by integrating data from accelerometers mounted on the mobile node [9]. Kalman Filters applications for fusing data have also been have also been used widely in the literature to fuse data from multiple sensors [10], [11]. Following these methods, we designed an approach to fuse the data from an accelerometer on the mobile node with the received signal strength data.

While there are several reports in the literature that are based on simulation results, we found the following reports that were based on experiments [12], [13], [14]. Thus, the main focus of this work was to design and carryout experiments to understand how much precision could be achieved for indoor position estimation when using commercial WSN nodes that provide RSSI estimates, with no additional hardware, and when the data from a local accelerometer were fused with the data from the RSSI measurements.

The remainder of this paper is organized as follows. In Section 2 we describe the problem precisely and present the design used. After describing the localization approach in Section 3, we present results from our experiments in Section 4. Finally, we present our conclusions and next steps in Section 5.

2 PROBLEM STATEMENT AND DESIGN

In this section we describe the system model, the development platform, the measurement inputs, their calibration, and how Kalman Gain was computed.

2.1 System Description

The system comprises a single mobile mote and four anchor motes. Each mote was equipped with wireless transceiver that
can periodically transmit data. The mobile mote transmitted data with a known transmission power to the anchor motes. The motes, located within the transmission range of the mobile mote calculate the received RSSI values. The mobile mote was also equipped with accelerometer which was used to record the mote’s inertial data.

2.1.1 Development Platform: The IRIS mote was used as the platform to carry out the experiments. This mote has an 8-bit ATMega1281 microcontroller that is optimized for low power operations and can be easily programmed using the C language. The wireless communication between nodes is based on an AT86RF230, IEEE 802.15.4 compliant, transceiver operating in 2.4GHz ISM band. The microcontroller and the transceiver communicate via the SPI protocol. This mote was used both as the anchor nodes and as the mobile node.

2.1.2 System Model: The state of the mobile node was modeled in terms of its position, velocity and acceleration in a two-dimensional space \( \mathbf{X}_k = [x, y, v_x, v_y, a_x, a_y]^T \).

Due to the non-linearities in the measurements, both for RSSI and for acceleration, Extended Kalman Filter was used to minimize errors. A discrete time model for the system was formulated as

\[
\mathbf{X}_k = \mathbf{f} (\mathbf{X}_{k-1}) + \mathbf{w}_k
\]

where \( \mathbf{X}_k \) is the state vector at the time \( k \); the state transition function, \( \mathbf{f}(.) \), was used to estimate the future state of the mobile node and \( \mathbf{w}_k \sim N(0, \mathbf{Q}_k) \) is a random variable that represented the noise with zero mean and covariance matrix \( \mathbf{Q}_k \). Here,

\[
\mathbf{f} (\mathbf{X}_{k-1}) = \mathbf{A}_k \mathbf{X}_{k-1}
\]

where \( \mathbf{A}_k \) is given by

\[
\mathbf{A}_k = \begin{bmatrix}
1 & 0 & dt & 0 & dt^2/2 & 0 \\
0 & 1 & 0 & dt & 0 & dt^2/2 \\
0 & 0 & 1 & 0 & dt & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

where \( dt \) represents the time interval for each step, \( k \). The covariance \( \mathbf{Q}_k \) of zero-mean Gaussian noise \( \mathbf{w}_k \) is given by

\[
\mathbf{Q}_k = \begin{bmatrix}
dt^4/4 & dt^3/2 & 0 & dt^2/2 & 0 & 0 \\
0 & dt^4/4 & dt^3/2 & 0 & dt^2/2 & 0 \\
0 & 0 & dt^4/4 & dt^3/2 & 0 & dt^2/2 \\
0 & 0 & 0 & dt^4/4 & dt^3/2 & 0 \\
0 & 0 & 0 & 0 & dt^4/4 & dt^3/2 \\
0 & 0 & 0 & 0 & 0 & dt^4/4
\end{bmatrix},
\]

where \( q = \sigma^2 \). This covariance matrix captures the effect of different factors that could cause changes in mobile node directions. The measurement \( \mathbf{Z}_k \) relies on current system state

\[
\mathbf{Z}_k = h (\mathbf{X}_k) + \mathbf{v}_k
\]

where \( \mathbf{v}_k \sim N(0, \mathbf{R}_k) \).

2.2 Measurement Inputs

As already noted, RSSI and acceleration measurements were used to estimate the position of the mobile node. Both these measurements had to be calibrated before being used.

2.2.1 Calibration: The purpose of calibration is to choose suitable parameters that correspond to the actual devices being used in the experiment.

The acceleration data were gathered using a tri-axial accelerometer, LIS344AL [15]. The signal from this device was formulated as

\[
\mathbf{acc}_k = \mathbf{a}_k - \mathbf{g}_e + \mathbf{b}_{a,t} + \mathbf{v}_{a,t},
\]

where \( v_{a,t} \sim N(0, \sigma^2) \). Samples were collected from the accelerometer when the mobile node was stationary. These samples were used to account for the effect of gravity and measurement errors that arise because of temperature sensitivity, improper mounting of the sensor, or other unknown issues.

The strength of the received signal is peculiar to the specific environment in which it is received. This is usually measured in units of decibels per unit distance (dBm). The Friis equation relates the power of the received signal, \( P_R \), (in Watts), the power of the transmitted signal, \( P_T \), (in Watts), the receiver’s antenna gain \( G_R \), the transmitter’s antenna gain \( G_T \), the signal wavelength, \( \lambda \), the distance \( d \) in meters and the signal propagation constant \( n \) for the environment as

\[
P_R = P_T \frac{G_T G_R \lambda^2}{(4\pi)^2d^n}.
\]

From this relationship, using a standard reference distance of 1 m, and standard conversions from Watts to dBm, we obtain

\[
\text{RSSI} = -(10 \log_{10} d - A).
\]

To calibrate RSSI measurements, we need to calculate \( A \) and \( n \) for the environment. We collected RSSI values by moving the mobile node over a 10 m distance in steps of 0.5m. We obtained the average RSSI value at the mobile node, for each step, using 100 samples, from each anchor node.

The average power of the RF signal in dBm was computed as \( P_R = \text{RSSI} - 45 \) the value of \( A \) was obtained by measuring the average RSSI value with distance 1m between the anchor nodes and the mobile node. The calibrated propagation constant \( n \) was computed as

\[
n = \frac{-\text{RSSI}_i - A}{10 \log_{10} d_i}
\]

where \( d_i \) is the euclidean distance between anchor node \( i \) and mobile node. The final value of \( n \) is obtained by averaging the values obtained from above for each anchor node.

2.2.2 Measurement Model: Measurements, \( \mathbf{Z}_k \), were incorporated into the system as indicated in Equation 4; here, \( h(.) \) is the observation function that provides estimates for the expected measurements when the system is in state \( \mathbf{X}_k \). The measurement noise vector, \( \mathbf{v}_k \), was modeled as a normally distributed random variable with zero mean and covariance matrix \( \mathbf{R}_k \). We set \( \mathbf{R}_k \) to a diagonal matrix since we assumed that the measurement errors were independent.

\[
\mathbf{R}_k = \begin{bmatrix}
diag(\sigma^2_{a_x} & \sigma^2_{a_y} & \sigma^2_{dBM_{1,k}} & \sigma^2_{dBM_{2,k}} & \ldots & \sigma^2_{dBM_{4,k}})
\end{bmatrix}
\]
Matrix $R_k$ characterizes the errors between measured and propagation based models, and inaccuracies relating to measured acceleration. Our system used RSSI measurements $P_i$ obtained as described earlier. The measurements $a_x$ and $a_y$ contained provided additional information regarding the node’s state that was fused to improve the accuracy of the position estimation.

The measurement vector for the RSSI data, $Z_k^i$, and for accelerometer $Z_k^a$ is given by

$$Z_k^i = [P_1 \ldots P_4]^T$$

$$Z_k^a = [0 \ 0 \ 0 \ a_x \ a_y]^T$$

The observation function was derived assuming a log-normal propagation model applied to each receiver. However since the measurement is non-linear, the measurement matrix $H_k$ in the correction step must be replaced by Jacobian. The observation function $h(X_k)$ and corresponding Jacobian $H_k$ are given by

$$h(X_k) = -\begin{bmatrix} A - 10\eta \log_{10}(\frac{d_i}{d_{0i}}) \\ A - 10\eta \log_{10}(\frac{d_j}{d_{0j}}) \\ A - 10\eta \log_{10}(\frac{d_k}{d_{0k}}) \\ A - 10\eta \log_{10}(\frac{d_i}{d_{0i}}) \end{bmatrix}.$$  \hspace{1cm} (10)

$$H_k = \frac{\partial h^{(i)}}{\partial x_k}$$ \hspace{1cm} (11)

### 2.3 Kalman Gain Computation

To estimate the state of the system $X_k$ at each time step, the Kalman Filter considers a Gaussian probability density to represent the estimated state. At each time step, predicted system state $X_k^{(+)}$ is calculated from the previous estimated system state $X_{k-1}$.

If the measurement $Z_k$ is available, its effect on new state $X_k^{(+)}$ has to be determined. Following the approach in [?], we obtained

$$X_k^{(+)} = X_k^{(-)} + K_k[Z_k - h(X_k^{(-)})],$$  \hspace{1cm} (12)

$$P_k^{(+)} = [I - K_k H_k]P_k^{(-)}.$$  \hspace{1cm} (13)

$$K_k = \frac{P_k^{(-)}}{H_k^T H_k + V_k R_k V_k^T}.$$  \hspace{1cm} (14)

The matrix $K_k$ denotes the Kalman Gain in step $k$.

### 3 Localization Approach

In this section, we describe the localization approach; we first describe how the position was estimated using the inertial sensors and RSSI. Finally, we describe how we fuse the two estimates for position into a single estimate.

#### 3.1 Position Estimation using inertial sensors

The tri-axial accelerometer provided linear acceleration of the mobile node along three orthogonal axis w.r.t. the frame of the mobile node. Since we measured the acceleration, we computed the position by integrating the acceleration twice over the duration when the mobile node is moving. However, the errors in the accelerometer measurements and those introduced by numerical integration tend to accumulate during time producing large errors in the estimate of the position. Further, as discussed in the previous section, accelerometers also measure the effect of gravity and this must be removed in order to obtain the real acceleration of the mobile node.

#### 3.2 Position Estimation using RSSI

To estimate the unknown position of mobile node, at least three anchor nodes must be able to detect and measure mobile node’s signal strength. Each anchor node stores its position coordinates and value of RSSI received from the anchor nodes. The distance between mobile node and each anchor node is calculated from the measured RSSI as

$$d = 10^{\frac{4 - 10 \log_{10}(\text{RSSI})}{10}}.$$  \hspace{1cm} (15)

We use Trilateration to estimate the position of the mobile node $(x_0, y_0)$ based on estimated distances $d_i$ between mobile node and of at least three anchor nodes $(x_i, y_i)$.

Fig. 1: Intuitively, the trilateration approach involves drawing circles with the anchor nodes as the center; for each anchor node, the distance estimated using RSSI is the radius of the circle. The mobile node lies at the intersection of these circles.

The actual distance between the mobile node and anchor node is given by

$$r_1 = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

$$r_2 = \sqrt{(x_0 - x_2)^2 + (y_0 - y_2)^2}$$

$$r_3 = \sqrt{(x_0 - x_3)^2 + (y_0 - y_3)^2}$$

$$r_4 = \sqrt{(x_0 - x_4)^2 + (y_0 - y_4)^2}.$$  \hspace{1cm} (16)

The error $e_i$, between the estimated distance $d_i$, and the actual distance, $r_i$ is given by

$$e_i = d_i - \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}. \hspace{1cm} (17)$$
Ideally we would like to minimize this error

\[ e_i = d_i - \sqrt{(x_i - x_1)^2 + (y_i - y_1)^2} = 0. \]  

(18)

Simplifying, we get

\[ 2x_0(x_k - x_1) + 2y_0(y_k - y_1) = a_i^2 - a_k^2 - x_1^2 - y_1^2 + x_k^2 + y_k^2. \]  

(19)

This equation is linear and can be represented as

\[ Ax = B \]

where

\[
A = \begin{bmatrix}
2(x_k - x_1) & 2(y_k - y_1) \\
2(x_k - x_2) & 2(y_k - y_2) \\
\vdots & \vdots \\
2(x_k - x_{k-1}) & 2(y_k - y_{k-1})
\end{bmatrix}
\]

(20)

\[
B = \begin{bmatrix}
d_1^2 - d_k^2 - x_1^2 - y_1^2 + x_k^2 + y_k^2 \\
d_2^2 - d_k^2 - x_2^2 - y_2^2 + x_k^2 + y_k^2 \\
\vdots \\
d_{k-1}^2 - d_k^2 - x_{k-1}^2 - y_{k-1}^2 + x_k^2 + y_k^2
\end{bmatrix}
\]

and

\[ x = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \]

The least squares solution for \( Ax = B \), is given by

\[ x = (A^T A)^{-1} A^T B \]

Hence, the estimated position of the mobile node can be computed from above equation.

### 3.3 Fusing the Position Estimates

The approach to measurement fusion used in our model is to weight the individual measurements from each sensor and then track those fused measurements by a Kalman Filter to obtain an estimate of the state vector. Since the measurement noise is independent for accelerometer sensor and RSSI, the equation for fusing the measurement vectors \( Z_k^1 \) and \( Z_k^2 \), for minimum square estimate is given by

\[ Z_k = Z_k^1 + R_k^1 (R_k^1 + R_k^2)^{-1} (Z_k^2 - Z_k^1), \]  

(22)

where \( R_k^1 \) and \( R_k^2 \) are the covariance matrices of the measurement vector of sensor 1 and sensor 2. The measurement noise covariance matrix of fused measurement \( Z_k \) is derived by

\[ R_k = [(R_k^1)^{-1} + (R_k^2)^{-1}]^{-1}. \]  

(23)

The fused estimate measurements are then used to estimate the state vector \( X_{k|k} \) by applying extended Kalman Filter equations.

### 4 Results

In this section, we present results from our experiments to estimate the position of the mobile nodes.

#### 4.1 Experimental Setup

Experiments were carried out in an indoor environment to estimate the accuracy of the position estimates. All the antennas on the motes were fixed at an angle of 90 degrees at the mounting surface. The average RSSI values from the mobile node were collected at a base station.

The physical dimension of the test-bed is 9m x 10m where a total of 4 anchor nodes were placed at four corners of the room. The experiment was conducted when interference from human activities was minimal. Before the experiment was conducted, several preparatory steps were performed. First, the mobile node was rotated slowly in all orientations to examine the orientation effect. After that, the signal propagation constant was calculated for each anchor node in this experiment environment. During the experiment, mobile node was manually moved at steps of 0.5m. The accuracy of the system was found by comparing the position estimated by this system with the predicted position.

#### 4.2 Parameter Determination

Values for \( n \) and \( A \) were determined as described in the preceding section. To model the propagation characteristics of the receiver antennas, an off-line calibration phase was conducted. 100 RSSI measurements were taken at each place in 0.5m increments after placing the mobile node at various distances from the anchor nodes. The resulting model parameters are represented in Table I. Calibration results based on several static locations reveal the drawback of uniform computation as propagation constants are different for different anchor nodes in RSSI-Distance conversion formula. The variation of signals is due to attenuation of the signal on the medium surrounding the reference point.

#### 4.3 Localization Accuracy

The mobile node and the anchor nodes were programmed to exchange data packets; from each message, RSSI, LQI, Acceleration values and node ID were extracted. The Average, Maximum and Minimum values was calculated for each anchor node and results were sent to the base station.
TABLE I: Path Loss Constants

<table>
<thead>
<tr>
<th>Node ID</th>
<th>A</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>-46.86</td>
<td>2.45</td>
</tr>
<tr>
<td>13</td>
<td>-54.02</td>
<td>1.64</td>
</tr>
<tr>
<td>14</td>
<td>-45.51</td>
<td>2.74</td>
</tr>
<tr>
<td>15</td>
<td>-44.70</td>
<td>3.13</td>
</tr>
</tbody>
</table>

The ids of the anchor nodes are shown in the first column; for each anchor node, the transmitted power (in dBm) at a distance of 1m and the path loss constant obtained from calibration are shown.

The average RSSI of all anchor nodes vs Distance between the anchor and mobile node in indoor environment is shown in the Figure 4. We observe that RSSI decreases with increasing distance and has larger variation due to effect of fading and shadowing. Due to the presence of walls each of the signal reflected takes a different path. So even a slight change in node’s position resulted in a significant difference in received power.

Figure 5 depicts the distance traveled by the mobile node as obtained from the results of the accelerometer. The graph represents the acceleration along x-axis where position and velocity estimates along x-axis are obtained by double and single integration of the acceleration values respectively. Vibrations were kept minimum by performing the experiment on a smooth surface. The position estimation with information from accelerometer is susceptible to errors because of integration errors.

After calculating the estimated distances, these distances were compared with actual ones so that the errors can be evaluated. Figure 6 shows a comparison of fusion and no fusion distance with predicted distance for reference node 14. The average variation of the fusion filtered distance from the actual distance is reduced, which implies that the accuracy is improved compared with individual sensor position estimation.

Figure 7 shows the co-ordinates obtained by trilateration. The average localization error without fusing data from the two sources was 2.885m; this error error was reduced to 0.62m after fusion.

5 CONCLUSIONS

We presented an approach to estimate the position of a mobile node in an indoor environment that used an Extended Kalman Filter. The position estimate was obtained using RSSI values collected by exchanging beacon messages with anchor nodes, and accelerometer data sensed on the mobile node. We compared the accuracy of the position estimate that was obtained without fusing data from the two sources and by fusing the data based on experimental results. The results showed that fusing the estimates obtained using RSSI and acceleration data reduced the absolute position error from
Fusing the data from the accelerometer with the RSSI data helps to reduce the variation in the distance estimate with respect to the actual position.

2.88m to 0.62m. Thus, we can conclude that fusing data from multiple sources is indeed a viable strategy to improve the position estimates.

The experimental activities revealed that a large number of parameters must be manipulated to achieve high-resolution position estimates using such an approach. In particular, the orientation of the nodes, the transmission power used, calibration and other environmental effects affect the fidelity of the estimates. If the transmission power is too high, the differences in RSSI values are not significant. Several experiments were conducted to derive the value of $n$ and $A$ and this value can be different for each anchor node. In the future, we are interested to carry out a more comprehensive set of experiments with a larger number of anchor nodes. In addition, we would also seek to understand how fusing multiple inertial sensors affects the accuracy of the position estimates.

REFERENCES