Detecting Frequency from Randomly Sampled Data Implementation of random sampling in BRATUMASS

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Abstract—In this article, a system which implements random sampling theory is presented - the system obtains each sample after a random time interval, and it is a part of the Brest Tumor Microwave Sensor System (BRATUMASS) [1]; It is designed to refine the data of BRATUMASS. The first part introduces the system in the following aspects - the components of the system, how the signal is obtained, and the algorithm we used to calculate the spectrum of non-uniformly sampled data. The second part introduces a set of experimental performances based on random sampling method to explore the features of random sampling; the signals used in the experiment are single frequency sinusoidal waves, mixed sinusoidal waves, and a piece of bass music waves [2], since the random sampling method is a prototype and not integrated with BRATUMASS yet. The data from BRATUMASS is a uniformly sampled data interpolated with the same mechanism - random time interval interpolation, and it will be the next step of this study.

Keywords: Compressive Sensing, Microwave Imaging, Random Sampling

1 Introduction

Breast cancer usually comes from breast tumor which could later become worse and convert to breast cancer. Thus the earlier breast cancer is detected, the more likely to practice permanent cure. BRATUMASS is developed to detect breast cancer at an earlier stage, more treatable stage. Thus, In BRATUMASS, the analysis of the data becomes significantly important. In BRATUMASS, the frequency resolution is such important in discriminating different breast tissue. Here is random sampling comes into the picture - random sampling is a method of compressive sensing [3]. It features a signal to information conversion. Due to limited of information energy in the Nyquist-Shannon theorem, random sampling reduces the number of samples without much perceptual loss. This quality is ideal in the developing of a portable cost-effective device. More importantly, the signal to information conversion, if successful, would increase the information to noise ratio while processing BRATUMASS’s data and improve the imaging resolution of BRATUMASS. This work is to explore the features of random sampling and to prepare for its implementation in BRATUMASS.

2 Glance of the system

The Randomly Sampled Data system consists of sampling module, FIFO to USB converter, data display and analysis platform on PC.

A. Sampling Module: This module is based on an Arduino [4]. Timer, ADC module and random number generator mentioned below are all integrated in this chip. Timer is set according to a sequence of random number. ADC module is triggered by the compare match of the timer and the value of compare match register is updated in each of the compare match interrupt routine. Thus the time interval between two neighboring sample is controlled by a random number generated by algorithm. Both ADC data and the random time interval is send to PC through FIFO to USB module.

B. FIFO to USB Converter: Stores data from the sampling module before computer reads it and sends message from computer to sampling module [5].

C. Data display and analysis platform on PC: Sends instructions to and reads data from the USB port. Display waveform and stores data for analysis.

3 Spectrum calculation

The anti-aliasing properties of random sampling make it possible to represent signal from a relatively small set of data. This work is to explore the features of random sampling and to prepare for its implementation in BRATUMASS.
Assuming that $x(t)$ is a band limited signal, $X(f)$ is Fourier transform of $x(t)$, sampling interval is $T$, total number of sample is $N$, then $NT$ is the total sampling time. Let $x(n)$ be the uniformly sampled data, $x(t_s) \{n = 1,2,3,...\}$ be the randomly sampled data, $X_0(f)$ be the Fourier transform of $x(t_s)$. We have:

$$X_c(f) = \int_0^{NT} x(t)e^{-j2\pi ft} dt \quad (1)$$

$$X_u(f) = \sum_{n=1}^{N} x(n)e^{-j2\pi fn} \quad (2)$$

$$X_d(f) = \sum_{n=1}^{N} x(n)e^{(-j2\pi fn)/\Delta t} \quad (3)$$

4 Random number generation

Random number of three kinds of distribution is used in this experiment; they are Uniform, Normal and Rayleigh distribution. These random numbers are interpreted as time interval between neighbouring samples. For comparison, Nyqvist style samples (identical time interval between neighbouring samples) is also taken and analysed. Therefore we have 4 groups of data for one target signal. HIST of a typical series of random number is shown in Fig. 1. These numbers are generated on data acquisition board and then transmit to PC.

5 Signal and spectrum

Although perfect in simulation [6], practices in experiment shows limits while distinguish signal frequency at low sampling frequency. During the experiment, while sampling frequency is lower than the signal frequency, the spectrum of the signal failed to distinguish signal frequency. What caused this limit and how to improve this remains to be explored. However, distinguish of signal frequency lower than sampling frequency is successful. Experiments and results will be discussed in this part.

Five groups of signal are selected as target signal in this experiment. They are 233Hz, 678Hz, (233.5+233.6) Hz, (678.5+678.6) Hz sine waves and a piece of bass music wave. As is shown in Fig. 2, first we tried to distinguish a single sine wave, then a mix of two sine waves with close frequency, finally a bass music wave.

Fig. 2. Signal in time domain

5.1 Single sine wave

As is shown in Fig. 3, this sampling method is successful in distinguishing single sine wave. Aliasing is suppressed but frequencies with small amplitude are appearing where aliasing frequencies should be. This signal is obtained at an average sampling rate of 800Hz.

Fig. 3. Spectrum of single sine wave 233Hz and 678Hz in one figure

5.2 Mixed sine wave

In mixed sine wave experiment, a same amount of samples (16000 samples) is obtained from a mixed sine wave of 233.5Hz and 233.6Hz. The average rate of random sampling is 800Hz and the sampling rate of Nyquist sampling is 2000Hz. Thus the time window for random sampling is 20 seconds, and its frequency resolution should be $1/20=0.05Hz$. While the time window for Nyquist sampling is 8 seconds, so the frequency resolution should be $1/8=0.125Hz$. In other words, lower sampling rate means longer observation time window for signal when the data amount is limited. And the longer the observation time the higher the frequency resolution. The spectrum of this experiment shows that random sampling shows better frequency resolution at the same number of samples, as it is in Fig. 4 that random
sampled data successfully distinguished mixed sine wave while Nyquist fails to distinguish two frequency components.

5.3 Bass music wave

We also had a bass music wave experiment to explore how this mechanism fits sound wave with rich frequency components in it. This bass music wave is a wave(.wav) file with sampling rate of 44100Hz. Original bass music wave and its FFT of is showed in the last line in Fig. 5. Most of its energy is in frequency components within 1000Hz. All the sampling rate or the average of sampling rate in this bass music experiment is 800Hz.

The spectrum from random sampled data is no big difference from the signal’s fft, except for the aliasing around 700Hz. The anti-aliasing propriety seems to be weakened when signal has rich main frequency components, shown in the Fig.5. The spectrum of Uniform, Normal, Rayleigh and Nyquist is roughly symmetric at an axis of 400Hz. All spectrum calculated from random sampled data is aliasing just like the spectrum calculated from Nyquist style sampled data. However the anti-aliasing propriety of random sampling is not totally gone. The amplitude of higher frequency, which is the aliasing frequency, is slightly lower than that of its symmetric frequency in Fig. 5.

6 Refinement of BRATUMASS’s data

A uniformly sampled data from BRATUMASS is interpolated with random time interval. The signal is obtained at 500Hz. It is interpolated at an average rate of 5000Hz. As is shown in Fig. 6, The samples interpolated between the original samples follows the linear relationship. Because the target signal in BRATUMASS is in 0-50Hz, the spectrum between 0 and 50Hz is compared in the third line of Fig. 6. Different deviation from 5000Hz in this experiment is to explore the influence of different $\sigma$ while calculating the time interval of interpolation. The spectrum of signal after interpolation deviates from the spectrum before interpolation as the deviation grows. As is shown in Fig. 6 that the subtraction of the two fluctuates as the deviation grows. The result might be different while using larger $\sigma$ or different mean. How different distribution of random numbers affect the signal spectrum is still to be explored.

7 Conclusion

This Randomly Sampled Data system successfully distinguishes the spectrum of sine wave and bass wave. It shows advantage in improving the resolution of spectrum and in the suppressing of aliasing. However, how the distribution of random numbers affects the signal spectrum and how to relate the distribution of random interval and the information energy is still to be explored.

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