Direction of Arrival Estimation Using Particle Swarm Optimization-based SPECC

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Abstract

This paper proposes a novel direction of arrival (DOA) estimation scheme using particle swarm optimization (PSO)-based SPECC (Signal Parameter Extraction via Component Cancellation). The proposed algorithm is a PSO-based optimization method and extracts the amplitudes and incident angles of signal sources impinging on a sensor array in a step-by-step procedure. On the other hand, other algorithms extract those parameters at the same time. Proposed algorithm is very fast, robust to noise and has a high resolution in DOA estimation. Simulation results using artificially created data show the accuracy in the angle estimation and robustness to noise.

Key words: particle swarm optimization, direction of arrival, signal parameter extraction via component cancellation

1. Introduction

Estimation of signal direction of arrival (DOA) from the received data by array antenna is a critical issue in radar, sonar and communication systems. A variety of techniques for DOA estimation have been proposed. The well-known methods are the maximum likelihood (ML) technique [1], the multiple signal classification (MUSIC) [2], the root-MUSIC [3], the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [4], the genetic algorithm (GA)-based method [5], the maximum likelihood estimation using particle swarm optimization (PSO) [6] and the evolutionary programming (EP)-based signal parameter extraction via component cancellation (SPECC) [7]. Each algorithm has a strength and weakness relative to each other.

In this paper, a novel DOA estimation algorithm, PSO-based SPECC is proposed. The previously developed algorithms, such as ML, MUSIC, root-MUSIC, ESPRIT, GA-based method and ML using PSO extract the parameters (amplitudes and DOAs) of all source signals at the same time. Whereas, in the EP-based SPECC and the PSO-based SPECC, the parameters of each source signal out of multiple signals impinging on a sensor array are extracted in a step-by-step procedure. For the optimization of cost function, the PSO is used instead of EP. Recently, a PSO algorithm has been applied to electromagnetic optimization problems [8 – 10], because the basic algorithm of PSO is very simple and easy to implement.

Our algorithm is fast, robust to noise and accurate. In the simulation using artificially created data, we conduct some tests to verify the robustness to noise, and accuracy of the proposed PSO-based SPECC algorithm.

2. Particle Swarm Optimization

Traditionally, the gradient-based methods are used for complex function optimization. But, the gradient-based methods have a problem of local minima because of the characteristic of local search behavior. For this reason, the global optimization algorithms, such as the genetic algorithms (GA), the evolutionary strategies (ES), the evolutionary programming (EP), and particle swarm optimization (PSO) have emerged as efficient and robust search methods.

PSO is an optimization technique that offers attractive benefits to the user, including the fact that the algorithm can be easily understood and implemented. Compared to evolutionary optimization methods, PSO is based on the simulation of the social behavior of bird flocks and fish schools [8]. In PSO, individual particles in a swarm represent potential solutions, which move through the problem search space finding an optimal or good solution. In Fig. 1, the flowchart of typical PSO routine is shown in detail.
First, we define an $N$-dimensional vector space and a population of $N_p$ particles is assumed to evolve in this vector space. Each particle is assigned the following position and velocity vectors, respectively:

$$\mathbf{x}_i(t) = [x_i^1(t) \ x_i^2(t) \ ... \ x_i^N(t)]$$  \hspace{1cm} (1)

$$\mathbf{v}_i(t) = [v_i^1(t) \ v_i^2(t) \ ... \ v_i^N(t)]$$  \hspace{1cm} (2)

where $i = 1, 2, ..., N_p$.

We also postulate two more variables $\mathbf{x}_{pbest}$, the local particle best, and $\mathbf{x}_{gbest}$, the global best. The key idea of the classical PSO algorithm is how to predict the best update for the position in the next iteration. The particle dynamics utilizes its own experience and that of its neighbors. The idea is to change the velocity component in a manner that the increments are proportional to the difference between the current position of the particle and the local and global best, respectively. The particle dynamics are accomplished by the following two equations:

$$\mathbf{v}_i(t) = \omega \mathbf{v}_i(t - 1) + \rho_1 (\mathbf{x}_{pbest} - \mathbf{x}_i(t)) + \rho_2 (\mathbf{x}_{gbest} - \mathbf{x}_i(t))$$  \hspace{1cm} (3)

$$\mathbf{x}_i(t + 1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)$$  \hspace{1cm} (4)

where $i = 1, 2, ..., N_p$; $\rho_1 = r_1 c_1$ and $\rho_2 = r_2 c_2$; $c_1$ and $c_2$ are the cognitive and social factors, respectively. $r_1$ and $r_2$ are two statistically independent random variables uniformly distributed between 0 and 1; $\omega$ is the inertia factor. In this paper, we select $c_1$ and $c_2$ are 1.5 and $\omega = 0.5$.

![Flowchart of PSO](image)

**Fig. 1. Flowchart of PSO**

### 3. PSO-based SPECC

The proposed PSO-based SPECC algorithm extracts the desired number of signal sources in order from the highest energy to the lowest energy. Therefore, the individual’s dimension becomes very low compared to the GA-based method [5] and PSO-based method [6] which extract parameters of all signal sources simultaneously. In the methods of [5] and [6], each individual is composed of DOAs, amplitudes, and relative phases of whole signal sources, whereas in our algorithm, each individual is composed of only one signal source parameters. As the dimension of the individual vector increases, premature convergence to local minimum is more likely to occur and the convergence time is much longer.

In this paper, we first consider a linear array with $P$ sensor elements as in Fig. 2. Then, the $M$ narrow-band signals are assumed to be received by the linear array antenna.

![Geometry of linear array antenna with two signal sources](image)

**Fig. 2. Geometry of linear array antenna with two signal sources**

If the first array element is assumed as reference point, the complex signals received by the $p$th element can be expressed by

$$y_p(k) = \sum_{m=1}^{M} A_m(k) \cdot \exp \left[ j \frac{2\pi}{\lambda} (p - 1) d \theta_m \right] + n_p(k)$$  \hspace{1cm} (5)

where $\lambda$ is the signal central wavelength, $d$ is the distance between the array elements, $\theta_m$ is DOA of the $m$th source signal, $A_m(k)$ is the complex amplitude of the $m$th source signal at $k$th time sample, and $n_p(k)$ is the additive noise of $p$th array element at $k$th time sample. We obtain the average received signal of $p$th array element using $K$ time samples to reduce the noise by the equation (6)

$$y_{p,\text{ave}} = \frac{1}{K} \sum_{k=1}^{K} y_p(k)$$  \hspace{1cm} (6)

The detailed PSO-based SPECC algorithm is as follows:

**STEP 1. (Initialization)** Set $m=1$, where $m$ is the index of iteration to extract the $m$th signal source, and define the received average signal at the $p$th array element as $y_{p,\text{ave},m}$ for $p=1, 2, ..., P$, where $P$ is the number of array elements.

**STEP 2. (Parameter extraction)** Obtain the complex coefficients $A_m$ and $\theta_m$ that minimize the following cost function $F_m$ of the $m$th iteration using the PSO subroutine:

$$F_m = \sum_{p=1}^{P} \left| y_{p,\text{ave},m} - A_m \cdot \exp \left[ j \frac{2\pi}{\lambda} (p - 1) d \theta_m \right] \right|^2$$  \hspace{1cm} (7)
In the PSO subroutine, the particle position vector is composed of the real and imaginary parts of $A_m$ and $\theta_m$. Terminate the PSO subroutine when the available execution time has passed because we do not know the minimum value of $P_m$.

**STEP 3.** (Component cancellation) Subtract the components of the determined signal sources in Step 2 from $y_p^{\text{ave},m}$ and obtain $y_p^{\text{ave},m+1}$, $p=1,2,\ldots,P$, as follows:

$$y_p^{\text{ave},m+1} = y_p^{\text{ave},m} - A_m \cdot \exp \left[ i \frac{2\pi}{\lambda} (p-1) \cdot d \cdot \sin \theta_m \right]$$  \hspace{1cm} (8)

**STEP 4.** (Termination check) Let $m=m+1$. Return to Step 2, unless the desired $M$ components are extracted.

As explained in [7], the general SPECC algorithm recursively estimates a DOA and amplitude of each source signal. During each iteration, the highest energy source in the remained average signal ($y_p^{\text{ave},m}$) is determined, and its DOA and amplitude are regarded as parameters of each source signal. After determining one signal source, the SPECC algorithm subtracts the determined signal components from the complex remained signal ($y_p^{\text{ave},m}$) and obtain $y_p^{\text{ave},m+1}$ which is the remained average signal used for the next iteration. This procedure is repeated until the residual energy is below the predefined threshold related to noise level or the iteration index ($m$) reaches the predefined (or estimated) value $M$. In this paper, we assume that $M$ is known or pre-estimated. Typically, for an estimation of the total signal sources, AIC [11] and MDL [12] could be used, but they have a high computational cost and may fail in noisy environments. Unlike the MUSIC or root-MUSIC, the false estimation of the number of signal sources ($M$) does not affect the accuracy of extracted parameter values in our algorithm. Therefore, our algorithm can even be used for a sufficiently large $M$ case and observe the magnitude of extracted source signal. If the signal magnitude becomes relatively small, we can consider this signal as noise and stop the SPECC algorithm.

### 4. Simulation results

To verify the performance of the PSO-based SPECC algorithm, we use the artificially created data. We consider two source signals which are located in 45° and 135°. The amplitudes and DOAs of two signal sources are shown in Table I. The number of array element $P=10$ and inter-element distance $d=\lambda/2$.

<table>
<thead>
<tr>
<th>number</th>
<th>DOA [degree]</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>1.0 + j0.0</td>
</tr>
<tr>
<td>2</td>
<td>135</td>
<td>1.0 + j0.0</td>
</tr>
</tbody>
</table>

Table I. Amplitudes and DOAs of two signal sources

Fig. 3 shows the extracted parameters of the proposed algorithm in the noise-free condition. The number of time samples ($K$) is assumed as 1. You can see that the two signals sources at 45° and 135° are accurately extracted in the proposed algorithm. Estimated DOAs (45.02° & 134.21°) and its amplitudes (0.9919+j0.0212 & 0.9762-j0.1412) are very accurate.

In next simulation, we added zero-mean white Gaussian noise to the received signals to test robustness to noise.

![Fig. 3. Proposed algorithm, noise-free, $M=2$, $K=1$](image)

![Fig. 4. Proposed algorithm, SNR=15 dB, $M=2$, $K=1$](image)
Fig. 5. Proposed algorithm, SNR=15 dB, M=2, K=10

Fig. 4 and Fig. 5 show the extracted DOAs of two source signals using PSO-based SPECC algorithm with a SNR of 15 dB when the number of time samples (K) is one and ten, respectively. As the number of time samples (K) is increased, the average signal (\(y_p^{\text{ave}}\)) has a lower noise level. Therefore, we can obtain more accurate DOAs.

5. Conclusion

We proposed a novel high-resolution DOA estimation method which is called PSO-based SPECC algorithm in this paper. The proposed algorithm has the high-resolution, robustness to noise, and good accuracy. Our algorithm also converges to the optimum value very fast compared to the conventional method which extracts all DOA parameters at the same time. In the simulation results, we verified these characteristics using the artificially created noise-free and noisy data. Future works include the performance comparison of our method with the well-known methods such as root-MUSIC or EP-based SPECC, etc.

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7. References


