

# Automatic Coronary Artery Segmentation Based on Matched Filters and Estimation of Distribution Algorithms

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**Abstract**—This paper presents an estimation of distribution algorithm (EDA) for improving the vessel detection performance of Gaussian matched filters (GMF) in X-ray angiographic images. The GMF method is governed by three discrete parameters and one continuous parameter. The optimal selection of the GMF parameters is highly desirable to maximize the detection rate of blood vessels in different types of medical images. The proposed optimization of these four parameters is carried out by applying a population-based method. From all the potential solutions found by EDA, the area ( $A_z$ ) under the receiver operating characteristic (ROC) curve is used as fitness function to obtain the best GMF parameters and the corresponding Gaussian response. The detection performance of the proposed method is compared with those obtained using five different GMF methods of the state-of-the-art and the ground-truth vessels hand-labeled by a specialist. The experimental results applying the proposed method demonstrated high detection rate with  $A_z = 0.9113$  using a training set of 40 angiograms and  $A_z = 0.9343$  with a test set of 40 angiograms.

**Keywords:** Automatic segmentation, Estimation of distribution Algorithm, Gaussian matched filters, Genetic algorithms, Vessel enhancement

## 1. Introduction

Coronary angiography is a specialized X-ray procedure for diagnosing and treating coronary artery disease. In recent years, the development of efficient and accurate computational methods has become essential for computer-aided diagnosis in cardiology. In general, there are two main drawbacks for automatic segmentation of vessels from coronary angiograms; nonuniform illumination along vessel structures and low contrast between vessels and image background. Due to these drawbacks, the vessel enhancement also called vessel detection problem plays an important and challenging role in most of the state-of-the-art coronary artery segmentation methods.

In literature, several methods have been proposed for automatic detection of coronary arteries from X-ray angiographic images. Most of the proposed automatic vessel detection methods are performed in the spatial image domain of the input angiogram such as single-scale top-hat operator [1], multiscale top-hat operator [2], [3], hit-or-miss transform [4],

Hessian matrix [5], [6], [7], and Gaussian matched filters (GMF) [8], which have been used in different types of clinical studies including retinal image segmentation and registration [9], [10].

The GMF method is a spatial template matching technique used in the detection of different blood vessels. It works on the assumption that the shape of blood vessels can be approximated by a Gaussian curve as matching template. This Gaussian template is rotated at different angles and then convolved with the input image to form a filter bank of oriented responses. The maximum response at each pixel is recorded to obtain the final enhanced image. The GMF method has four main parameters, which have to be tuned to obtain the highest detection performance. The parameter  $L$  that determines the length of the vessel segment,  $\sigma$  that defines the spread of the intensity profile,  $T$  which is the position where the Gaussian curve trails will cut, and  $\theta$  that represents the angular resolution of the filter bank.

Since the GMF was introduced, many researchers have suggested different values for each parameter of the filter. Kang et al. [11], [12], [13] applied the GMF taking into account different values for  $\sigma$  and angular resolution. Al-Rawi et al. [14] proposed extend the range of the variables  $L$ ,  $T$ , and  $\sigma$  obtaining the best values by using an exhaustively deterministic search method and the area ( $A_z$ ) under the receiver operating characteristic (ROC) curve. Cinsdikici and Aydin [15] used the original parameters of the method, just modifying the angular resolution. Al-Rawi and Karajeh [16] applied the population-based method of genetic algorithms in order to select the best  $L$ ,  $T$ , and  $\sigma$  values for vessel detection keeping constant the angular resolution. The performance of the population-based method working together with the Gaussian matched filter is more accurate according to the tests than the empirically determined methods.

The population-based methods represent an effective way to solve discrete or continuous optimization problems. Recently, the Estimation of Distribution Algorithms (EDAs) have begun to attract more attention for solving global optimization problems. EDAs are stochastic methods consisting on a set of potential solutions called population that incorporate statistical knowledge to solve optimization problems [17], [18]. In EDAs a probabilistic model of the potential solutions is constructed at each generation, and the new solutions are generated from this model until a stopping

criterion is satisfied. EDAs have proven to be efficient in solving optimization problems such as cancer chemotherapy optimization [19], image segmentation [20], and proving to be more effective than genetic algorithms in discrete optimization systems [21].

Because of the importance of the Gaussian matched filters in many of the aforementioned segmentation methods, an optimization process for selecting the most suitable parameters is required. In this paper we propose the use of the Univariate Marginal Distribution Algorithm (UMDA) from the family of EDAs for improving the detection performance of the Gaussian matched filters. This proposed method addresses the problem of detecting coronary arteries in X-ray angiographic images. The optimization method is performed over the four GMF parameters, where  $L$ ,  $T$ , and  $\theta$  are discrete values, and  $\sigma$  represents a continuous value. The vessel detection performance of the proposed method is compared and analyzed with those obtained using five, previously described, state-of-the-art vessel detection methods by using the area  $A_z$  under the ROC curve.

The remainder of this paper is organized as follows. In Section 2, the basics of the Gaussian matched filter and Univariate Marginal Distribution Algorithm are introduced. In Section 3, the parameter optimization process is presented and analyzed. The experimental results are discussed in Section 4, and conclusions are given in Section 5.

## 2. Background

In this section, the fundamentals of the Gaussian matched filters and the univariate marginal distribution algorithm are described in detail.

### 2.1 Gaussian matched filters (GMF)

The Gaussian matched filters method [8] has been used for detecting blood vessels in different types of medical images. The main idea of GMF is to approximate the shape of blood vessels in the spatial image domain utilizing a Gaussian curve as matching template, which can be defined as follows:

$$G(x, y) = -\exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right), \quad |y| \leq L/2, \quad (1)$$

where  $L$  is the length of the vessel segment to be detected in pixels, and  $\sigma$  represents the average width of blood vessels, which uses an implicit parameter  $T$  (in pixels) to define the position in the template where the Gaussian curve trails will cut. This Gaussian kernel  $G(x, y)$  is rotated at different angular resolutions ( $\theta$ ), obtaining  $\kappa = 180/\theta$  oriented filters. These filters are convolved with the input image, and for each pixel the maximum response over all orientations is preserved to obtain the filtered resulting image.

In order to obtain the best detection performance of the GMF, the discrete parameters of  $L$ ,  $T$ , and  $\theta$ , and the

continuous parameter of  $\sigma$  have to be optimized. In Figure 1, an X-ray coronary angiogram is illustrated along with its hand-labeled image (ground-truth) as drawn by a specialist. The Figure 1(c) presents the Gaussian template as it was defined in the work of Chaudhuri et al. [8] with values  $L = 9$ ,  $T = 13$ ,  $\theta = 15$ ,  $\kappa = 12$ , and  $\sigma = 2.0$ , which have been used successfully in different segmentation applications [9], [10], and the enhancement result of the Gaussian filter method is given in Figure 1(d).

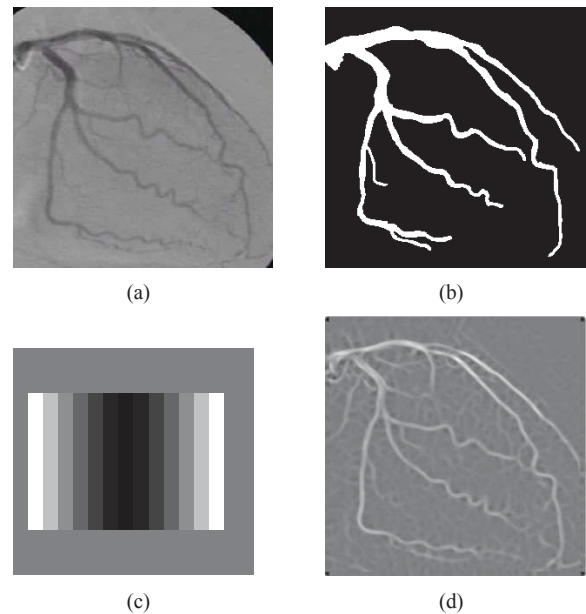


Fig. 1: (a) Original X-ray coronary angiogram. (b) Ground-truth of angiogram in (a). (c) Gaussian template with the predefined parameters of Chaudhuri et al. [8]. (d) Resulting enhanced image using the angiogram in (a) and the template in (c).

### 2.2 Estimation of Distribution Algorithms (EDAs)

Estimation of distribution algorithms represent population-based methods that incorporate statistical information of potential solutions to solve optimization problems in discrete and continuous domain [17], [18]. EDAs are similar to evolutionary computation (EC) techniques since they use a population of potential solutions called individuals, selection operators, and binary encoding. The main difference between EC methods and EDAs is the fact that the crossover and mutation operators are not required by EDAs, instead, they build a probabilistic model based on global statistical information of promising individuals at each generation. This probabilistic model is used to generate new potential solutions by inferring statistical dependencies between the variables. In the present work, the Univariate Marginal Distribution

Algorithm (UMDA) has been adopted because of ease of implementation and because it works ideally for linear problems with not many significant dependencies [21], [22].

UMDA uses a binary codification of the problem, and it generates a probability vector  $\mathbf{p} = (p_1, p_2, \dots, p_n)^T$  to build the probabilistic model at each generation in order to create new individuals for each variable independently, where  $p_i$  is a probability rate. The key idea of UMDA is to approximate the probability distribution of the potential solutions in  $\mathbb{P}_t$  as the product of the univariate frequencies computed from a subset of individuals assuming that all the variables are independent [23]. The steps of selection of promising solutions, estimation of probability distribution and creation of new individuals represent the evolutionary process of UMDA. To perform the selection step, the individuals in the search space  $\Omega$  have to be arranged according to fitness value, then selection probability  $s$  is computed through proportional selection as follows:

$$\mathbb{P}^s(x) = \frac{\mathbb{P}(x)f(x)}{\sum_{\tilde{x} \in \Omega} \mathbb{P}(\tilde{x})f(\tilde{x})}. \quad (2)$$

The second step consisting in the estimation of a joint probability  $\mathbb{P}$  is calculated as follows:

$$\mathbb{P}(x) = \prod_{i=1}^n \mathbb{P}(X_i = x_i), \quad (3)$$

where  $x = (x_1, x_2, \dots, x_n)^T$  represents the binary value of  $i$ th bit in the potential solution, and  $X_i$  is the  $i$ th value of the random vector  $X$ . Finally, the third step generates new individuals applying the estimated probability distribution, which is evaluated by the fitness function through generations, and these three steps are performed until a convergence criterion is satisfied.

According to the above description, UMDA can be implemented by the following procedure:

- 1) Establish number of generations  $t$ .
- 2) Generate  $n$  individuals randomly initialized.
- 3) Select a subset of individuals  $S$  of  $m \leq n$  according to a selection method.
- 4) Calculate the univariate marginal probabilities  $p_i^s(x_i, t)$  of  $S$ .
- 5) Generate  $n$  new individuals by using  $p(x, t + 1) = \prod_{i=1}^n p_i^s(x_i, t)$ .
- 6) Stop if convergence criterion is satisfied (e.g., stability or number of generations), otherwise, repeat steps (3)-(5).

### 3. Optimization of the Gaussian matched filter

Since the GMF method was introduced by Chaudhuri et al. [8], many researchers have suggested different values for each parameter of the filter. The original work of Chaudhuri

et al. fixed these parameters as  $L = 9$ ,  $T = 13$ ,  $\sigma = 2.0$ , and  $\theta = 15$ , obtaining  $\kappa = 12$  oriented filters. Kang et al. [11], [12], [13] applied the GMF with an angular resolution of  $\theta = 30$  degrees, obtaining 6 different kernels, with an average vessel width of  $\sigma = 1.5$ . Cinsdikici and Aydin [15], applied the original parameters of the GMF method, just modifying the angular resolution to  $\theta = 10$ , obtaining  $\kappa = 18$  oriented filter responses. Al-Rawi et al. [14] proposed extend the range of variables to  $L = \{7, 7.1, \dots, 11\}$ ,  $T = \{2, 2.25, \dots, 10\}$ ,  $\sigma = \{1.5, 1.6, \dots, 3\}$ , and keeping constant  $\theta = 15$ , with  $\kappa = 12$  oriented filters. In this method, the best detection performance is obtained through an exhaustively search over all possible combinations, and using the area ( $A_z$ ) under the receiver operating characteristic (ROC) curve. On the other hand, in order to avoid the exhaustively search to obtain the best GMF performance, Al-Rawi and Karajeh [16] applied the population-based method of Genetic Algorithms (GAs) to select the best  $L$ ,  $T$ , and  $\sigma$  values keeping constant the angular resolution. The detection results acquired by the population-based method are promising to reduce the number of evaluations and obtaining superior performance than the empirically defined methods.

Due to the suitable performance of the population-based method, in the present work the univariate marginal distribution algorithm has been adopted to perform the optimization task in X-ray coronary angiograms. The search space of the GMF variables was defined according to the aforementioned methods and taking into account the analyzed coronary angiograms as  $L = \{8, 9, \dots, 15\}$ ,  $T = \{8, 9, \dots, 15\}$ , and  $\sigma = [1, 6]$  with an step size  $\Delta = 0.001$ . The number of oriented filters was set as  $\kappa = 12$  with angular resolution  $\theta = 15$ . Similar  $A_z$  results were obtained with  $\kappa = 15, 20, 30, 45$ , and 60 filters. For further analysis, only  $\kappa = 12$  was applied. The binary encoding of the UMDA population is set to 18 bits, where 12 bits are used for  $\sigma$  parameter, 3 bits for  $L$  parameter and the remaining 3 bits for  $T$  parameter. The objective function to be maximized is the area  $A_z$  under ROC curve which represents the true-positive fraction (TPF) against false-positive fraction (FPF). TPF is the rate of vessel pixels in the ground-truth image that are correctly detected by the method, also known as sensitivity metric. FPF is defined as the rate of nonvessel pixels that are incorrectly classified as vessel pixels by the computational method. The area under ROC curve is one for perfect detection, and zero otherwise.

### 4. Experimental results

The proposed GMF-UMDA method was tested on a computer with an Intel Core i3, 8GB of RAM, 2.13 GHz processor through Matlab software version 2014b.

The database used in the present work consists of 80 X-ray coronary angiograms of size  $300 \times 300$  pixels of

27 different patients. Each coronary angiogram was hand-labeled by a specialist. Ethics approval was provided by the Cardiology Department of the Mexican Social Security Institute. To assess the detection performance of the computational methods, the database was divided into two subsets of images. The first subset consists of 40 angiograms, which is used as training set for tuning purpose, and the second subset of the remaining 40 angiograms is used as the test set for evaluation of vessel detection methods.

The performance of the proposed method is compared with five GMF of the state-of-the-art for automatic vessel detection. In Figure 2, the detection performance of the methods over a subset of the training angiograms is presented. These ROC curves are obtained by concatenating the filtered angiograms of the training set as only one large image, and applying the set of parameters discussed above. The genetic algorithm used in the work of Al-Rawi and Karajeh [16], is applied with the same set of parameters as: population size = 30, crossover fraction = 0.7, mutation fraction = 0.3, elite = 1, with an heuristic multi-point as crossover method. UMDA is applied with a population size = 30 and selection rate = 0.6. Both population-based methods employ 40 generations as stopping criterion. The comparative analysis suggests that the GMF-UMDA method provide superior performance in vessel pixel detection than the comparative five methods in the 40 training angiograms. To visualize the detection results of the comparative analysis, in Figure 3 the Gaussian filter response of the six methods is introduced. By visual analysis it can be observed that the proposed method presents in general, a higher discrimination of false-positive pixels and higher intensity in vessel pixels than the comparative five methods.

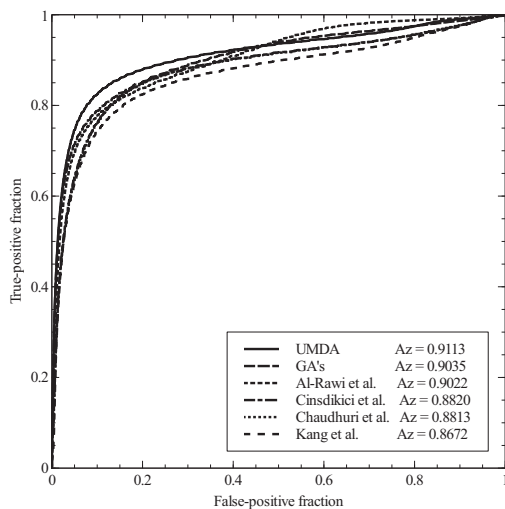


Fig. 2: Comparison of ROC curves for vessel detection with the training set of 40 angiograms, using the proposed method and the comparative methods.

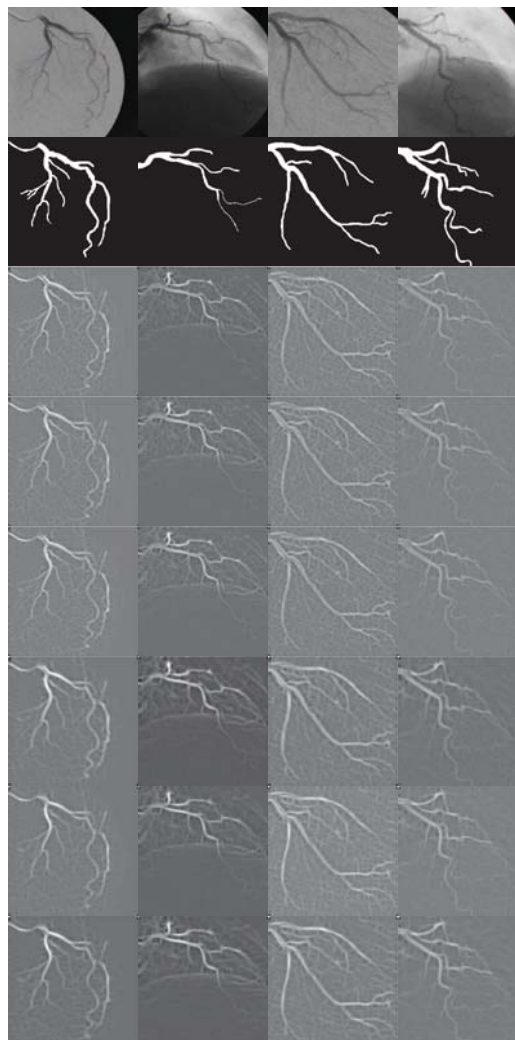


Fig. 3: First row: subset of angiographic images from the training set. Second row: ground-truth of the images in first row. The remaining six rows present the Gaussian filter response of the methods of Kang et al. [11], [12], [13], Chaudhuri et al. [8], Cinsdikici and Aydin [15], Al-Rawi et al. [14], Al-Rawi and Karajeh [16], and proposed GMF-UMDA method, respectively, applied on the angiograms in first row.

From the comparative analysis of  $A_z$  values over the training set of angiograms, the best GMF detection parameters of the two population-based methods were acquired. The genetic algorithm found as best parameters  $L = 14$  pixels,  $T = 13$  pixels, and  $\sigma = 5.370$ . UMDA found as best parameters  $L = 15$  pixels,  $T = 15$  pixels, and  $\sigma = 2.414$ . In Table 1, the detection performance of the analyzed methods is presented. The  $A_z$  rate of the empirical methods of Kang et al., Chaudhuri et al., and Cinsdikici and Aydin shows in general low performance. These three methods use similar values of  $L$ ,  $T$ , and  $\sigma$ , and the main

difference among them is the number of oriented filters, which were defined as  $\kappa = 6, 12, 18$ , respectively. The exhaustively method of Al-Rawi et al. [14], presents superior performance than the empirical methods. This is mainly due to the search space in which the method is optimized. The best GMF parameters obtained by the method were found as  $L = 11$ ,  $T = 8$ , and  $\sigma = 1.9$ . To perform the GA and UMDA strategies, the search space for three variables of the GMF method was extended in comparison with the method of Al-Rawi et al. discussed above. Due to this fact, both evolutionary methods present a superior performance than the exhaustively strategy. The detection results obtained by the proposed method using UMDA presents the highest  $A_z$  rate over the test set of 40 angiograms.

Table 1: Comparison of detection performance with the test set of 40 angiograms, using the proposed and comparative methods.

Method	Area under ROC curve ( $A_z$ )
Proposed GMF-UMDA	<b>0.9343</b>
GMF based on GA [16]	0.9239
Al-Rawi et al. [14]	0.9232
Cinsdikici and Aydin [15]	0.8934
Chaudhuri et al. [8], [9], [10]	0.8918
Kang et al. [11], [12], [13]	0.8843

Moreover, to illustrate the robustness of the UMDA against genetic algorithm for improving the Gaussian matched filter performance, in Table 2 an statistical analysis is shown. This analysis was performed with 30 runs over the test set of angiograms, where the mean and standard deviation values show that UMDA is more stable and with solutions closer to the mean than the best solutions found by the GA strategy.

Table 2: Statistical analysis with 30 runs of the GA and UMDA methods over the test set of 40 angiograms.

Method	Max.	Min.	Mean	Std. Dev	Median
GMF-GA [16]	0.9239	0.8596	0.8960	0.0148	0.9007
GMF-UMDA	<b>0.9343</b>	<b>0.8598</b>	<b>0.9101</b>	<b>0.0121</b>	<b>0.9106</b>

Generally, after applying a detection method, thresholding strategies are then used to classify vessel structures as white pixels and background information as black pixels. Although ROC analysis is used to quantify the vessel detection performance of the methods, also it is used to define a threshold value obtained with the best trade-off between true-positive and false-positive fractions. Figure 4 presents a subset of angiograms from the test set, which is filtered by the GMF method based on GA and UMDA strategies, and then it is thresholded by using the best classification value from ROC analysis. By this thresholding strategy, the segmentation

results obtained from the proposed GMF-UMDA method show an appropriate rate of true-positive pixels with low rate of false-positive pixels compared with the GMF-GA method.

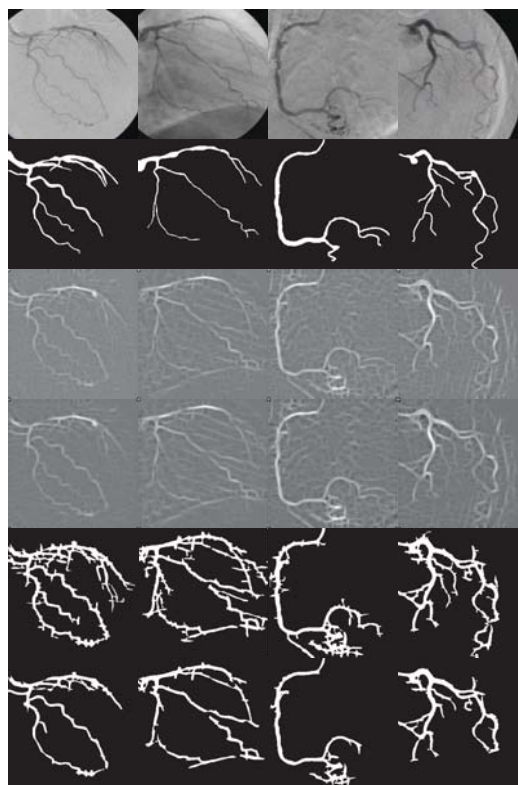


Fig. 4: First row: subset of angiographic images from the test set. Second row: ground-truth of the images in first row. Third row: Gaussian filter response based on GA strategy. Fourth row: Gaussian filter response based on proposed UMDA strategy. The remaining two rows present the thresholded responses of GA and UMDA by ROC analysis respectively.

The detection methods based on Gaussian matched filters of the state-of-the-art discussed above, provide suitable performance based on the area  $A_z$  under ROC curve. However, different comparative analysis reveal that the improved GFM method based on the univariate marginal distribution algorithm is robust and suitable for segmenting vessels in coronary angiograms. The results have also shown that the detection performance obtained from the proposed method is suitable for computer-aided diagnosis considering the coronary arteries hand-labeled by a specialist.

## 5. Conclusion

In this paper, an estimation of distribution algorithm has been applied for improving the Gaussian filter performance in coronary artery detection from X-ray angiograms. The use of UMDA produces a number of potential different Gaussian filters. All the candidate solutions were evaluated

by computing the area under ROC curve, and the best GMF parameters found were set as  $L = 15$  pixels,  $T = 15$  pixels, and  $\sigma = 2.414$ . The performance of the proposed strategy has demonstrated to be more efficient compared with five GMF methods of the state-of-the-art achieving  $A_z = 0.9113$  with a training set of 40 angiograms. According to the experimental results, the proposed GMF method using UMDA as optimization strategy can lead to higher accuracy and efficiency than the comparative methods obtaining  $A_z = 0.9343$  with the test set of 40 angiograms. In addition, considering the images hand-labeled by specialist and the segmentation results obtained from the improved GMF based on UMDA, it can be highly suitable for computer-aided diagnosis in cardiology. As future work, we plan to study continuous optimization methods, in order to apply our method to detect vessel abnormalities and prevent coronary artery stenosis.

## Acknowledgment

This research has been supported by the National Council of Science and Technology of México (Project Cátedras-CONACYT No. 3150-3097). The authors would like to thank the Cardiology Department of the Mexican Social Security Institute, UMAE T1 León, for making this research possible providing us the sources of coronary angiograms and for valuable clinical advice.

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