# How can Augmented Reality favor the learning of Calculus?

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**Abstract -** *This paper presents the use of augmented reality for educational purposes within a grounded innovation process in college for the calculus curriculum. The aim of the application developed is offering students an opportunity to deal with spatial visualization skills. An activity performed in spring semester 2014 in a Calculus I course with 20 students shows the positive impact regarding the interaction with mathematics in this mode. The AR Application from 2D to 3D was provided to students in tablets prepared before with the whole activity. As part of the data collection involving the multiple source of information that a case study includes, a brief survey with open-ended questions was applied. We share the descriptive analysis of its results which make us believe that the abstraction feature of mathematics feels reachable when a virtual object stands in front of our own space and time reality.*

**Keywords:** Augmented reality, Calculus, learning, innovation.

## 1 Background

We work at Monterrey Institute of Technology and Higher Education, a private educational institution in México whose educational model cares for the integration of technology in the learning process. Our particular concern is Mathematics education, and as part of an innovation program, we have been combining the use of emerging technologies and educational methods in order to achieve a better math learning. Today we identify ourselves as TEAM, *Tecnología Educativa para el Aprendizaje de las Matemáticas*. We took on the challenge of creating a multi-discipline team that could lead an innovation process for the learning of Calculus in our Campus Monterrey.

Nowadays technology brings the opportunity to transform mathematics curriculum. The way we interact with math knowledge is changing. The algebraic paradigm is not the only way to present Mathematics anymore. Today we can admit that the visual aspects of Mathematics are available everywhere and within our reach, just as our mobile devices are. Why not thinking about a classroom with plenty of images showing the beauty of mathematics knowledge?

Technology changes the way we learn, and students in this millennium are already prepared to live learning experiences through visual and gestural interaction. As part of TEAM we are confident of the advantages in this new paradigm under construction. We are also conscious of the challenge we have in order to make the most of technology with an educational goal inside the higher education institutions. We must be aware of new technology development and, as a matter of fact, we truly enjoy knowing about it. We seek to transform the learning of mathematics through an enjoyable interaction with technology.

The experience we have about Mathematics Education includes the educational research that allows a grounded innovation process in the calculus discourse for college. Three PhD dissertations in Mathematics Education and several Master degrees have been dedicated to build a curriculum innovation in our institution. The new approach promotes the transfer of calculus knowledge to the different specialty branches for engineering, favoring the student's understanding of the application of this knowledge solving real life problems [1, 2 & 3].

The practice with the new didactic for the teaching and learning of Calculus at engineering programs is possible by the development of a training program with teachers at Campus Monterrey. Teachers live the experience with the authors about the way to deal with the textbooks and electronic resources putting at their service. Discussions and reflections about the educational process invite them to make use of technology in classroom. The textbooks collection Applied calculus; mathematical competencies through contexts I, II, and III offer students and teachers a new structure to be introduced to calculus notions and procedures [4, 5 & 6].

Even when this redesign of calculus discourse integrates technology, like spreadsheets and specialized graphical software, now as TEAM we inquire about the introduction of the new emergent technologies. We want to focus learning on the development of certain skills that have not been explicit in the curriculum but nevertheless are important when you deal with mathematics. We believe that in the teaching of calculus some cognitive skills have been taken for granted. Some of them result basic for the understanding of mathematics and useful in problem solving processes. Spatial visualization, for

example, has been assumed as an innate skill in the students. Experience with the teaching of solids of revolution may question if this is so.

We could identify spatial visualization as a cross-curriculum content and took on the task of designing an educational resource improving its development. Augmented Reality technology has been our choice to begin our research about its relative advantages.

## 2 An Augmented Reality Application

Last year we presented at VARE 2013 (Virtual and Augmented Reality in Education International Conference) a prototype of Augmented Reality educational resource. We worked on the design to serve the purpose of supporting the development of spatial visualization skill in students of Calculus I, II and III at college level. The prototype included the part corresponding Calculus I. We are, at the time, working on Calculus II and III.

For VARE presentation we included a case study where 30 students were invited to get to know the application outside the classroom. They didn't know how to use it or how it was done. The experience allows us to understand the students' attitude when interacting with the technology and the devices it included. It also gave us information about certain improvements already done now [7].

In general terms, we found the product acceptance and positive impact on students regarding interaction with mathematics in this mode. We were looking forward to continue with the remaining two parts of the product. Now TEAM moves to complete the Application with the sponsor of Novus Calls 2013 and 2014 that our institution offers in order to promote innovation projects. Figure 1 and 2, respectively, show the graphical user interfaces for the cover and the thematic content of the app.

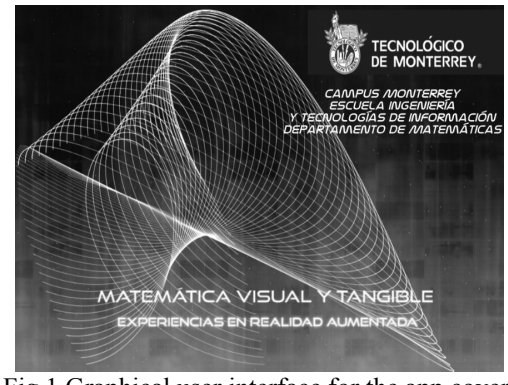


Fig.1 Graphical user interface for the app cover.


Fig. 2 Graphical user interface for the app thematic content.

Dealing with mathematics is dealing with abstractions, a kind of entity that is conceived in our mind through a cognitive process. Building an Augmented Reality system involves producing a virtual object superimposed on a given space in real time. Some abstractions in mathematics give place to an abstract object, but one that now Augmented Reality could represent. Our aim using AR is to provide our mind with a way to conceive a three-dimensional virtual reality combined with a real-time interaction; this way our perception takes place through the 3D object, a mathematical abstraction taking a visual and tangible form.

The first part of the application proposes a way to conceive the 3D object (surface) from an original 2D object (curve). A parabola, sine and circular shapes will go through a process of repetition in parallel planes because of the effect of a parameter *k* introduced in the algebraic representation of each curve. We are looking for the activation of cognitive processes in the student enabling the development of the spatial visualization skill. Figures 3, 4 and 5 show the considered shapes in the app and their associated algebraic representations.

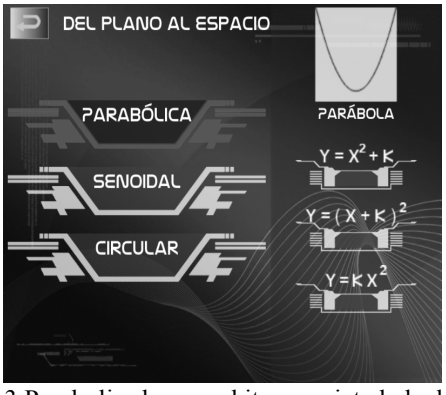


Fig. 3 Parabolic shape and its associated algebraic representations.

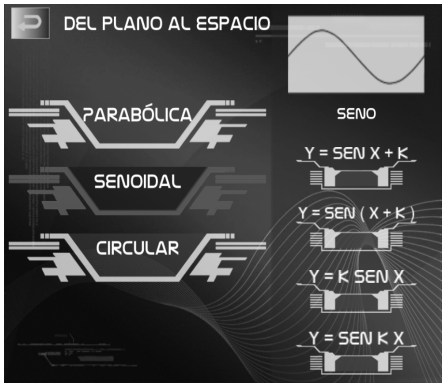

Fig. 4 Sine shape and its associated algebraic representations.

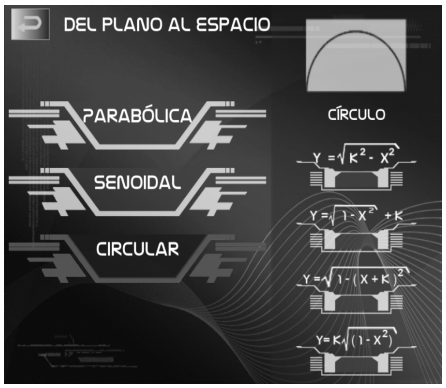

Fig. 5 Circular shape and its associated algebraic representations.

The intention to include RA could certainly be at the service to provide motivational elements for the learning of mathematics, and this also may involve the improvement of student's interest for this science. However, the purpose we mainly seek in our projects is to focus on the didactic benefit of providing a useful knowledge with a new sense of accessibility for a learning experience.

This perspective suggests a new role of technological resources in the learning process. Several authors have been producing appropriate frameworks to study the benefits of didactics involving the use of technology resources. Our perspective shares the conception of digital technology acting as a *mediator* between the user and the mathematical knowledge. The purpose of the interaction between user and technology should favor a co-action that promotes a thinking process that affects learning. Technology should act as a kind of *cognitive partner* for the student, fostering mathematical understanding. Our perspective about digital technology takes into account the knowledge that the discipline of Mathematics Education organizes now in cognitive and sociocultural theories [8, 9 & 10]

# 3 Introducing the AR Application in in Calculus I

During spring semester 2014 we had the opportunity to work with students in the Calculus I course in charge of one of the authors at Campus Monterrey. We designed an activity to perform in one class, during an hour and a half. The design considered working in teams of 3 students each. All the teams had their interaction with the AR Application from 2D to 3D because we provided them with tablets that were prepared before in order to complete the experience at classroom. The activity was videotaped and each particular team had a recorder functioning to capture the talk they had during the activity. This information has not been yet analyzed but we like to share in this paper the final part of the activity, when the students answered a brief survey we gave them in paper.

The survey we applied is based in the one we report in VARE before, when we interviewed the students that were invited to the use of the Application in May 2013. We wanted to confirm if the information collected then still is given now, with the students in Calculus I 2014 class. We act according with a qualitative case study, certainly now we will no offer the complete an in-depth description and analysis of the case but, the information given on the survey is quite useful. We decided to share it as part of the data collection involving the multiple sources of information that the case study should report [11 & 12]. Figure 6 shows students using the app during the course of Calculus I.


Fig. 6 Students using the app during the course of Calculus I.

Next we will give the review of such information involving a five-question survey that was generated and asked to 20 students. The survey contained open-ended questions that address the interaction between respondents and the application, its usability and its content.

A descriptive analysis of participants' answers has been made and it's presented below accompanied by the corresponding question.

1. Do you consider this application would help you develop any mathematical skill? Which skill would it help you develop?

After asking students if the application could help them develop any mathematical skill, 33% of respondents affirm that it was useful to have a better spatial view of mathematics while 28% think the application could be an alternative way to interact with math. Another 22% are sure that the application helps them to get a real perspective of mathematics, while the last 33% think the application helps the students to better understand and interpret plots and graphics.

2. Could you mention any advantage of the fact that the application is available in an electronic tablet?

Concerning the advantage of having the application available in tablet, 60% of participants think portability and practicality are some advantages, while another 15% think user comfort and the ease of use because of their previous experience with the device are advantageous features. 10% of respondents think the easy access to tablets is favorable, and the same amount of students (10%) supports the idea of using the application as an alternative tool because of its interactivity. 5% think there's no advantage at all.

3. Do you think the use of this application could allow learning? Did you learn something using it?

About the learning perceived, 30% think the use of this application is useful to recognize forms, figures and mathematical structures in nature, while 25% appreciate the 3D visualization that the app generates. 20% of participants think the use of the application helps to differentiate between function types (parabolas, circular and sine), 15% favor the way the app interacts with the content, while the other 10% affirm the use of the application contributes to learn tech development, programming and graphics.

4. Please tell us what you think about the interface and the way to interact with the application.

After asking about the interaction interface, 59% of the students think the application is easy to use and understand, 18% favor the attractive graphical user interface and 14% mention the application need a little tweak, as it's functional but it still has some bugs that can be fixed. 9% of participants express the need to have a shortcut to take a screenshot, while some others (2) would prefer an iOS environment instead of Android.

5. Do you think this application could be useful to acquire knowledge in your career (field of work)? Please give us an example.

Finally, the students were asked if the use of application could be useful in their field of work, and there were many different responses, mostly because all students are in multiple careers. Some think 3D graphics treatment could be achievable with the application, or to establish a link between mathematics and technology. Some other think the application could be useful in acoustics, to observe sound waves, or to better view and understand blueprints. Other students think it could be useful to analyze logistics procedures, or some measurements in industrial machines, to see the behavior of a certain reaction or as an editing software of some sort for their needs, as a visual programming tool or just to use it along the teaching of mathematics. A single student answered the application was not useful at all in their career (Economy & Laws). Figure 7 shows students working during the case study.

Fig. 7 Students working during the case study.

# 4 Concluding remarks

Mathematics is highlighted because of the abstract nature of their notions and concepts, and the cognitive and learning difficulties that accompany its presentation and understanding confirm a didactic problem that deserve to be working on. Augmented Reality technology stand out for its potential as an attractive alternative when working to create real Mathematics content for students. The abstraction feature of Mathematics feels reachable when a virtual object stands in front of our own space and time reality. Data obtained promise that our work deserves to continue. Improvement of

mathematics education should consider the integration of emergent technologies as Augmented Reality.

The current technological development asks for its use in education, and mathematics education has a great opportunity to be fortified. The interdisciplinary team that worked on the first call of Novus 2012 at Tecnológico de Monterrey has been expanding with graduate students, and continues to seek the means to consolidate as TEAM: Educational Technology for Learning Mathematics. Today we continue looking for the ways to overcome our attitude of innovation based in research education working with students of our Educational Innovation Doctorate Program.

We are aware that innovation in education could be considered as such by the mere fact of using technology. But being responsible for an attitude of innovation in the use of digital technology in math education requires a constant updating in new technology development and a continuous attitude of educational researcher. The educational innovation that provides TEAM interested is concerned with curriculum innovation. In this sense, the content of learning and the means by which we learn it are always being subjected to a continuous questioning. Educational research is becoming a way of consolidating this TEAM.

# 5 Acknowledgements

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