Performance Improvement in Subcarrier QAM Free Space Optical Links in the Presence of Atmospheric Turbulence and Pointing Error with Spatial Diversity and Beam Optimization

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Abstract — Free-space optical communication suffers from irradiance variations caused by random atmospheric temperature fluctuations and pointing error due to jitter and misalignment between transmitter and receiver. In this paper, the simulation results for the average symbol error rate performance of Free Space Optical links over gamma-gamma turbulence fading channels in the presence of pointing error with subcarrier QAM are studied and numerical results are obtained using spatial diversity and beam optimization. The received signal at the receivers is combined using maximal ratio combining. Furthermore, optimum beamwidth is calculated taking into account various metrics such as the electrical signal-to-noise ratio, the normalized jitter, and the SER. The results obtained can be a useful outcome for FSO system designers in order to limit pointing error effects and achieve an optimum performance.

Index Terms — Diversity Reception, Free-space optics, Fading, MRC, Pointing error, QAM

1. INTRODUCTION

Free-space optical (FSO) communication has emerged as a promising and commercially viable technology in today’s communication infrastructures. It can provide high-speed links for a variety of applications; can be considered as a supplement or an alternative to RF for the next generation broadband in order to support large bandwidth, unlicensed spectrum, excellent security, and quick and inexpensive setup. However, atmospheric turbulence produces scintillation of the transmitted optical beam at the receiver end, severely degrades the link performance. Due to line-of-sight (LoS) connection and the directional reception of narrow FSO beams another cause of concern is the pointing error (PE) which arises due to misalignment between the transmitter and receiver due to weak earthquakes, dynamic wind loads resulting in sway of high rise buildings that causes vibrations of transmitted beam and thus misalignment.

An appropriate selection of the modulation technique is vital to circumvent turbulence induced fading. Though the on-off keying (OOK) scheme is the simplest and extensively used modulation technique, but it requires an adaptive threshold scheme to perform optimally in atmospheric turbulence induced fading. This adaptive threshold is complex to implement and practically not suitable. Therefore, it is a reasonable approach to use modulation techniques that carry the information in the phase or the frequency of the RF carrier signal. The phase shift keying (PSK) or the quadrature amplitude shift keying (QAM) based subcarrier intensity modulation (SIM) requires no adaptive threshold scheme, thereby offering superior performance compared to OOK in the presence of atmospheric turbulence induced fading channels. They require lower bandwidth and have higher spectral efficiency as compared to binary modulation schemes. Further, M-ary QAM delivers better BER performance due to more distance between constellation points for values of M >4. However, the penalty paid is higher SNR to attain a desired BER performance. Further, multiple RF SIM is the preferred choice when increased capacity is more important than the power requirement. Also, QAM is becoming the standard for 4G wireless communications, which is another driver, in order to have seamless integration with the system.
Diversity schemes with different combining systems have received much attention in RF due to their ability to mitigate the performance degradation of multipath fading through diversity. The random pointing error in FSO systems has a profound effect on the BER performance of the atmospheric fading channel. This can also be improved by using diversity and optimum combining techniques at the receiver. In the present paper, therefore, study is carried out using diversity with single-input multiple-output (SIMO) and maximal ratio combining (MRC) for the FSO link affected by turbulence fading and pointing error.

The remainder of this paper is organized as follows. The channel, system model and beam optimisation are described in Section II followed by simulation results and conclusions in section III and IV, respectively.

2. CHANNEL AND SYSTEM MODEL

A. Channel Model

The channel state $h$ models the random attenuation of the propagation channel. In our model, $h$ arises due to two factors: atmospheric turbulence $h_a$ and pointing errors $h_p$. The combined optical channel model is defined as:

$$h = h_a h_p$$  \hspace{1cm} (1)

In (1), ‘$h$’ is the normalized channel fading coefficient considered to be random but constant during the symbol duration. The coefficients $h_a$ and $h_p$ are random with distributions discussed in the following paragraphs.

The atmospheric turbulence follows the Gamma–Gamma distribution which recently has emerged as a useful turbulence model as it has excellent fit with measured data over a wide range of turbulence conditions [1]. According to the Gamma–Gamma model, the irradiance ‘$I$’ can be modeled as weak eddies induced irradiance fluctuation modulated by strong eddies induced irradiance fluctuation. The probability density function (PDF) of a normalized Gamma–Gamma random variable $I$ is given as [1].

$$f_{h_a}(h) = \frac{\alpha^\alpha \beta^\beta \Gamma(\alpha+\beta) (\frac{\alpha \beta}{\alpha+\beta})}{\Gamma(\alpha)\Gamma(\beta)} h^{\alpha-1} (1 + \frac{\alpha}{\beta}h)^{-\alpha-\beta} K_{\alpha-\beta}(2\sqrt{\alpha \beta}h)$$  \hspace{1cm} (2)

where $\Gamma(.)$ is the Gamma function and $K_{\alpha-\beta}(.)$ is the modified Bessel function of the second kind of order $\alpha-\beta$. The shaping parameters $\alpha$ and $\beta$ are related to the Rytov variance ($\sigma_r^2$) and are given by [1]

$$\alpha = \left[ \exp \left( \frac{0.49\sigma_r^2}{(1+1.11e^{12/5})^{3/5}} \right) - 1 \right]$$  \hspace{1cm} (3)

$$\beta = \left[ \exp \left( \frac{0.51\sigma_r^2}{(1+0.69e^{12/5})^{3/5}} \right) - 1 \right]$$  \hspace{1cm} (4)

Pointing errors which can arise due to mechanical misalignment, errors in the tracking system, or due to mechanical vibrations present in any system, is composed of two components [2]: a fixed error, called boresight, and a random error, called jitter. The spatial intensity profile of the beam is assumed to be Gaussian with a beam waist ‘$w_0$’ at the receiver plane at a distance ‘$z$’ from the transmitter with a circular aperture of radius ‘$r$’. The probability density function of the pointing error is given by [2]

$$f_{h_p}(h_p) = \frac{\gamma^2}{\kappa_0^2} h_p^{\gamma-1} e^{-\frac{\gamma^2}{\kappa_0^2} h_p^2}, \hspace{0.5cm} 0 \leq h_p \leq A_0 \hspace{1cm} (5)$$

where,

$$\gamma = w_{seq}/2\sigma_s \hspace{1cm} \text{ (6) }$$

$h_p$ represents fraction of the power collected by the detector, $\sigma_s$ is the pointing error displacement standard deviation due to jitter at the receiver. $w_{seq}$ is the equivalent beam width at the receiver. The parameters $w_{seq}$ and $A_0$ are given in [2].

B. System Model

The block diagram of the transmitter of subcarrier FSO system for a single RF carrier and employing QAM-SIM is shown in Fig.1. At the transmitter, the serial data signal is converted to two parallel streams and the radio frequency signal is first pre-modulated with the data signal $d(t)$. After proper biasing, this RF signal $s(t)$ is used to modulate the irradiance of a continuous wave optical laser beam. The transmitted irradiance will have the following waveform:

$$i(t) = P[1 + \xi(t)], \hspace{1cm} \text{ (7) }$$

with $\xi(t)$ representing the random pointing error.
where ‘P’ is the transmitted optical power which is normalized to unity and ‘ξ’ is the modulation index satisfying the condition -1 ≤ ξ ≤ 1 in order to avoid over-modulation. Here QAM is chosen as the electrical modulation format with the in-phase and quadrature-phase of the RF signal. The output signal of the QAM modulator can be written as [3]

\[ s(t) = s_i(t) \cos(2\pi f_i t) - s_q(t) \sin(2\pi f_i t) \]  

(8)

In Eq. (8), \( s_i(t) = \sum_{j=0}^{\infty} a_j(t) g(t - jT_s) \) and \( s_q(t) = \sum_{j=0}^{\infty} b_j(t) g(t - jT_s) \), where \( a_j(t) \) and \( b_j(t) \) are the in-phase and quadrature components of the \( j \)th data symbol, respectively, \( g(t) \) is a shaping pulse, and \( T_s \) is the symbol interval.

At the receiver shown in Fig.2, in a spatial diversity system, aperture area of each detector in the N-receiver system is assumed to be A/N, where \( A \) is the aperture area of detector under single-transmitter single-receiver link and the background radiation noise on each link with detector diversity is 1/N that of a SISO link resulting in \( \{n_i(t)\}_{i=1}^{N} \sim N(0, \sigma^2/N) \).

This approach is particularly valid for the case where noise from the background radiation is the dominant source as is the case for FSO. By assuming identical PIN photodetector on each link, the individual detector output is given by [4]

\[ i_{\text{ri}}(t) = \frac{\mathcal{R}}{N} h \left\{ 1 + A_j g(t) \cos(\omega_{c j} t + \theta_j) \right\} + n_i(t) \]  

(9)

where, \( i = 1, 2, 3, \ldots, N \). ‘\( \mathcal{R} \)’ is the responsivity of the photodetector, \( n(t) \) is the additive white Gaussian noise (AWGN).

After removing the DC bias, the electrical signal is first demodulated by an electrical QAM demodulator. It is then sampled to recover the transmitted data. The MRC combiner weights each output signal \( \{i_{\text{ri}}(t)\}_{i=1}^{N} \) from each link by gain \( \{a_i\}_{i=1}^{N} \) proportional to the received intensity. The weighted signals are then co-phased and coherently added to obtain the combined output current. In a SIMO MRC system, channel state information (CSI) is perfectly known at the receiver. The received signals are combined in such a way that the signal-to-noise ratio (SNR) at the receiver combiner output is maximized.

C. Beam Width Optimisation

To overcome the problem of misalignment and keep line-of-sight between the transmitter and receiver, the beamwidth and transmitted power need to be increased. However, a wide beamwidth increases the required signal-to-noise ratio (SNR), leading to increased cost and complexity, whereas, a narrow beam may result in the signal outage. A beam width which is larger than receiver aperture, reduces the SNR due to spreading of optical power. On the other hand a beam width which is smaller than receiver aperture radius would result in the increase of ambient noise, reducing the effective SNR. Hence, a proper optimization of the beamwidth is required.
3. SIMULATION RESULTS

The RF SIM-QAM SIMO FSO system described above is simulated using Matlab with single subcarrier. The system parameters and that of the atmospheric turbulence and pointing error (considering a fast tracking system with negligible bore sight error) are given in Table 1.

The simulation has been carried out for low turbulence ($\sigma_l^2=0.2$) and moderate turbulence ($\sigma_l^2=1$) by varying jitter standard deviation $\sigma_s$ for each case.

The pulse shaping function is assumed to be rectangular in the simulation.

Table 1: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Data rate</td>
<td>1Mbps</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td>1550nm</td>
</tr>
<tr>
<td>Distance between Tx and Rx</td>
<td>1Km</td>
</tr>
<tr>
<td>Receiver diameter</td>
<td>8cm</td>
</tr>
<tr>
<td>Jitter standard deviation, $\sigma_s$</td>
<td>4cm, 6cm, 8cm</td>
</tr>
<tr>
<td>Variance of Turbulence</td>
<td>$0.2 \leq \sigma_l^2 \leq 1$</td>
</tr>
<tr>
<td>Number of receivers</td>
<td>2</td>
</tr>
</tbody>
</table>

In Fig.3, we have shown the performance improvement of the FSO SIMO system over the single-input-single-output (SISO) system. Fig. 3 has plots of the SISO system with and without PE. It is observed that the performance of the FSO system degrades considerably by a value of 9dB because of pointing error at a SER of $10^{-4}$ for the SISO system. With receiver diversity of order two and MRC combining, a diversity gain of the order of 7-13dB is achievable for jitter values of $8cm \geq \sigma_s \geq 4cm$ at a SER of $10^{-4}$.

In Fig. 4, once again, similar order of diversity gains has been obtained for the FSO system with PE for moderate turbulence conditions.

Next, we have obtained the results for the optimisation of the beam. From Fig.5 we observe that the value of optimum beam-width remains constant even on varying the turbulence from low to a high value keeping the pointing error jitter.
constant. The minimum beam-width remains unchanged for different atmospheric turbulence at 1.7 for a fixed PE. Therefore, turbulence has no effect on the optimum beam-width required.

**Fig.5.** SER vs $w_2/r$ curves for QAM-SIM at fixed SNR and jitter standard deviation with varying turbulence.

In Fig.6, it can be seen that by keeping the turbulence constant at a moderate level and varying the pointing error jitter standard deviation by only a small amount, the optimum beam width required has grown very large in size (from 1.6 times to 2.85 times the receive aperture radius). This is due to the fact that a higher jitter results in larger variations in radial displacement of the incoming beam center demanding a larger beam width for optimum SER performance.

**Fig.6.** SER vs $w_2/r$ curves for QAM-SIM at fixed SNR and turbulence with varying jitter standard deviation.

4. **CONCLUSION**

In this paper, SER performance of QAM-SIM FSO link over Gamma-gamma atmospheric turbulence channels with receiver diversity and MRC has been investigated. The effect of varying the pointing error standard deviation on the SER performance for low and moderate turbulence has been obtained. It is concluded that the degradation in SER due to turbulence induced fading with PE can be reduced considerably by employing receive diversity. Also, the performance of the FSO link with pointing error has been investigated with the beam-width radius. The effect of varying the pointing error standard deviation and turbulence on the beam-width has been obtained. The effect of misalignment may be reduced by using broad transmitted beam but this would be at the expense of higher transmitting power. In this regard, beam width optimization was found to be a viable option for improving SER performance along with automatic tracking systems. It is concluded that, increase in turbulence doesn’t have any effect on optimum beam width required whereas increase in pointing error jitter increases the optimum beam width required.

5. **REFERENCES**


