A Dynamical System Approach to Compute the Evolution of Daily Commuting Traffics Interacted with Travel Time Information

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Abstract - In this paper, a day-to-day flow evolution model is developed for a vehicular network equipped with Intelligent Transportation Systems (ITS) providing road users with pre-trip travel time information. Daily route swapping process of commuting traffic is considered and formulated as a dynamical system. It means that a single user will change to a less costly path tomorrow compared to his travel time over today’s route and travel time of other routes detected today by ITS. The result tells that the proposed model has the ability to provide the evolution of traffic states that helps to design control strategies and to mitigate daily congestion not just based only on equilibrium state. A simple network is provided to illustrate the volume-shifting among competing paths and the asymptotic behavior of system variables.

Keywords: Dynamical System, Lyapunov Stability, Day-to-Day Traffic Assignment, Intelligent Transportation Systems (ITS)

1 Introduction

This paper attempts to formulate how traffic information distributed by ITS operations influences the temporal trajectory of network flows in a theoretical viewpoint. Based on similar behavioral assumptions of minimal-travel-time seeking (MinTTS) and ordinal perception of travel-time-updating process (OPTTU)[1-4], the interactions between road users and ITS operator are decomposed into four parts: the travel time induced path flow dynamics (PFDTT), the demand induced path flow dynamics (PFDQ), the travel time induced demand dynamics (DDTT), and the predicted travel time dynamics for an origin-destination (O-D) pair (PTTDOD). PFDTT describes the collective results of user’s daily travel decision by transfer to less congested path with path travel time information provided from ITS services. The other three components, PTTDOD, DDTT, and PFDQ, are concentrated on the evolutionary behavior of system variables predicted by ITS operators to act as a benchmark in guiding whole systems towards an expected and preferable status. For paper space saving, detailed literature review and stability analysis [5] are omitted in this version of paper.

2 Modeling network dynamics

The basic stimulus-response structure incorporates three main implications in this study. They are: all responses and stimuli discussed hereinafter are macroscopic and divided into two clusters, users and ITS operator; response \( \hat{R}_i^t \) is defined as the time change rate of system variable with one day lag and denoted as a function of stimulus \( St_i^t \) and sensitivity \( Se_i^t \); and stimulus is specified as the difference between experienced (or observed) status \( \tilde{x}_i^t \) and expected (or predicted) status \( \tilde{x}_i^t \) of a system variable \( i \) at day \( t \).

\[
\hat{R}_i^t = F_i(Se_i^t, St_i^t),
\]

\[
St_i^t = \tilde{x}_i^t - \tilde{x}_i^t.
\]

2.1 User dynamics

User dynamics defined in this paper is the travel time induced path flow dynamics (PFDTT). By similar proposition of [4], the intention of developing PFDTT is extremely straightforward that if users wish to improve travel time tomorrow, they might select a faster path than today. The stimulus defined in the following formulation is the collective effects of travel time difference [6] at day \( t \) between two statuses of each user \( i \) selecting path \( p \), the “experienced” status \( c_{i,p}^t \) and the “expected” status \( \hat{c}_w^t \) (minimal travel time) for O-D pair \( w \).

\[
\hat{h}_{p,PFDT}^t = -\alpha_p \left( \sum_{i=1}^{k_p} (c_{i,p}^t - \bar{c}_w^t) \right) = -\alpha_p h_{p}(c_{p}^t - \hat{c}_w^t). \tag{3}
\]

With full information of travel time, Equation (4) is the result of ordinal perception process which determines the target
routes of flow shifting, from path \( p \) to path \( q \) if \( c'_q < c'_p \). It is assumed that the amount of shift is weighted by travel time \( \omega_{p \rightarrow q} \) to reflect the congestion effect.

\[
\dot{h}_{p \rightarrow q}^t = -h_{p \rightarrow q}^t \omega_{p \rightarrow q} \quad (4)
\]

### 2.2 ITS operator dynamics

With similar adaptive and learning process presented previously, ITS operators utilize the detected information to compare and update three system variables. The first one of ITS operator dynamics is demand induced path flow dynamics, \( PFD_D \). It comes from the difference between “observed” path flow \( h_w^t \) and “expected/predicted” demand \( D_w^t \) for an O-D pair \( w \) weighted with sensitivity \( \alpha_{D_w} \) and travel time \( \gamma_{t_w} \).

\[
\dot{h}_{p \rightarrow q}^t = \alpha_{D_w} (D_w^t - h_w^t) \gamma_{t_w} \quad (5)
\]

The second ITS operator dynamics is the predicted travel time dynamics for an O-D pair \( (PTTD_{OD}) \). The difference between “observed” path flow and “expected/predicted” demand for an O-D pair \( w \) generates a simultaneous effect on \( PTTD_{OD} \) to reflect an expected increase of \( c_w' \) if path flow is predicted to increase, i.e. if \( D_w^t > h_w^t \).

\[
c_w' = \beta_w (D_w^t - h_w^t) \quad (6)
\]

The last ITS operator dynamics is the travel time induced demand dynamics \( (DD_{TT}) \). Demand of an O-D pair is allowed to be a little disturbed purely by travel time fluctuations. It is formulated as the response due to the difference between the minimal path travel time (an “observed” status) and the predicted travel time (an “expected” status) of an O-D pair.

\[
D_w^t = \alpha_w (c_w' - c_w) \quad (7)
\]

### 3 Numerical results

A five links network with strict monotone cost function with respect to link flow is used to evaluate the proposed model numerically. There is one O-D pair connected by three paths in this example network. Figure 1 shows the effects of path flow dynamics (path flow switching) that come from the net volumes simultaneously generated by \( PFD_{TT} \) and \( PFD_D \). And the trajectory seems to be convergent as time evolves.

Table 1 shows the selected results of system variables at the initial time, at the 20th time step, and at the steady state respectively. The three path travel times and the predicted travel time of O-D pair are all the same which reveals the user equilibrium [7]. The predicted demand for an O-D pair is also equal to the sum of path flows at steady state.

<table>
<thead>
<tr>
<th>Path 1</th>
<th>Initial Flow State at ( t=200 )</th>
<th>Initial Travel time State at ( t=200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>40.55</td>
<td>91.74</td>
</tr>
<tr>
<td>45</td>
<td>41.12</td>
<td>93.74</td>
</tr>
<tr>
<td>10</td>
<td>4.96</td>
<td>94.60</td>
</tr>
</tbody>
</table>

Predicted travel time of O-D pair \( w \)

<table>
<thead>
<tr>
<th>Demand of O-D pair ( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.00</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of path flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>85.00</td>
</tr>
</tbody>
</table>

### 4 Conclusion

The proposed nonlinear system considering both user and operator dynamics is organized as four parts: travel time induced path flow dynamics \( (PFD_{TT}) \), predicted travel time dynamics for an O-D pair \( (PTTD_{OD}) \), travel time induced demand dynamics \( (DD_{TT}) \), and demand induced path flow dynamics \( (PFD_D) \). A simple numerical example is provided to demonstrate the shifting effect among path flow. By collecting path-flow shifting information, traffic control strategies can then be simulated in transition state but equilibrium state which usually appeared in historical studies. That is essential to make the evaluation of the effectiveness and efficiency of ITS-related deployments more reasonable.

### 5 References