PAPR Reduction of Coded OFDM Signals

Pedro Bento†‡, Marko Beko§¶, João Nunes†‡, Marco Gomes†‡, Rui Dinis†‡ and Vitor Silva†‡

*Instituto de Telecomunicações (IT), Portugal
†Department of Electrical and Computer Engineering, University of Coimbra, 3030-290 Coimbra, Portugal
‡FCT-UNL, 2829-516 Caparica, Portugal
§Universidade Lusófona de Humanidades e Tecnologias, Lisbon, Portugal
¶UNINOVA – Campus FCT/UNL, Caparica, Portugal

Abstract—A wide range of techniques have been proposed to reduce the Peak-to-Average Power Ratio (PAPR) of Orthogonal Frequency Division Multiplexing (OFDM) signals with different trade-offs between PAPR reduction and Bit Error Rate (BER) performance degradation. Usually these techniques are studied in an uncodded context and over an ideal Additive White Gaussian Noise (AWGN) channel, and a compromise between PAPR and BER degradation is achieved depending on the adopted constellation.

In this paper, we go further and compare several PAPR reduction techniques for Quadrature Amplitude Modulation (QAM) constellations under a typical coded OFDM scenario with time-dispersive channels. Our results show that low complexity techniques as clipping can have a good trade-off between PAPR reduction and BER degradation close to the one of the best techniques analyzed, even in a scenario with multipath time-dispersive channel and for relatively large QAM constellations. Once that is shown that clipping is a good choice for PAPR reduction, it opens a subject in the study of PAPR reduction techniques in order to improve its behavior. We also consider the PAPR reduction as a constrained optimization problem which, although too complex to solve, provides an approximate bound on the achievable PAPR.

Index Terms—OFDM, PAPR, Clipping and Filtering (CF), BER

I. INTRODUCTION

A key feature of a communication system is to make efficient use of available power for data transmission. One of the most spectrally-efficient modulations is the Orthogonal Frequency Division Multiplexing (OFDM) scheme, which is widely used in most broadband wireless systems [1], [2] and currently being recommended for future 5G systems [3], [4]. However, it is widely recognized that one of the main drawbacks of OFDM signals is their high envelope fluctuations and so a high Peak-to-Average Power Ratio (PAPR) which lead to amplification difficulties [1]. A wide variety of techniques was proposed to reduce the envelope fluctuations of OFDM signals. These include multiple signal representations such as with Partial Transmit Sequences (PTS) and Selective Mapping (SLM) techniques [5]–[8], clipping techniques [7]–[11], Tone Reservation (TR) techniques [7], [8], [12]–[14], among many other techniques.

PTS and SLM techniques have the advantage of not leading to performance degradation, since the transmitted signals are not distorted (actually, these techniques might require some side information, and transmitting it might lead to a slight spectral and power efficiency decrease). However, the computational complexity increases significantly when we want to reduce substantially the envelope fluctuations of the transmitted signals. On the other hand, clipping techniques are very simple and flexible, although the signal distortion might lead to significant performance degradation. TR techniques can have limited performance degradation since only the reserved tones are modified (in that case, the performance degradation is essentially due to the power spent on the reserved tones), unless we also modify data subcarriers. They can be particularly interesting for large constellations. However, since only a fraction of the subcarriers is effectively used for data transmission we have some degradation in the spectral efficiency, which is higher if we want to have signals with very low envelope fluctuations. Moreover, it is not always easy to obtain the optimum symbols for the reserved tones.

The achievable PAPR with a given technique is hard to obtain, although we can formulate the PAPR reduction as an optimization problem and employ powerful math tools to solve it [13], [15], for example by minimizing the PAPR conditioned to a given Error Vector Magnitude (EVM) or minimizing the EVM conditioned to a given PAPR target. Although this usually leads to non-practical PAPR reducing methods (we need to solve a complex optimization problem for each transmitted block), this has the advantage of providing some reference on the achievable PAPR.

The study of PAPR reducing techniques that lead to performance degradation is usually performed for an uncoded transmission in an ideal Additive White Gaussian Noise (AWGN) channel, which makes sense since the PAPR is essentially a transmitter problem, regardless of the channel. In fact, for small constellations, e.g., Binary Phase Shift Keying (BPSK) or Quadrature Phase Shift Keying (QPSK), the signal distortion usually is not high enough to lead to significant Bit Error Rate (BER) degradation. However, when we consider larger constellations like 16-Quadrature Amplitude Modulation (QAM) or 64-QAM this is no longer true. In that case, we should consider both the channel effects and the adopted channel coding scheme.

Some previous works [16], [17] have considered coded

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OFDM however, they only have considered a single PAPR reducing technique over an ideal AWGN channel. Therefore, in this paper we compare several PAPR reducing techniques, when multipath time-dispersive channels are considered and appropriate channel coding schemes are employed. We focus our study in the most well-know PAPR reducing techniques, such as SLM, PTS and Clipping and Filtering (CF), as well as the constrained optimized PAPR reduction as to obtain a reference on the achievable PAPR.

II. PAPR REDUCTION OVERVIEW

OFDM signals have high envelope fluctuations which brings some difficulties in amplification, therefore it is necessary to keep the envelope fluctuations of the signal below a certain level. In order to do this, a measure of the envelope fluctuations has to be used and the most widely used is PAPR.

The PAPR is defined as the ratio of the peak power of the signal to its average power [8]. Mathematically, the PAPR of the $m^{th}$ OFDM symbol is written in time domain as:

$$PAPR(x_m[n]) = \frac{\max_{0 \leq n \leq \ell N-1} |x_m[n]|^2}{E[|x_m[n]|^2]} ,$$

where $E[.]$ is the expectation operator, $N$ the number of subcarriers and $\ell$ is the oversampling factor that it must be at least 4 in order to ensure that the difference between continuous time and discrete time PAPR is negligible as it is shown in [18]. Then if PAPR is limited to a certain level, envelope fluctuations will be limited too and it is possible to achieve better results in amplification. Therefore lots of research has been made to reduce PAPR and in this section we will briefly present the concepts of some PAPR reduction techniques: CF, PTS, SLM, TR and constrained optimized PAPR.

Let $X \in \mathbb{C}^N$ be an original OFDM frequency-domain symbol and $\hat{X} \in \mathbb{C}^N$ the PAPR optimized frequency-domain symbol using $N$ subcarriers. The original, $x$, and the optimized, $\hat{x}$, OFDM time-domain symbols are obtained by Inverse Fast Fourier Transform (IFFT) with $\ell$-times oversampling, i.e.,

$$x = \text{IFFT}_\ell(N)(X) = AX$$

$$\hat{x} = \text{IFFT}_\ell(N)(\hat{X}) = A\hat{X},$$

where the matrix $A \in \mathbb{C}^{\ell N \times N}$ is the first $N$ columns of the corresponding Inverse Discrete Fourier Transform (IDFT) matrix.

The OFDM subcarriers are usually divided into three disjoint sets: data subcarriers, free subcarriers and pilot subcarriers, with cardinalities $d_{sub}$, $f_{sub}$ and $p_{sub}$, respectively, so that $d_{sub} + f_{sub} + p_{sub} = N$, where $N$ is the total number of subcarriers. For simplicity, pilot subcarriers will not be considered here, although the results can be easily generalized to systems with pilot subcarriers. In addition, in order to identify on a symbol $X$ the data subcarriers, it will be use a diagonal matrix $S \in \mathbb{R}^{N \times N}$ with $S_{kk} = 1$ when the $k^{th}$ subcarrier is reserved for data transmission and $S_{kk} = 0$ otherwise, i.e. data subcarriers can be obtained by the product $SX$.

A. Clipping and Filtering

CF [7]–[11] is the simplest way to reduce PAPR. This technique clips every sample of the signal above a certain defined level. Therefore, the clipped version of $x$ can be expressed as

$$\tilde{x}_k = \begin{cases} x_k & \text{if } |x_k| < A_{CL} \\ x_k & \text{otherwise} \end{cases} ,$$

where $A_{CL}$ is the amplitude of clipping level. However, the Clipping Ratio (CR) defined as the amplitude of clipping level, $A_{CL}$, normalized by the Root Mean Square (RMS) value of OFDM signal, $\sigma = \sqrt{\|x\|^2}$ is more suitable to use since it adapts the clipping level from symbol to symbol.

Although CF is a simple technique, it causes signal in-band distortion which results in a degradation in BER performance and it also causes out-of-band radiation. In order to reduce the out-of-band radiation, filtering is used but, unfortunately, this leads to a peak regrowth and the obtained signal may exceed the desired clipping level [8]. Therefore, an iterative process was proposed in [10], [19], [20], designed by Repeated Clipping and Filtering (RCF), that may require a few number of iterations that should be done in order to obtain the desired PAPR reduction. However, this process increases the computational complexity.

B. Selective Mapping

SLM technique [5], [7], [8] is a symbol scrambling technique based on the fact that an OFDM symbol can be scrambled by a certain number of different sequences and, then, the symbol with the lowest PAPR is chosen to transmit. In more detail, the transmitter generates $U$ phase sequences (vectors) expressed as

$$P^u = [p_0^u, p_1^u, ..., p_{N-1}^u]^T , u = 1, 2, ..., U ,$$

with the same length of the original OFDM symbol $X$, where each element of the phase vector, $p_k^u$, $k = 0, 1, 2, ..., N - 1$, is randomly selected from a finite set of phase factors, e.g. $\{-1, 1, -j, j\}$. Each of these vectors, $P^u$, is then point-wise multiplied by $X$ and the resultant symbol with the lowest PAPR, expressed as

$$\tilde{X} = [X_0 p_0^u, X_1 p_1^u, ..., X_{N-1} p_{N-1}^u]^T ,$$

is chosen. To ensure that the unmodified symbol is in the set of choices, $P^1$ is the all-one vector.

When SLM is performed, the $U$ phase sequences are stored at both the transmitter and the receiver. To perfectly recover the signal at the receiver, the transmitter must send side information to the receiver telling which phase sequence is used. If side information is incorrectly detected, the whole data information will be lost. Although side information is so important, its use leads to losses in spectral and power efficiency reducing the data transmission rate, mainly, because it needs a strong protection [7]. However, in [21] it is proposed a SLM technique without explicit side information.

For each OFDM symbol, it is necessary to execute $U$ IFFT operations and $\lceil \log_2 U \rceil$ bits must be passed as side
information (where \([y]\) denotes the lowest integer greater than \(y\)). Therefore, this technique may not be feasible, due to its computational complexity, which increases when \(N\) is large, and mainly, when \(U\) is increased in order to achieve a substantial PAPR reduction.

\(\text{C. Partial Transmit Sequence}\)

PTS technique [5]–[8] as SLM is a scrambling technique. The difference between them is that the first only scrambles groups of subcarriers, while the latter scrambles independently all subcarriers [1]. Therefore, SLM produces signals that are asymptotically independent, while signals generated by PTS are interdependent [22]. With this property, PTS can avoid some of the complexity of the several full IFFT operations, which results in an advantage over SLM [5].

PTS technique partitions each OFDM symbol into \(V\) disjoint subblocks with equal size as follows:

\[
X_v = [X_{v,1}, X_{v,2}, \ldots, X_{v,N-1}]^T, v = 1, \ldots, V,
\]

(5)

and

\[
X = \sum_{v=1}^{V} X_v.
\]

(6)

This partition can be one of three kinds: adjacent, interleaved, and pseudo-random. Among these, the latter has been found the one that provides the best performance [23]. In either partition the subcarriers in each subblock are independently rotated by a phase factor, \(b^v\), in order to minimize the PAPR of the combined signal. The phase factor is selected from a finite set of phase factors, e.g. \([-1,1,-j,j]\), with length \(W\). Hence, the optimized time-domain OFDM symbol is

\[
\bar{x} = \sum_{v=1}^{V} b^v X_v.
\]

(7)

The PAPR reduction given by this technique depends on the number of subblocks, \(V\), the number of allowed phase factors, \(W\), and the subblock partitioning. However, the search complexity of this technique increases with the number of subblocks, and most importantly, it increases exponentially with the number of phase factors. Therefore, their selection is usually limited to a set with a finite number of elements [8]. Furthermore, this technique has the inconvenient of requiring the transmission of side information to the receiver telling which are the phase factors used for a correct decoding of the transmitted symbols, which reduces spectral and power efficiency as in the case of SLM. Thus, in PTS is required \(V\) IFFT operations and \(\log_2 W(V-1)\) bits of side information for each OFDM symbol.

\(\text{D. Tone Reservation}\)

In TR technique [7], [8], [12]–[14], some of the subcarriers, called frees, are not used to transmit information data. Once these subcarriers are free, they can take values that minimize PAPR, without introducing distortion in data subcarriers since all subcarriers are orthogonal. In some applications, there are subcarriers with SNR too low for sending information, therefore, they can be set as free subcarriers and used for PAPR reduction [8].

Let \(C\) denote the frequency-domain vector that contains the information of free subcarriers and that will be added to data vector, \(X\), in order to reduce the PAPR. For definition of the TR technique, \(X\) and \(C\) lie in disjoint frequency subspaces, resulting in the following signal

\[
\bar{x} = \text{IFFT} \{X + C\}.
\]

(8)

The free subcarrier values are found solving a convex optimization problem (see Section II-E) and their locations are established \(a\) priori\) between the transmitter and the receiver.

As data subcarriers don’t suffer distortion, this technique has no BER degradation, however this performance must have a penalty added once the free subcarriers require additional power and reduce the spectral efficiency. This penalty is given, in decibels, by

\[
\mu = 10 \log_{10} \left( \frac{||S\bar{x}||^2}{||X||^2} \right),
\]

(9)

where \(||S\bar{x}||^2\) is the power of data subcarriers and \(||X||^2\) is the total power of the OFDM symbol.

\(\text{E. PAPR Reduction as Constrained Optimization Problem}\)

Convex optimization has recently emerged as an efficient tool for reducing the PAPR of OFDM signals [13], [15]. This can be explained partially by the fact that convex optimization methods can efficiently compute global solutions to large scale problems in polynomial time. Furthermore, convex optimization approaches show advantages over the classical RCF approach [10]; see [13], [15] for more details.

An efficient way to reduce the PAPR is by distorting the OFDM constellation [24], [25]. The level of distortion is measured by the EVM and should be kept at a minimum, since a larger EVM value leads to a BER performance degradation. A single OFDM symbol’s EVM is mathematically defined as,

\[
\text{EVM} = \frac{||S(\bar{X} - X)||^2}{||SX||^2}.
\]

(10)

The PAPR can be further reduced by assigning a portion of energy to the free subcarriers [24], [25], i.e. a TR technique is performed. In this case, it must be taken into account the FCPO that measures the value of free subcarriers power and it is given by,

\[
\text{FCPO} = \frac{||(I_N - S)\bar{X}||^2}{||SX||^2}.
\]

(11)

The FCPO should be kept small since it measures the fraction of power "wasted" in the free subcarriers, which are not used to carry information. In this technique, we will focus on minimizing EVM for given PAPR and FCPO thresholds, i.e.,

\[
\text{minimize } \text{EVM} \quad \bar{X} \in \mathbb{C}^N
\]

(12)
subject to
\[ \text{PAPR} \leq g_1 \quad \text{FCPO} \leq g_2, \]
where \( g_1 \) and \( g_2 \) are PAPR and FCPO thresholds, respectively. It is straightforward to see that the optimization problem (12)–(14) is equivalent to
\[ \begin{align*}
\text{minimize} & \quad \| S (\tilde{X} - X) \|^2 \\
\text{subject to} & \quad \tilde{X}^H (M_i - T_{g_i}) \tilde{X} \leq 0, \quad i = 1, \ldots, \ell_N, \\
& \quad \tilde{X}^H (I_N - S_{g_2}) \tilde{X} \leq 0,
\end{align*} \]
subject to
\[ M_i = \ell N A^H e_i e_i^T A, \quad T_{g_i} = g_1 A^H A, \quad S_{g_2} = (g_2 + 1) S \]
where \( M_i = \ell N A^H e_i e_i^T A, T_{g_i} = g_1 A^H A, S_{g_2} = (g_2 + 1) S \)
and \( e_i \) represents the \( i^{th} \) column of the identity matrix \( I_{\ell N} \).

The EVM optimization framework (15)–(17) results in a nonconvex optimization problem since the matrices \( I_N - S_{g_2} \)
and \( (M_i - T_{g_i}) \), for \( i = 1, \ldots, \ell_N \), are indefinite; in other words, all the constraints are nonconvex. As most nonconvex problems, our problem is NP-hard and, thus, difficult to solve.

Alternatively, we can introduce an optional constraint that will keep the EVM below some preset threshold. This corresponds to a more challenging and realistic scenario where PAPR, FCPO and EVM are simultaneously constrained. In that case, the EVM optimization can be formulated as
\[ \begin{align*}
\text{minimize} & \quad \epsilon \\
\text{subject to} & \quad \epsilon \in \mathbb{R}, \quad \tilde{X} \in \mathbb{C}^N \\
& \quad \text{EVM} \leq \epsilon \text{ EVM}_{\text{max}}, \\
& \quad (13), \quad (14), \quad \epsilon \leq 1,
\end{align*} \]
where \( \text{EVM}_{\text{max}} \) is the maximum allowed EVM. Note that the optimization problem (12)–(14) is different from the one in (18)–(20), since the search space for \( \tilde{X} \) in the former is larger than in the latter.

Although addressing the PAPR reduction of OFDM symbols as an optimization problem is a non-practical method, this has the advantage of providing some reference on the achievable PAPR, when the problem is properly formulated. Results obtained from formulation (18)–(20) will be used as comparison reference of the CF, SLM and PTS techniques previously described.

### III. PERFORMANCE EVALUATION

In this section we perform a comparison of several PAPR reduction techniques on a coded OFDM transmission scenario when time-dispersive channels are considered. To this end, the best of the author’s knowledge was not done before. We considered an OFDM signal with \( N = 64 \) subcarriers and oversampling factor \( \ell = 4 \) under 16-QAM and 64-QAM constellation using Gray mapping rule. OFDM signal pass through different channels: AWGN and a time-dispersive channel with 32 paths. The coding scheme was used was a (1664, 840) LDPC code (coding rate near 1/2), and the bits are interleaved inside each codeword. At the receiver, perfect channel estimation is assumed.

Results presented concern to the following techniques: CF, SLM, PTS and OPT, where OPT denotes the constrained optimized PAPR reduction technique. OPT was developed in two different ways, with free subcarriers, i.e. TR (OPT-TR), and without them (OPT). The analysis takes into account the following parameters: PAPR reduction, BER and the trade-off between them.

The CF technique was performed with 1 and 10 iterations, with the latter being denoted from now on as RCF, and the acronym CF referring to a single iteration. The CR was chosen to guarantee a good performance trade-off between PAPR reduction and BER, and was set to 1.4 and 1.7 (in linear units) for 16-QAM and 64-QAM, respectively.

In the case of SLM technique we multiplied each OFDM symbol by 32 different phase factor vectors with length \( N \) and values randomly chosen from the set \( \{-1, 1\} \) for 16-QAM and 4.5dB for 64-QAM and they will be used as reference, while the remaining techniques were tuned taking into account the trade-off between PAPR reduction and BER. At a clipping probability of \( 10^{-5} \), all techniques perform a considerable reduction in PAPR, however, as we will see in Figs. 3a and 3b, this results, in most cases, in a decrease in BER performance. Although this decrease is unbearable in OFDM uncoded as we can see in Fig. 2a, it can be bearable, even in a dispersive channel with multipath, if we use OFDM coded as we see in Fig. 2b.

In fact, by analyzing Fig. 2b we can observe that all the PAPR reduction techniques perform close enough from the original OFDM transmission with unconstrained PAPR, i.e. no more than 1.2dB and 2dB for 16-QAM and 64-QAM respectively, thus with a power penalty much lower than the corresponding power gain achieved on restricting the PAPR. This shows that the use of an adequate coding scheme leads a great performance improvement even in a dispersive channel. It also shows that all techniques can have a similar performance when coding is used.

Thus, although PAPR reduction can lead to a decrease in BER performance which is a undesirable effect, this can be accepted in cases where this decrease is lower than the improvement in PAPR. Therefore, a BER curve shifted by the required amplifier’s backoff given the PAPR level at a certain clipping probability is more informative, showing the trade-off between PAPR reduction and BER performance.

Figs. 3a and 3b illustrate this trade-off for a time-dispersive channel, which is the most suitable real world situation and, \( E_{\text{peak}} = E_b + \text{PAPR(dB)} \) with PAPR level chose to a clipping probability of \( 10^{-3} \) (see Figs. 1a and 1b).
Analyzing Figs. 3a and 3b we can observe that techniques with higher complexity based on constrained optimization have the best results. However, in opposite direction, RCF technique has also good performance too with a very low complexity. Considering that we want to perform PAPR reduction in real-time, RCF with coded OFDM and the right parameters can have considerable good results even in a dispersive channel and, as so, it is a good candidate technique. The use in real-time applications rises another question for RCF, the latency of doing such iterative method. For this reason CF had been tested too and the results are very satisfactory mainly in 64-QAM modulation.

IV. Conclusion

In this paper, we go further than previous studies in PAPR reduction techniques and compare them for Quadrature Amplitude Modulation (QAM) constellations under a typical coded OFDM scenario with time-dispersive channels and not only
for an ideal AWGN channel. Our performance results show that when OFDM is performed with an appropriate coding scheme, the most suitable technique for PAPR reduction, considering the implementation complexity, the response in a real-time situation and the trade-off between PAPR reduction and BER degradation, is clipping and filtering, with only 1 to 2.5 dB from the reference, even for relatively large QAM constellations.

Therefore, this work opens a subject in the study of PAPR reduction techniques. Once that is shown that CF is a good choice for PAPR reduction, it should be further studied in the future. To improve the performance of CF, coding and equalization schemes to tackle the distortion of clipping, and not only the channel effects, must be studied. If the effect of distortion of clipping can be reduced in the received signal, a better BER performance could be achieved and CF could have a trade-off between PAPR reduction and BER degradation even closer to the other techniques presented here. Also, the number the iterations of RCF may be studied in order to find an optimum value.

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