Structural and Percolation Models of Intelligence

Dmitry Zhukov[1], **Irina Samoylo**[2], **James William Brooks**[3] **and Victoria Hodges**[4]
[1] Professor, Head of the Department of Regional Systems of Education Quality Management, Institute of Higher Education Quality, National University of Sciences and Technology (NUST "MISiS"), Moscow, Russia
[2] Professor, Department of Medical and Biological Physics, I.M. Sechenov First Moscow State Medical University, Moscow, Russia
[3] Consultant, Salem International University, Salem, West Virginia, USA
[4] Consultant , Department of Medical and Biological Physics, I.M. Sechenov First Moscow State Medical University, Moscow, Russia

Abstract **-** *This paper discusses the questions of the application of Percolation Theory with the purpose of estimating the number of neuronal synaptic connections sufficient for a productive intellectual activity. The use of the percolation approach in the description of human intellectual activity can be practically useful for solving problems of the creation of effective models of artificial neural networks, as well as for the development of sensitive methods for diagnostics of neural networks of the brain in autism and hyperactivity, and for the development of information security systems.*

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Introduction

A child is asked by his/her parents to choose a ball out of several toys. Both the parents and the child are in delight – the choice has been made correctly! Their delight is absolutely pertinent here: after all, in order for a round shaped toy to appear in the child's hand, his/her brain had to perform the most complicated intellectual work. Therefore, it is clear that discussing such a complex and multi-valued phenomenon as intelligence, it is imperative to be concerned about one of general questions of the human brain functioning and structure: the neural network granted by nature and certainly, by parents to each newborn. It was established experimentally that the number of neural connections in a cerebral cortex of a newborn is quite small. This number makes only a few percent of the neural network connections of an adult person's brain. However, in the process of child's development, the number of connections between neurons in his/her brain grows very productively and reaches its maximum by the age of six years old. In the subsequent stages of human development, there is a reduction in their neural network: the quantity of the synaptic links decreases and then it stabilizes [1]. The process of the synapses' dying off and stabilization is, probably, one of the basic mechanisms by means of which, the experience changes the structure of the brain in the course of its formation [2 – 5]. How much more reasonable is the process of reduction of the synaptic connections between neurons of the brain? What number of the synaptic connections between neurons is sufficient for a productive intellectual activity? Let us try to find the answers to these questions with the help of the methods of numerical modeling, based on the ***percolation model of human intellectual activity.***

To begin with, it is necessary to consider the basic principles of the structure and functioning of the human brain's neural network.

According to Santiago Ramon y Cajal's neuron doctrine, which is the basis for our understanding of the brain, neuron is the main structural and functional element of the brain. The dendrites of the neuron are used to obtain signals, and axons are used to transmit signals to other neurons. Signal transmission is carried out only on the special sites, the synapses, and each neuron interacts only with certain neurons. It is important to consider that in real biological structures the number of available neurons is from 10 to 100 billion, each of which has from 10 to 1000 connections with other nervous cells (**a multi connectivity condition**).

The theory of the organization of the nervous system leads to the conclusion that the brain cells, neurons, are grouped in a very complex network infrastructure, thanks to which its work is carried out. The cortex functional features are determined by the distribution of cortical nerve cells (neurons) and their connections within the layers and columns. The convergence of the impulses from various sense organs is possible. According to the modern ideas, similar convergence of diverse excitations is a neurophysiological mechanism of the integrative brain activity, i.e. the analysis and synthesis of the reciprocal activity of an organism. It is also significant that neurons are combined into complexes, which apparently realize the results of the convergence excitation in separate neurons.

Ultimately, complex interactions of all parts of the brain determine the diversity of human behavior and intellectual activity.

Abilities of the brain in information processing

John Griffith [6] made a rough calculation that if a person continually remembered information with the speed of 1 bit per second throughout 70 years of their life, then the total of 10^{14} bits would be accumulated in their memory. It is approximately equivalent to the amount of the information stored in Encyclopedia Britannica. In fact, every second the human brain receives about 20 bits of information, and during 14 hours, it can process 18 billion bits. To store this amount of information, a person needs only one-thousandth of all nerve cells of the brain. According to various estimates, the amount of information that a person can remember for a lifetime is up to 10^{21} bits. A person is able to recall any necessary information in the tenth fractions of a second, which requires the search speed of about 50 billion bits per second. It should be noted that processing of such amount of information could be provided only by a parallel operation of the nervous structures. ***In terms of data storage and data retrieval, these structures have a very efficient topology***.

Formalized structural model of intellectual activity

A characteristic feature of the structure of the cerebral cortex is the ***oriented horizontal and vertical distributions*** of its constituent nerve cells (neurons) in the layers and columns. Thus, the cortical structure is notable for its spatially ordered arrangement of the functioning units and connections between them.

The management of intellectual processes as well as the management of motor skills can be carried out by several hierarchically subordinated rings of the connected neurons, which among themselves distribute the roles according to the hierarchy of their abilities. One part of neurons plans only general ways of realization, and the following rings of neurons are responsible for the details of the execution. However, neurons can interact not only vertically but also horizontally, forming horizontal rings. Thus, this interaction can be carried out by the principle of the branched network, where ***the trajectories of the transmission of the nerve signals look like loops***: the same signal can return to the starting point several times. The main type of the direct and reverse connections of the neo-cortex is the vertical bundles of fibers that bring the information from the subcortical structures to the cortex and send it back to the cortex. Along with the vertical links, there are intra-cortical or horizontal bundles of associative fibers extending at various levels.

The set of neurons, their communication and the topology of their connections during the process of learning and saving various images and objects, form a certain subnet of knobs. Those knobs are responsible for the process, which created them and further on, it responds for the identification of the given process (or an object).

The formalized topological model of the neural network of the brain may have the form shown in Figure 1: the arc–shaped lines represent non–overlapping connections between distant neurons, and the straight segments are the connections with the nearby nerve cells. Neurons can have both types of connections: those which are conditionally lying in one plane (one layer), and vertical connections between the nerve cells belonging to different layers (they are indicated by numbers 1, 2, 3, 4, and 5 in Fig. 1). For example, the chain of neurons indicated as **a-c-d-e-n** in Fig.1 can be included in one of the vertical layers, and the chain, which is indicated as **o-i-p-s-m-k** can be included in a horizontal layer. ***It is very important to note that <u>the same neuron</u> can simultaneously belong to both various horizontal and vertical layers (and the chains of the connected neurons themselves can be conditionally called horizontal and vertical rings)***.

The structure shown in Fig. 1 can be conditionally called as a random network with multiple connections between the knobs (neurons).

Let us analyze to what degree the structure represented in Fig. 1 corresponds to the existing data of the structure and operation of the human brain.

The speed of the nerve impulse is significantly lower than the speed of the transmission of electrical signals at their contacts. Therefore, practically, the brain of any person loses in speed to computers, but it considerably surpasses them in its intellectual activity because of the ***associativity of the brain's work*** and almost infinite ***degree of the parallelization of the information processing***. If there is an available network composed of 10 to 100 billion of neurons with the total number of connections of 10^{12} pieces, it can be divided into subnets (for example, the size of 10^4 – 10^6 knobs). Given that some of the neurons or their groups can be parts of different subnets, then at a rough estimate, the number of possible combinations can make:

$$C_{10^4}^{10^{12}} = \frac{10^{12}!}{10^4! \cdot (10^{12} - 10^4)!} \qquad C_{10^6}^{10^{12}} = \frac{10^{12}!}{10^6! \cdot (10^{12} - 10^6)!}$$

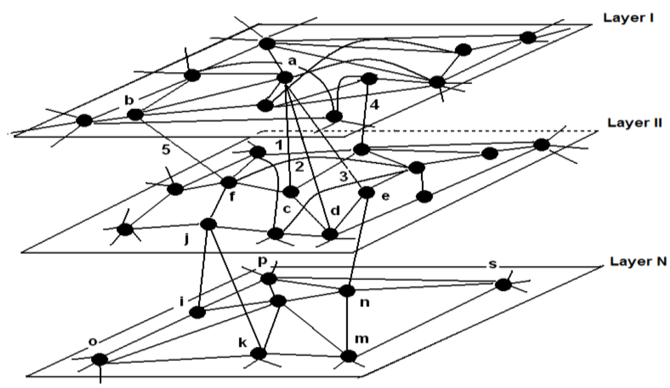

Fig. 1. Formalized model of brain topology

It is possible to consider each of such structures as a separate processor (using the Computer Science language). In this case, at a very rough estimate, there will be about 10^{11} to 10^{12} units working in parallel, which is practically not achievable (even in the distant future) for a computer built on semiconductors. For the structure shown in Fig.1, the increasing number of knobs and connections between them should lead to an increase in the number of possible subnets or (using the language of Computer Science again) parallel processors. The structure shown in Fig. 1 is not only multilayered, but ***it is also characterized by having many different connections among its knobs***. Therefore, at the reduction of its size, a removal of one part of the structure (or leaving only a small part of it) will not affect its topological properties. If the network's work is defined also by its topological properties, then its reduction in size in qualitative terms will not affect its work (the quantitative detection of accuracy may deteriorate or time will increase). All the above is consistent with the existing data proving that the localization of functions in primary areas of the brain is duplicated in such a manner that each smallest area contains the information about the whole object.

The percolation model of intellectual activity

For image recognition purposes, we propose the following model in this paper. A signal is transmitted to the input of the neural network. The incoming signal is compared with the images previously stored in its subnet. This may result in some active (or excited) states of the neurons. If between the input and output layers of the network or in any of its sub-networks, an unbroken chain of unexploded and excited neurons appears, then the stored in the given subnet image is recognized. Otherwise, the image cannot be recognized. Then the synthesis of a certain amount of concentration of RNA and the protein coding information takes place in the neurons involved in the recognition. The above described process, creates one more subnet. The new subnet saves the new image without deleting the old ones. Considering that the number of possible created subnets is very high (about 10^{11}), it allows to store a large number of various images. Besides, given that during the recognition, the process will be taking place in parallel within all structures, then the speed of the search and access to the information should be reasonably high, despite a considerable volume of the stored data.

It is possible to name the route created through the active cells (they can be called the knobs of the subnet) as filtering or percolation. For regular structures, Percolation Theory is a well-developed area. However, the structures represented in Fig. 1 have a random irregular topology, and the study of percolation processes in them is a challenging task that can only be solved by the methods of numerical modeling.

From the point of view of a mathematician, Percolation Theory should be carried to Probability Theory on graphs. There is quite a number of monographs on both theoretical and applied aspects of percolation [7-10].

To explain the basic tenets of Percolation Theory, let us take a square network and paint over in black a part of knobs (see Fig. 2). One of the questions, which can be answered by Percolation Theory here, is: At what portion of n^c of the colored knobs does their black chain connecting the upper and lower sides of the net (the chain of connectivity), arise? For a grid of the finite size, such chains may occur at different concentration (see Fig. 2). However, if the size of grid *L* is directed to infinity, then the critical concentration becomes quite definite. Such critical concentration is called *the percolation threshold.*

The square grid is only one possible model. One can consider percolation on the triangular and hexagonal grids, trees, three-dimensional lattices (for example, on a cubic one that is in space with the dimension more than three). The grid does not necessarily need to be regular; it is possible to consider the processes on random lattices.

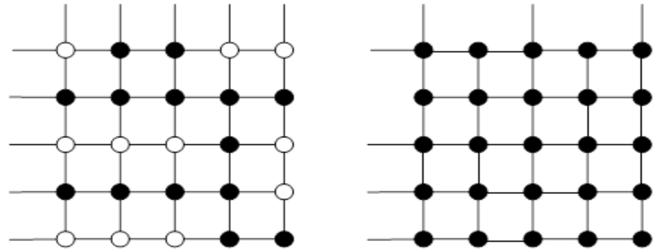

Fig. 2. Percolation on a square lattice

Let us consider percolation in random networks with multiple paths between the knobs having the form shown in Fig. 1. Let us choose any two knobs, **A** and **B,** on the opposite layers of the network and then let us start randomly activate certain other knobs. Obviously, if there are many activated knobs, a situation may occur at which between the two arbitrarily selected knobs **A** and **B,** there will be at least one "open" path (the path formed by the activated knobs). Using the methods of numerical modeling with statistical averaging of the obtained results from separate experiments, it is possible to determine at what proportion of the activated knobs (the percolation threshold) the conductivity between knobs **A** and **B** appears in the network**,** and how it depends on the average number of connections per single knob. Table 1 presents the results of numerical modeling of identifying the percolation threshold for random networks with multiple paths between the knobs (see Fig.1) with a various average number of connections per knob. (See Table 1)

Table 1.

Network type	Average of connections per knob in the network of the final size within the given structure	Portion of the activated knobs, at which conductivity occurs in the network (**n_c - percolation threshold**)
Random network with multiple paths between the knobs	2,36	0,515
	2,82	0,425
	3,29	0,365
	4,70	0,270
	4,75	0,250
	6,15	0,150
	6,17	0,185
	6,75	0,175
	9,41	0,170
	10,02	0,150
	10,31	0,130
	10,69	0,135
	11,07	0,115
	13,10	0,115

Since an increase of an average number of connections per knob in the network leads to a substantial increase in time and computing resources expenses, in numerical modelling it was decided to choose the area from 2.5 to 15 connections per knob. Figure 3 shows the graphic dependence of the results introduced in Table 1.

Figure 3 demonstrates that at the increase in the average number of connections per knob in the network, the percolation threshold begins to pursue its certain minimum value. Thus, the received results suggest that there is no need to carry out numerical modelling for large values of the average number of connections per knob. Instead, it is possible to linearize the results and extrapolate them to higher values. (See Fig.3)

As the graphic kind of the dependence in Figure 3 reminds the exponent, then it can be described by the function of the following kind: $P(x) = P_0 e^{-z}$, where $P(x)$ – is the value of the percolation threshold with the average number of connections per knob equal to some value x, and z=1/x, P_0 – is the value of the percolation threshold at infinite number of connections per knob. As Figure 4 shows, the results presented in Table 1 are linearized well in the co-ordinates: ln P(x) – z = 1/x (natural logarithm of the percolation threshold is the reciprocal of the average number of connections x per knob), which supports the use of the function of the following kind: $P(x) = P_0 e^{-z}$. (See Fig.4)

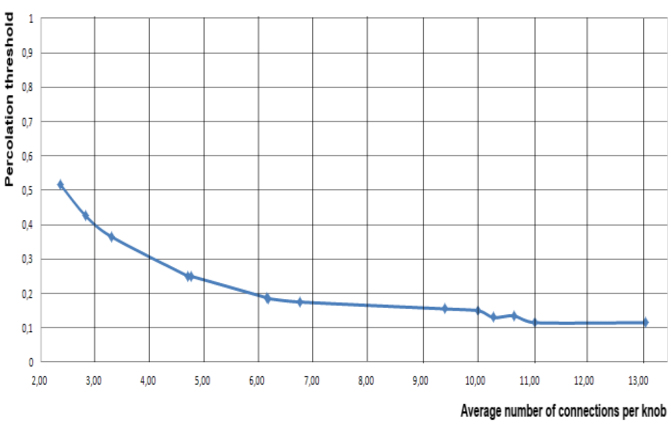

Fig. 3. Dependence of the size of the percolation threshold in a random network on average number of connections per knob

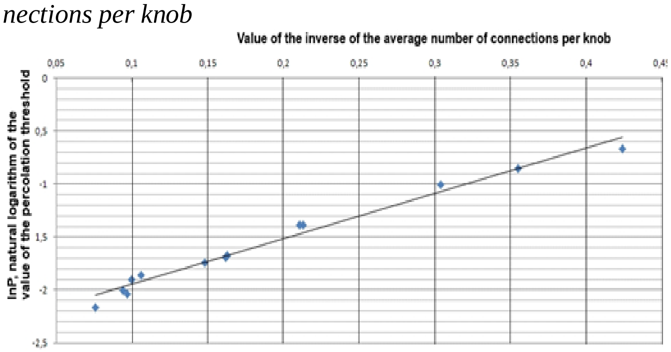

Fig. 4. Dependence of the logarithm of the value of the percolation threshold in a random network on the value of the inverse of the average number of connections per knob.

The dots in Figure 4 present the experimental data and the solid line corresponds to linear relationship:

y = 4.2882z – 2.3766, with very high correlation coefficient equal to 0.98. At z=1/x = 0 (corresponding to the case when x = ∞) we get: y=lnP_0 = –2.15, and the value of the percolation threshold at an infinite large number of connections per knob P_0 will be equal to 0.093. Thus, for a random network with an infinite large number of connections per knob, it is enough to have an equal share of the activated neurons equal to 0.093 of the total number, in order for the conductive chain of knobs to appear and for the network of knobs to recognize the presented image. With the average number of connections equal to 100, the percolation threshold is equal to 0.097, and at 10 it is equal to 0.143.

The obtained results demonstrate that a substantial increase in random network connections with an average of more than ten connections per knob, hardly changes the percolation threshold, and from the point of view of the use of the biological resources, this type of increase is unfavorable for neural structures. Therefore, the reduction of neuro-synaptic redundancy of the network is an inevitable step in the formation of a neural network of the human brain.

The use of the percolation approach can be practically helpful in solving problems in the development of more efficient models of artificial neural networks as well as in the development of sensitive methods of diagnostics of the neural networks of the brain in autism and hyperactivity. It can also be used in the design of information security systems that can detect false knobs, through which leak or substitution of information can occur.

We also consider that another potentially interesting area for the percolation model application is its use in the development of crossbar nanocomputers (while designing them a considerable redundancy of interconnections of conductive nano lattices is put in). The network reduction based on the percolation approach will allow building an optimal architecture of the keys and raising the system's resistance to defects.

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