

# Heat Exchanger Network (HENET): A parallel GA/SA algorithm

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**Abstract** - Heat exchanger network (HEN) synthesis has been a well-studied subject over the latest decades. Many studies and methodologies were proposed to make possible the energy recovery. Based on simulated annealing and genetic algorithm, this paper presents an efficient simultaneous synthesis method that provides the optimal networks in a two-level procedure. Genetic algorithm is used by an evolutionary algorithm to manage HEN topology and simulated annealing is used to manage heat load distribution among exchangers. This two-level method is applied to solve one benchmark that includes 28 instances with different dimensions, from 3 to 39 streams. For every instance, this benchmark includes the best result obtained with a state-of-the-art algorithm from the literature, which has been useful in this work to compare our proposed hybrid algorithm with the best existing approaches. The results of this study show that, in some situations, the hybrid approach is able to derive networks that are more economical than those from the known solutions in the literature.

**Keywords:** heat exchanger network synthesis; genetic algorithm; simulated annealing; hybridization; optimization; benchmark.

## 1 Introduction

The problem of synthesizing optimal network configurations has received considerable attention in the literature in the last few decades, so as for its significant impact on energy and cost saving in industry. The objective is to design a heat exchanger network that minimizes total annualized cost (TAC) as the sum of annualized investment cost and annual operating cost with the given sets of streams and utilities.

The complexity of the HENs has a combinatorial nature. For a fixed number of streams, there are a wide range of possibilities of combinations among exchangers. However, the number of possible HEN configurations that contains the minimum utilities consumption is smaller than the entire number of configurations. This restriction ensures finding a HEN with the minimum utilities to a presented minimum temperature of approach ( $\Delta T_{min}$ ).

For addressing the HEN synthesis problem most of methods can be grouped into three different lines, which are thermodynamic based approaches, mathematical

programming methods and metaheuristic optimization methods.

On one hand, thermodynamic approaches based on the pinch analysis by Linnhoff and Flower [1] and Linnhoff and Hindmarsh [2] are most commonly used. Pinch analysis method is flexible and provides an overview of the problem. It creates the problem into a sub-problems based on the concept of pinch and with various targets, which are then solved sequentially. The targets include minimum approach temperatures,  $\Delta T_{min}$ . They will illustrate for the cumulative cost of the heat exchanger network, and certain way they can define the optimal level of  $\Delta T_{min}$  or can be used as an instrument of the optimization progresses. A review on this method was collected by Shenoy [3].

On the other hand, mathematical programming methods could solve the problem with and without decomposition. With decomposition, commonly called sequential approach, the reduction of computational complexity is found. This method usually have mixed integer linear programming (MILP) or non-linear programming (NLP) formulation. The most well-known works are the transshipment model Papoulias and Grossmann [4], the explanation of Biegler [5], and superstructure model Floudas [6].

Without decomposition, frequently called simultaneous approaches, near global optimal solutions are found by mixed integer non-linear programming (MINLP) formulations. Non-convex terms as the LMTD of heat exchangers, the energy balances for mixers and splitters and the non-linear area cost function make the solution of these models much more difficult. For this reason, some simplifications in the problem must be done in order to reduce its complexity. For instance, the stage-wise network superstructure proposed by Yee [7] make the assumption of isothermal mixing for streams. Chen approximation of LMTD [8] term is usually used to avoid numerical difficulties, when the approach temperatures of both sides of the exchanger are equal. Other well-known approximation was made by Paterson [9]. Ciric et al [10] collected a review of mathematical programming method.

Finally the third line, metaheuristic optimization methods, such as Simulated Annealing (SA), have been applied by Athier et al [11], Tabu Search (TS) by Lin and miller [12] and Genetic Algorithm (GA) by Lewin et al. [13], another contribution of their work is to introduce the concept

of 'HEN level' for structure representation which was then used in a two-level synthesis method of HENs combining harmony search (HS) and sequential quadratic programming (SQP) [14]. Differential evolution (DE) algorithm for synthesis of HENs have been proposed by Yerramsetty and Murty [15] and also utilized the structure representation similar to 'HEN level'. A particle swarm optimization (PSO) method and a GA/PSO algorithm have been presented by Silva et al. [16] and Huo et al. [17] respectively. These techniques are able to solve complex problems without being limited by non-linearities, non-convexities and discontinuities of the models. All of them are robust and can find near optimal solution by means of searching space within a reasonable time but they still have the difficulty in converging to the precise global optimal solution in the feasible region.

This paper proposes a benchmark that has 28 problems with the same characteristics. To test this benchmark, some tests with hybrid algorithm are conducted. In the proposed hybrid algorithm, stochastic methods are combined in a two-level approach to take the advantage of each method and compensate deficiencies of individual methods. This hybrid algorithm uses the GA and SA algorithm; GA is used to create for network structural, while the fitness of each structure is calculated by SA.

The remaining parts of this paper are organized as follow. In section 2, the mathematical formulation of HENS is presented. Section 3 describes the proposed two-level synthesis method in detail. Section 4 briefly describes available benchmark instances and also results obtained followed by conclusions in Section 5.

## 2 Mathematical Formulation

The HENs problem was first rigorously defined by Masso and Rudd [18] and its objective is to find a sequence of combining exchangers in pairs of streams, getting that the network either optimal in relation to the global cost.

Section 2 presents the mathematical formulation solved by the two-level simultaneous synthesis method in this paper.

### 2.1 HEN Structure Representation

A structure representation based on superstructure proposed by Yee [7] is presented, it has stages where only one exchange is allowed between a specific hot stream and a specific cold stream, however this stage wise superstructure allows a stream to split into several substreams (or branches) at each stage to exchange heat with other streams of the opposite kind. The utilities streams are placed at the ends of the sequence of stages.

A stage-superstructure with branch involving two hot and two cold streams along with cold and hot utilities is shown in Fig. 1.

For single heat exchanger with counter-current flow patterns, the feasibility of heat exchange temperature difference is rigorously required as shown in Fig. 2.

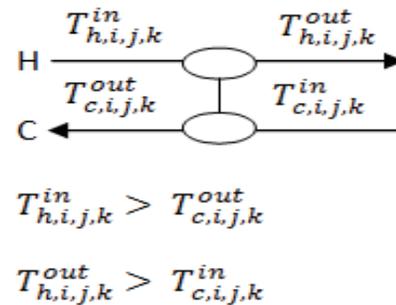


Figure 2: Modification of HEN representation

## 3 Method

A two-level method is proposed to optimize the binary and continuous variables, one is based on Genetic Algorithm (GA) [19] and other is settle on Simulated Annealing (SA) [20]. Both of them are in principle random methods generally used to solve large scale combinatorial optimization problems.

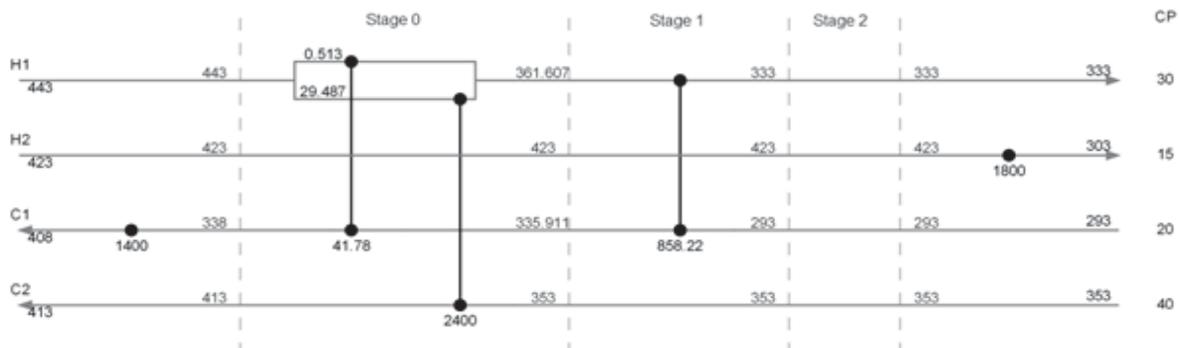


Figure 1: Stage-wise superstructure

GA is applied for binary variables optimization to search optimal network structure since it has been proven to be a powerful discrete variables optimization algorithm of combinatorial problem and SA algorithm is applied to the continuous variables optimization as well and designed to converge to the optimal heat distribution of each candidate structure with low computational effort [21, 22]. Fig. 3. Illustrates the basic concept of the two-level method.

SA can find good quality solutions in a neighborhood, but most it will get trapped in local minimum and takes longer to scope, while GA rapidly discovers the search space, but has difficulty in finding the exact minimum. For this reason a parallel GA/SA hybrid has been adopted in the present work where in the upper level, a series of candidate structures will be generated by specified strategy by GA and then sent to the lower level SA for solving minimum TAC until converging to an optimal HEN solution.

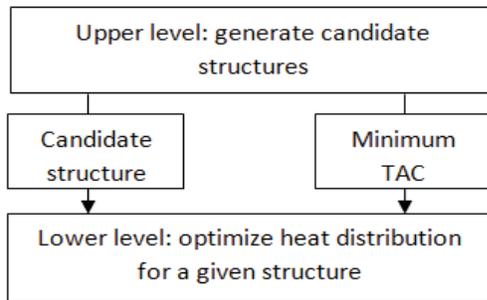


Figure 3: Basic concept of the two-level method

### 3.1 Genetic algorithm

Genetic algorithm is settling on the natural selection and genetic mutation in biological world. The genetic algorithm consists of three main operators: selection, crossover, and mutation. The individual with a better value of fitness (lower value of the objective function) has a greater chance to be selected to produce its offspring by crossover, or to return directly to the next generation. By using a crossover operation, two selected parents are combined to form their offspring. A mutation operation will introduce new genes into the population to avoid the evolution converging into a local optimum.

### 3.2 Simulated Annealing algorithm

Simulated annealing is organized according to the Monte Carlo simulation technique developed by Metropolis et al. and the theory of Markov [23] chains provides mathematical properties about its asymptotic convergence. The simulated annealing algorithm was firstly introduced to solve large combinatorial optimization problems by Kirkpatrick et al. [20] who drew an analogy between the annealing of a solid and the optimization of a complex system. For it accepts and rejects 'moves' generated randomly on the basis of a probability related to an 'annealing temperature', SA can accept uphill moves and consequently escape from a local optimum. Obviously, the accepted proportion of uphill moves increases with the annealing temperature  $T$ . Until a specified stop

criterion is satisfied, the annealing temperature is periodically reduced according to the annealing schedule. The higher the temperature, the larger the possibility of having accepted random moves. Therefore, the ability of this algorithm to escape from the region of poor local optima can be controlled by adjusting the annealing schedule.

### 3.3 Structure of the two-level method

The global optimization procedure can be summed up as follows. In the upper level, the candidate structure combined by 0 or 1 binary variables are evaluated based on the minimum TACs solved in the lower level. The current structure will be gradually improved by the genetic and simulated annealing mechanisms and finally converge to an optimal structure. The overall algorithm is illustrated in Fig. 4.

The optimization is started with a randomly initial population of structures that is produced by the GA. The structure search should be performed in a sufficiently feasible space to guarantee that the optimal structure is involved. A topological structure will be generated randomly as the first current structure, where  $z_{i,j,k} = 1$  denotes that a heat exchanger is matched between  $i$ th hot stream and  $j$ th cold stream at the  $k$ th stage. Each exchanger has an associated value,  $q_{i,j,k}$ , which represents the exchanged heat load in this exchange. In the beginning, this value is set to 0 and the whole heat load is carried out by the utilities. SA is the responsible of the optimization of these heat loads as long as the total annual cost is concerned. Notice that SA is not allowed to change  $z$  structure; SA only handles the values of heat loads related to each exchange for calculating TAC.

After the optimal heat distribution for a structure has been obtained by SA, GA handles it within a population in order to obtain the best structure for applying operators by roulette-wheel procedure and obtain offspring.

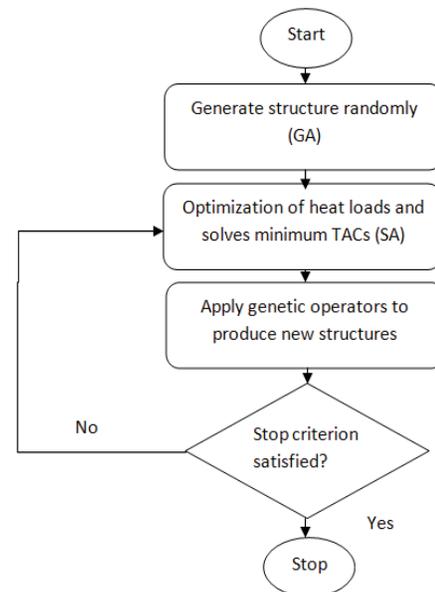


Figure 4: Flow diagram describing the structure of the two-level method

GA presents two probability model establish Monte Carlo [23] sampling respectively to create new structures (offspring), this is add or delete heat exchangers as random moves for a given structure. These two moves are equally probable to be performed when a candidate structure is generated. Genetics operations are taken into account in accordance with TAC. The better the TAC, the bigger the likelihood to be chosen to create the offspring. Roulette-wheel is the responsible of choosing the adequate structure that the offspring will have, once the parents are selected; only one genetic operation is applied. This operation is chosen randomly. There are two different possible operators, one-point crossover or mutation.

(i) One-point crossover to search the bigger solution space possible, combination of two structures selected by Roulette-Wheel is repeated to generate new offspring. Two parents, p1, p2 are combined by simple crossover to create two offspring, s1, s2, a half of the structure will go to one offspring and the other half to the other.

$$S1 = 0.5 * p1 + 0.5 * p2 \quad (28)$$

$$S2 = 0.5 * p2 + 0.5 * p1 \quad (29)$$

(ii) In addition to maintain the diversity of the population, the main purpose of this operator is to help prevent information loss in the evolution progress. Mutation of the parents is made as follow. Randomly an exchange of each parent is selected, and then these values are switched as follows.

$$1 \rightarrow 0$$

$$0 \rightarrow 1$$

Once the new structure is obtained, GA adds it to the population. Then, roulette-wheel updates the likelihood to choose parents. So, one child could be chosen in the next steps. The iteration ends when the second half of the structures is completely formed by offspring.

## 4 Cases and discussion

In order to verify the performance of our algorithm, the computation is conducted on a benchmark. The benchmark is composed of instances taken from the literature. The instances are organized according to the number of streams; they are sorted in ascending order.

### 4.1 Results

In this section, the instances of the benchmark are solved to test the performance of the presented two-level method. The results are compared to the best result obtained with a state-of-the-art algorithm from the literature. These results are summarized in Table 1.

The algorithm proposed is able to achieve five best results as done by others in the literature for HTN2, HTN5, HTN9, HTN21 and HTN22 problems.

Table 1: Summary results.

Method	Annual cost(\$/year)	Method	Annual cost(\$/year)	Method	Annual cost(\$/year)
HNT1		HNT11		HNT21	
Bjoerk2002	<b>76350</b>	Silva2010	<b>1624768</b>	Brandt2011	6110902
Huang2012	76327				
Huang2013	76327				
This work	76742.8	This work	1885667.4	This work	<b>5520273.5</b>
HNT2		HNT12		HNT22	
Bjoerk2002	52429	Bjoerk2002	<b>61295</b>	Agarwal2008	43728
This work	<b>47901.2</b>	This work	64891.4	Huang2013	43359
HNT3		HNT13		This work	<b>36479</b>
Isafiade2008	97211	Isafiade2008	1150460	HNT23	
Ponce-Ortega2010	<b>97079</b>	Ponce-Ortega2010	1121175	Petterson2008	<b>43331</b>
		Huang2013	<b>1115868</b>	Escobar2013	44081.4
This work	184798.7	This work	2152571.5	This work	43949.7
HNT4		HNT14		HNT24	
Bjoerk2002	<b>411746</b>	Bjoerk2002	96001	Wei2004	<b>43048</b>
		Huang2012	95643		
		Huang2013	<b>94742</b>		
		Huang2014	<b>94742</b>		
This work	434750.4	This work	126718.8	This work	44267.9
HNT5		HNT15		HNT25	
Escobar2013	470732.1	Petterson2008	<b>80962</b>	Khorasany2009	<b>5662366</b>
		Zamora1998	83400	Huang2012	5737274
				Huang2014	5733679
				Yerramsetty2008	5666756

Method	Annual cost(\$/year)	Method	Annual cost(\$/year)	Method	Annual cost(\$/year)
This work HNT6	<b>280451.4</b>	This work HNT16	1835556.8	This work HNT26	5673129.7
Isafiade2008	<b>311300</b>	Bjoerk2002	139083	Bjork2005	1530063.55
		Huang2012	128169	Escobar2013	<b>1524678.3</b>
		Huang2013	<b>123398</b>		
This work HNT7	570378.3	This work HNT17	288465.5	This work HNT27	1852969.9
Khorasany2009	<b>11895</b>	Isafiade2008	595100	Brandt2011	<b>6110902</b>
		Fieg2009	571698		
		Wei2004	571585		
		Toffolo2009	<b>570900</b>		
This work HNT8	14324.7	This work HNT18	593102.7	This work HNT28	71211663.5
Pettersson2008	84066	Khorasany2009	572476	Li2014	1805971
Zamora1998	87328	Huang2012	571657	Escobar2013	<b>1591070.1</b>
Yerramsetty2008	85972	Huang2013	<b>570362</b>	Huang2014	1937377
Toffolo2009	<b>82363</b>	Huang2014	612362		
Pariyani2006	85307				
This work HNT9	108245	This work HNT19	590016.4	This work	2620949.3
Ponce-Ortega2010	385346	Isafiade2008	<b>168700</b>		
This work HNT10	<b>376176.5</b>	This work HNT20	174307.7		
Ravagnani2005	117069.34	Chen2007	109765		
Chen2007	<b>109765</b>	Wei2004	<b>99524</b>		
		Huang2013	105403		
This work	154578	This work	109263.6		

## 5 Conclusions

A hybrid methodology for design and optimization of heat exchanger networks is presented. The HENs problem is solved by a two-level procedure, first GA is used to construct a HEN structure and then SA is employed to find optimum exchanger heat. Throughout the evolutionary process by the GA the structures of the individuals alter continuously. This is due to the genetic operations of structure crossover and mutation, respectively. In the lower level, the heat distribution of each candidate structure is optimized for minimum TAC by simulated annealing algorithm. The synthesis performance of this two-level method has been demonstrated using a benchmark. The assessed results indicate that the proposed algorithm is competitive with other forms of optimization algorithms. Combinations of heuristic based optimization methods for the efficient synthesis of HEN seem therefore very promising.

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