Robust stability criteria for Takagi-Sugeno fuzzy systems with sampled-data

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Abstract— *This paper presents the robust stability of Takagi-Sugeno(T-S) fuzzy systems with sampled-data. By constructing an augmented modified Lyapunov-Krasovskii functional, which includes three triple integrals, some less conservative results are obtained compared with the existing results. Numerical examples are presented to demonstrate the improvement of the proposed method.*

Keywords: Takagi-Sugeno fuzzy systems, time-varying delay, Lyapunov-Krasovskii functional, Linear Matrix Inequality

1. Introduction

The main advantage of T-S fuzzy model is that it can combine the flexibility of fuzzy logic theory and rigorous mathematical theory of linear system into a unified framework to approximate complex nonlinear systems [1-3]. On the other hand, sampled-data often appears in many dynamical systems such as biological systems, neural networks, networked control systems and so on. For a digital stabilization, sampled-data control design method can reduce the amount of stabilization information and increases the efficiency of bandwidth usage. Hence, the stability of T-S fuzzy systems with sampled-data has been studied by many researchers [3]. By construction of a modified augmented Lyapunov-Krasovskii functional, which includes three triple integrals, an improved stability criterion for guaranteeing the asymptotically stable is derived by using Wirtinger-based integral inequality [5], reciprocally convex approach [4], new delay-partitioning method. It should be pointed out that different with delay-partitioning method used in [3], we only divide the time interval into some subintervals, but not considered the cases that time-varying delay belongs to different subinterval, respectively. Finally, Numerical example is given to demonstrate the effectiveness of the proposed method.

2. Problem statement and preliminaries

Consider the following nonlinear system which can be modeled as TS fuzzy model with sampled-data:
Rule i: If $\theta_1(t)$ is M_{i1} and $\dots$ and If $\theta_n(t)$ is M_{in}, then

$$\dot{x}(t) = A_i x(t) + A_{di} x(t_k), i = 1, 2, \dots, r. \tag{1}$$

where $\theta_1(t), \dots, \theta_n(t)$ are the premise variables, M_{ij} is a fuzzy set, $i = 1, 2, \dots, r, j = 1, 2 \dots, n$. r is the index number of fuzzy rules, and $x(t) \in \mathcal{R}^n$ denotes the state of the system. A_i and A_{di} are the known system matrices and sampled-state matrices with appropriate dimensions, respectively.

In this paper, the sampled signal is assumed to be generated by using a zero-order-hold (ZOH) function with a sequence times $0 \le t_0 < \cdots < t_k \cdots < \lim_{k\to\infty} t_k = +\infty$.

$$t_{k+1} - t_k \le h.$$

Assume that $h(t)$ is a time-varying delay satisfying

$$0 \le h(t) \le h, \dot{h}(t) \le \mu, \tag{2}$$

where h, μ are known constants.

Using singleton fuzzifier, product inference, and center-average defuzzifier, the global dynamics of the delayed T-S system (1) is described by the convex sum form

$$\dot{x}(t) \;=\; \sum_{i=1}^{r} p_i(\theta(t))[A_i x(t) + A_{di} x(t - h(t))] \tag{3}$$

where $p_i(\theta(t))$ denotes the normalized membership function satisfying

$$p_i(\theta(t)) = \frac{w_i(\theta(t))}{\sum_{i=1}^{r} w_i(\theta(t))},\; w_i(\theta(t)) = \prod_{j=1}^{n} M_{ij}(\theta_j(t)), \tag{4}$$

where $M_{ij}(\theta_i(t))$ is the grade of membership of $\theta_i(t)$ in M_{ij}. For the sake of simplicity, let us define

$$\bar{A} = \sum_{i=1}^{r} h_i(\theta(t)) A_i,\; \bar{A}_d = \sum_{i=1}^{r} h_i(\theta(t)) A_{di}. \tag{5}$$

Now, the system (3) can be rewritten as

$$\dot{x}(t) \;=\; \bar{A}x(t) + \bar{A}_d(x(t - h(t)). \tag{6}$$

3. Main results

Let us consider the following Lyapunov-Krasoskii functional candidate

$$V(t) = \sum_{i=1}^{5} V_i, \tag{7}$$

where

$$V_1 = \begin{bmatrix} x(t) \\ \int_{t-\alpha h}^{t} x(s)ds \\ \int_{t-h}^{t-\alpha h} x(s)ds \end{bmatrix}^T \mathcal{P} \begin{bmatrix} x(t) \\ \int_{t-\alpha h}^{t} x(s)ds \\ \int_{t-h}^{t-\alpha h} x(s)ds \end{bmatrix},$$

$$V_2 = \int_{t-h(t)}^{t} x^T(s)Q_1x(s)ds,$$

$$V_3 = \int_{t-\alpha h}^{t} x^T(s)Q_2x(s)ds + \int_{t-h}^{t} x^T(s)Q_3x(s)ds,$$

$$V_4 = \alpha h\int_{-\alpha h}^{0}\int_{t+\alpha}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\alpha,$$

$$V_5 = (1-\alpha)h\int_{-h}^{-\alpha h}\int_{t+\alpha}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\alpha d.$$

By using above Lyapunov funtional, we can derive a stability criterion for delayed T-S fuzzy systems, and $e_i \in \mathcal{R}^{10n\times n}(i = 1, 2, \ldots, 10)$ are defined as block entry matrices(for example $e_4 = [0 \ \ 0 \ \ 0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$). The other notations are defined as:

$$\Xi_1^1 = \Pi_1^1\mathcal{P}[e_8 \quad e_1 - e_2 \quad e_2 - e_4]^T + (*),$$
$$\Xi_1^2 = \Pi_1^2\mathcal{P}[e_8 \quad e_1 - e_2 \quad e_2 - e_4]^T + (*),$$
$$\Xi_2 = e_1Q_1e_1^T - (1-\mu)e_3Q_1e_3^T,$$
$$\Xi_3 = e_1Q_2e_1^T - e_2Q_2e_2^T + e_1Q_3e_1^T - e_4Q_3e_4^T,$$
$$\Xi_4^1 = (\alpha h)^2e_8R_1e_8^T - \Pi_2^1\begin{bmatrix} \bar{R}_1 & \mathcal{S}_1 \\ * & \bar{R}_1 \end{bmatrix}\Pi_2^{1T},$$
$$\Xi_4^2 = (\alpha h)^2e_8R_1e_8^T - [e_1 - e_2]R_1[e_1 - e_2]^T - 3[e_1 + e_2 - 2e_7]R_1[e_1 + e_2 - 2e_7]^T,$$
$$\Xi_5^1 = ((1-\alpha)h)^2e_8R_2e_8^T - [e_2 - e_4]R_2[e_2 - e_4]^T - 3[e_2 + e_4 - 2e_7]R_1[e_2 + e_4 - 2e_7]^T,$$
$$\Xi_5^2 = ((1-\alpha)h)^2e_8R_2e_8^T - \Pi_2^2\begin{bmatrix} \bar{R}_2 & \mathcal{S}_2 \\ * & \bar{R}_2 \end{bmatrix}\Pi_2^{2T},$$
$$\Upsilon_1 = \Xi_1^1 + \Xi_2 + \Xi_3 + \Xi_4^1 + \Xi_5^1,$$
$$\Upsilon_2 = \Xi_1^2 + \Xi_2 + \Xi_3 + \Xi_4^2 + \Xi_5^2,$$
$$\bar{\Gamma} = [\bar{A} \quad 0 \quad \bar{A}_d \quad 0 \quad 0 \quad 0 \quad 0 \quad -I],$$
$$\Gamma_i = [A_i \quad 0 \quad A_{di} \quad 0 \quad 0 \quad 0 \quad 0 \quad -I].$$

Now we have the following theorem.

Theorem 1 For given scalars h, μ, the system (1) is globally asymptotically stable if there exist symmetric positive matrices $\mathcal{P} \in \mathcal{R}^{3n\times 3n}, \mathcal{M} \in \mathcal{R}^{2n\times 2n}, Q, R_1, R_2, N_1, N_2 \in \mathcal{R}^{n\times n}$, a positive scalar ϵ and any matrix $\mathcal{S}_j(j = 1, 2, 3, 4) \in \mathcal{R}^{2n\times 2n}$ such that the following LMIs hold for all $h(t) \in [0, h]$

$$(\Gamma_i^{\perp})^T\Upsilon\Gamma_i^{\perp} < 0, i = 1, 2 \qquad (8)$$

$$\begin{bmatrix} \bar{R}_1 & \mathcal{S}_1 \\ * & \bar{R}_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \bar{R}_2 & \mathcal{S}_2 \\ * & \bar{R}_2 \end{bmatrix} \geq 0, \qquad (9)$$

where $\bar{R}_1 = \begin{bmatrix} R_1 & 0 \\ * & 3R_1 \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} R_2 & 0 \\ * & 3R_2 \end{bmatrix}.$

Proof The detailed proof is omitted.

4. Numerical example

Consider the system with the following parameters

$$A_1 = \begin{bmatrix} -3.2 & 0.6 \\ 0 & -2.1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 1 & 0.9 \\ 0 & 2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}.$$

The maximum value of upper bound h compared with the results in [1-3] with different μ u are listed in Table 1.

Table 1: Upper delay bound h for different μ.

μ	0.03	0.1	0.5	0.9
[2]	0.7805	0.5906	0.5392	0.5268
[1]	0.8369	0.7236	0.7154	0.7014
[3]	0.8771	0.7687	0.7584	0.7524
Theorem 1 ($\alpha = 0.5$)	1.5835	1.2444	1.2216	1.1686
Theorem 1 ($\alpha = 0.6$)	1.5906	1.2698	1.2445	1.1852

5. Conclusion

The robust stability for T-S fuzzy systems with sampled-data has been investigated. Less conservative criteria have been obtained by employing new delay-partitioning technique, integral inequality and reciprocally convex approach. Numerical examples have been given to demonstrate the effectiveness of the proposed method.

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