Math Words and their Dendrograms

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Abstract
This document present a hierarchical clustering algorithm based on graph theory, which, from generation of a path from a given vertex, builds a math word and calculates a cluster under an index. This is made possible due to modification of Tarry's algorithm, by exchanging path elements. When the one dimensional clustering index is added to σ, it gives us, what I have called, Tarry's hierarchy. From the definition of net word, cycle, tree, tree word and vertex, a theorem on the relationship between vertices, lines, and letters of a labyrinth is shown, which allows the generation of words and their Dendrograms with the application of Euclidian distance. The practical use of these concepts is then shown, namely, that they can provides possibilities of connections in arrangements for telephone centrals.

Keyword: Dendrograms, Tarry, Math words, connected, labyrinth

Introduction
The first person who studied the combinatory properties of schemes was Leonard Euler in 1736; he studied the network of the seven bridges in Konigsberg, figure 1. He wrote, in Berlin in 1739, "*in Königsberg Pomeranie, they have a little island named Kneiphof, the river is divided into two and around the island there are seven bridges. You can arrange a network where by you only walk over one at one time*". He continued, "Is possible for everyone, but that not everyone has the capacity to do it" Euler (1739).

The modern theorem enunciated by Euler, demonstrated the necessity of the parity of the valence in each vertex: *A connected graphic is eulerienne if all vertex have degree pair*. Since then, graph theory has developed slow but steadily. Its principal contributors are G. Tarry, who wrote about labyrinths in 1886 and 1895 (see Tarry 1886 and 1895), D. Konig (1936), C. Berge (1957), with a book about graphs and hypergraphs, W. T. Tutte, with his studies about Hamiltonian networks (1976). S. Bollobas, who wrote about Hamiltonians cycles in regular graphs, and P. Rosenstielh, with *Existence d'automates finis capables de s'accorder bien qu'arbitrairement connectés et nombreux* (1966), and *Labyrintologie mathématique* (1971), (see Rosenstiehl, 1971), and *Les graphes d'entrelacement d'un graphe* (1976). Recently, works on graph theory, like *A performance comparison between graph and hypergraph topologies for passive star WDM light wave networks*, (1998) by H. Bourdin, A. Ferreira and K. Marcus; as well as the work by Gondran and M. Minoux, entitled *Graphs et Algorithms* (1979), have gained attention. More recently, the randomized Tarry algorithm has been discussed in the article named searching a graph by Shmuel Gal (2004), and by Urretabizkaya and Rodríguez (2004), who implement the Tarry algorithm for solving mazes of known structure (2004). Today, graph theory is used in branches of mathematics such like theory of groups, topology, and theory of numbers, data analysis and clusters.

Among those who have contributed the most to the development of theory and models in telephone connections, are A. K. Erlang, who implemented the well-known *Erlang* Probability

Density Function, as well as works on *Solutions of Some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges*, (1917 and 1918); A. and Elldin, with his work *Switch Calculations General Survey*, published by LM Ericsson in (1955). More recent works on the subject include, *Network Flows: Theory, Algorithms and Applications*, by R. V. Ahuja, T. L. Magnanti, and J. B. Orlin, (1993), and *An Algorithm for Hierarchical network design*, by G. R. Mateurs, F. R. B. Cruz, and H.P. L. Luna (1994).

Definitions and algorithms

Definition. A graph is a pair $G = (X, U)$ where X is a finite set, of vertices of G, each element of which is incident to two elements of another finite set V named the set of vertices, see Berge (1970).

Definition. A labyrinth, $\mathcal{L}$, is a finite set A not empty and of even cardinality, named set of words of $\mathcal{L}$ or alphabet, supplied of one involution l without an indeterminate point (i.e: without a tendency change) called by the prime operator:

$$\mathrm{l} \in A \text{ then } \mathrm{l}' \in A\,,\ \mathrm{l}' \neq \mathrm{l} \text{ and } (\mathrm{l}')' = \mathrm{l}$$

and one relation of equivalence (called “with the same right as” where the classes are “the points” of labyrinth, the letters of same class has the same right as the point) indicated by the application of the letters which belong the letters in the set X of class:

$$\mathrm{d}: A \longrightarrow X \quad : \quad \mathrm{d}(\mathrm{l}) = \mathrm{d}(k)$$

by < the right of l is equal to right of k >, where < l has the same right as k >, see Rosenstiehl (1971).

It is possible to speak of the left of letter too, making: $\mathrm{g}: A \longrightarrow X$ with $\mathrm{g}(\mathrm{l}) = \mathrm{d}(\mathrm{l}')\ \forall\ \mathrm{l} \in A$, as far as the labyrinth its expressed by triad (A, i, d).

Definition. A labyrinth (A, i, d) is said to be orientated, when one part A^+ of A such that:

$$\mathrm{l} \in A^+ \Leftrightarrow \mathrm{l}' \notin A^+ \quad \forall \quad \mathrm{l} \in A$$

Definition. Based on this definition of a labyrinth (A^+, i, d), σ is called a word of this labyrinth if it belongs to some of following classes:

- $\Delta_\alpha\ \forall\ \alpha \in X$ is called empty words of $\mathcal{L}$
- l with $\mathrm{l} \in A$ is called word-letter of the $\mathcal{L}$, and
- $\sigma = \mathrm{l}_1, .., \mathrm{l}_r, \mathrm{l}_{r+1}, \ldots, \mathrm{l}_p$ with $\mathrm{l}_r \in A \cup (\Delta_\alpha)\ \ \forall\ \alpha \in X$ we have $\mathrm{d}(\mathrm{l}_r) = \mathrm{g}(\mathrm{l}_{r+1})\ \forall\ r = 1, .., p\text{-}1$

Therefore, given X as the set of vertex, you can have left and right applications in all word of the labyrinth $\mathcal{L}$, defined as: $\mathrm{d}, \mathrm{g}: A \cup (\Delta_x)_{x \in X} \rightarrow X$ or $\mathrm{d}, \mathrm{g}: \mathcal{L} \rightarrow X$ with $\mathrm{g}(\Delta_x) = \mathrm{g}(\mathrm{l}_1)$ and $\mathrm{d}(\Delta_x) = \mathrm{d}(\mathrm{l}_p)$, i.e., $\mathrm{g}(\Delta_x) = \mathrm{d}(\Delta_x) = x$. And remember, σ is cyclical if $\mathrm{g}(\sigma) = \mathrm{d}(\sigma)$. For more elaborate definitions of tree, pure word and neutral word, see Golumbic (1980) and Bollobás (1978).

Definition. One tree A is a graph $G = (X, U)$ connected and non-cyclical.

Definition. σ of $\mathcal{L} = (A, \mathrm{i}, \mathrm{d})$ it is a pure word if inside σ there exists a occurrence of each letter of *A*.

Definition. If $\sigma \in \mathcal{L}$ and $\sigma \approx \Delta_\alpha$ $\forall$ $\alpha \in X$, σ is a neutral word in α, if the neutral element Δ_α is a neutral word in some α-particular.

Algorithm of Tarry

Definition. If $\mathcal{L}$ labyrinth is connected, we say Tarry's word of $\mathcal{L}$ all pure word $\Theta \in \mathcal{L}$ such that the entry tree *V* of Θ and the out tree *W* of Θ are opposites, i. e., $W = V\,'$.

Let $\mathcal{L}$ be the lexical of connected labyrinth $\mathcal{L} = (A, \mathrm{i}, \mathrm{g})$, with $l_1 \in A$. Let $V(\sigma)$ be the entry tree of σ, with σ a left factor of Tarry's word Θ to apply the following algorithm, see Tarry (1895):

T.0 To put $\sigma \leftarrow l_1$

T.1 If $\sigma\, l \in \mathcal{L}$ with $l \notin \sigma$ and $l\,' \notin V(\sigma)$ to put $\sigma \leftarrow \sigma l$

but

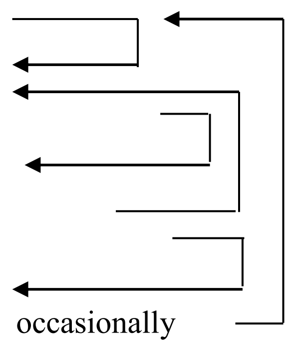
T.2 If $\sigma\, l \in \mathcal{L}$ with $l \notin \sigma$ to put $\sigma \leftarrow \sigma l$

if

but

T.3 Stop. Write $\Theta = \sigma$

Where $\Theta = \sigma$ is called Tarry's word



Note: The inversion of all entry letter is an out letter in $\Theta = \sigma$, where Θ is a cyclical word, and all letter of A haves the same right as the same left of Θ, see Casanova (1982).

Attachment index

The index of aggregation or level of classification of ways connected to the orientated labyrinth $\mathcal{L}$, begin with the first cycle or pleat of the Tarry's word. In this case, the index level is unit. When Tarry's word σ is formed, you always have to replace σ in the first place of the first obtained cycle.

Definition. If we let l_1 and $l_2 \in \mathcal{L}$ stand for two paths and let α stand for all letters with the same left $\mathrm{g}(\Theta)$, and β stand for all letter with the same right $\mathrm{d}(\Theta)$; the minimal distance d_M, the inferior ultrametric minimal distance over a point $x_0 \in l_1$ and $x_1 \in l_2$ is:

$$d_M(l_1, l_2) = \min \{ d(x_0, x_1) \mid x_0 \in l_1 \,\&\, x_1 \in l_2 \}$$

Remember, the algorithm is applied after finding the Tarry's word and his cluster by couples or pleats. To finish the last pleat, begin the letter's arrangement in σ. You must begin with the first pleat.

Conclusions

Since every problem demands a full solution using links that indicate the way to construct the algorithm solution within in a formal language capable of analyzing a system by which you can recreate the transit in its construction in any direction, we have presented here a new and modern form of Tarry's algorithm. This allows us to create a hierarchy based on a vertex, the levels of aggregation for the construction of the word of the circuit or labyrinth which applied to the something of the telephones, and lets us know configurations of connections with optimal telephone line use, fluidity and economy.

To date there has been no published algorithm of aggregation (hierarchical or otherwise), based on theory of graphs, that starting from the construction of a trajectory begun on a given vertex, which would construct a math word and calculate an aggregation under an index. The latter has been possible thanks to the modification made in Tarry's algorithm, through the exchange of elements, which has allowed a better arrangement in the union of pairs of letters and the construction of the math words. The one dimensional aggregated index applied to σ results in what I have called Tarry's hierarchy. The next step is to apply weight to aggregation algorithms.

There is no published clustering algorithm (whether hierarchical or not), based on graph theory which, from generation of a path starting on a given vertex, builds a math word and calculates clustering under an index. This has been possible by modification to Tarry's algorithm, through exchange of elements, which has allowed a better arrangement of letters coupling and the construction of a math word. The one dimensional clustering index applied to σ gives what I call Tarry's hierarchy.

References

Berge, C. 1970. *Graphes et hypergraphes*. Dunod.

Bollobás, B. (ed). 1978. *Advances in graph theory*. North-Holland. North-Holland ISBN: 0 7204 0843 1.

Bourdin, H., Ferreira, A., and Marcus, K.1998. "A Performance Comparison between Graph and Hypergraphs Topologies for Passive Star WDM Light wave Networks". *Computer Networks and ISDN Systems*, 8(30), pp. 805-819. Doi: 10.1016/S0169-7552(97)00125-6.

Casanova del Angel, F. 1982. "Introducción a la teoría de gráficas, a los laberintos y a sus palabras". *Boletín de Graduados e Investigación*, vol. I, núm. 3, pp. 23-51. IPN. México.

Euler, L. 1739. *Solutio Problematis ad geometriam situs pertinentis*. Mémoire de l'Academie des Sciences de Berlin.

Golumbic, M. Ch. 1980. *Algorithmic Graph Theory and Perfect Graphs*. Academic Press. ISBN: 0-444-51530-5.

Gondran, M and Minoux, M. 1979. *Graphes et algorithmes*, éditions Eyrolles, coll. « Dir. Ét. &Rech.EDF », 1979 (réimpr. 1985), 546 p. EAN13: 9782212015713.

Kőnig, D. 1936. *Theorie der Endlichen und Unendlichen Graphen: kombinatorische Topologie der Streckenkomplexe*. Leipzig: Akad. Verlag.

Tarry, G. 1886. *Parcours d'un labyrinthe rentrant*. Assoc. Franç. Pour l'Avanc. Des Sciences. pp. 49-53.

Tarry, G. 1895. "Le problème des labyrinths". *Nouvelles annales de Mathematiques*, vol. XIV.

Rosenstiehl, P. 1971. "Labyrinthologie Mathematique". *Mathematique et Sciences Humaines*, 9e année, Num. 33, 5-32.

Rosenstiehl, P. 1976. *Les graphes d´entrelacement d´un graphe*. Coll. Inter. CNRS nº. 260, Problèmes Combinatories et Théorie des Graphes, Orsay (Editions du CNRS, 1976) 359-362.

Shmuel Gal. 2004. Searching a graph. A working version.

Urretabizkaya, R and Rodríguez, O. 2004. Análisis e implementación de algoritmos para la solución de laberintos de estructura conocida. Universidad Autónoma de Querétaro. México.

W T Tutte. 1976. On Hamiltonian circuits. Colloquio Internazionale sulle. Teorie Combinatoire. Atti Convegni Lincei 17. Accad Naz. Lincei. Roma I, pp. 193-199.