A New Level Set Method for Biomedical Image Segmentation

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Abstract - This paper presents a new biomedical image segmentation method that applies an edge-based level set method. According to low contrast in biomedical images, we mainly focus on introducing the Laplace operator in external energy of level set method for accurately detecting object edge. A preliminary evaluation of the proposed method mainly performs on gallstone detection and extraction, mammographic image segmentation, iris inner location and polysaccharides extraction. Finally, the comparison experimental results demonstrate that our proposed approach potentially performs better than the representative level set method for biomedical image segmentation in terms of sensitivity, accuracy and specificity, with same initial contours.

Keywords: Biomedical Image Segmentation; Level Set Method; External Energy.

1 Introduction

Osher and Sethian first introduced level set method in 1988 [1]. In the past two decades, level set methods have seen the rapid development in many aspects within the image processing and computer vision field, such as Global Optimization, Etching, Deposition, and Lithography Development and so on [2], especially in image segmentation. In fact, image segmentation is one of the fundamental and significant tasks in image processing and computer vision. Among the applications in image processing, level set method has great potential for developing image segmentation algorithm. There are large amounts of algorithms and techniques that have been developed to solve image segmentation problems. Malladi et al. [3] introduced shape modeling with front propagation based on level set method to implement image segmentation. Typically, an image segmentation study applying level set approach has been performed independently by Caselles et al. [4]. They presented geodesic active contour models based on curve evolution, geometric flows and level set method [3,4]. The fundamental principle of level set method is to represent a contour as the zero level set of a higher dimensional function, usually called a level set function (LSF), and to formulate the motion of the contour as the evolution of the level set function based on a partial differential equation (PDE). It is crucial to keep the evolving level set function as an approximate signed distance function non-periodically during the evolution, especially in a neighborhood around the zero level set. However, this process needs re-initialization, which is an absolutely necessary step to realize level set function to a signed distance function during the evolution. Re-initialization scheme has been extensively used as a numerical remedy for maintaining stable curve evolution and ensuring desirable results. So far, the re-initialization process is quite complicated in computation and has stable side effects. Moreover, the level set function can develop shocks. To avoid these problems, it is necessary to re-initialize level set function periodically, but when to apply the re-initialization and how to make re-initialization achieve periodically during the curve evolution are still serious problems as mentioned in [5]. These problems also remain in [3,4]. Li et al. [6] presented a new variational level set method to force the level set function to close to a signed distance function and eliminate the re-initialization procedure. Li's scheme obtained good results for medical image segmentation. Furthermore, a significant research effort has been devoted to the design of effective images segmentation methods based on Li’s model over in recent years. The techniques based on Li’s scheme and C-V model for SAR image segmentation implied in [7]. Lately, Li et al. [8] introduced a distance regularized level set evolution (DRLSE) by incorporating a double-well potential function used in the geodesic active contour model [4]. By contrast, the DRLSE is more efficient than conventional level set formulations applying to image segmentation. Ni et al. [9] proposed an advanced variational formulation based on DRLSE, named ADRLE (advanced distance regularized level set evolution), that forces the level set function to be close to a signed distance function. The proposed method merges DRLSE equations and a signed pressure force function in [10].

However, most of biomedical images are often poor in contrast. Therefore, we improved the external energy of level set method based on Li’s model so as to urge the zero level set to be much closed to the region to be segmented, the experimental results and comparisons show that we improved method outperforms.

The following part of this paper is divided into five sections. Section II overviews the level set method which
gives a brief description of the traditional and variational level set method. Section III proposes the new algorithm for medical image segmentation. Section IV achieves the numerical implementation of our proposed method. Section V shows the experimental result by applying our approach to medical images. In the last section, a conclusion is made and some issues for further research are suggested.

2 Traditional level set model

To date, level set method has been rapidly developed. Since the variational level set model is a modified version of the conventional level set model, we review the conventional level set model briefly and then provide improvements on it for practical application.

2.1 Overview of traditional level set model

Actually, level set method is mapping from higher dimensional to lower dimensional. This method is able to express curves of complex topology and to handle topological changes automatically, i.e. naturally splitting and merging. Also, the level set method can efficiently perform numerical computations involving curves and surfaces on a fixed Cartesian grid. The level set method is based on curve evolution theory which can be expressed as follows:

\[ \frac{\partial C}{\partial t} = FN \]  

(1)

Where, \( N \), the inward normal vector to the curve \( C \), can be represented as \( N = -\nabla \phi / |\nabla \phi| \). \( F \) represents the speed function that controls the evolution of the curve \( C \). The level set method supposes that the curve \( C \) is represented by the zero level set of a level set function \( \phi(x,y) \).

\[ C(t) = \{(x,y) | \phi(x,y) = 0 \} \]  

(2)

Level set equation, the curve evolution equation of the level set function \( \phi \), can be written in the following general form, i.e. partial differential equation (PDE):

\[ \frac{\partial \phi}{\partial t} = F|\nabla \phi| \]  

(3)

Applying to the image segmentation, the speed function \( F \) mainly relies on the image data and the level set function \( \phi \). Although level set method has desirable advantages being applied to a wide range, there exists the problem that the level set function can develop sharp or flat shocks during the evolution. In term of this difficulty, a general numerical scheme known as re-initialization [1,10] is used periodically during the evolution.

\[ \frac{\partial \phi}{\partial t} = sign(\phi)(1-|\nabla \phi|) \]  

(4)

The re-initialization scheme considers level set function as a signed distance function to remain stable during the evolution process. However, this scheme may cause the zero level set away from the expected position [1], so this numerical scheme should be avoided.

2.2 Level set model without re-initialization

Li et al. [6] presented a new variational level set method to force the level set function to close to a signed distance function and completely eliminate the need of the re-initialization procedure. This variational formulation is associated with a penalty term that penalizes the deviation of the level set function from a signed distance function. The penalty term which plays a key role in the variational formulation is as follows:

\[ P(\phi) = \frac{1}{2} \int |\nabla \phi - 1|^2 dxdy \]  

(5)

Naturally, it makes function \( \phi \) satisfying \( |\nabla \phi| = 1 \) closed to the signed distance function. The penalty term eliminates the need for re-initialization and allows the use of a simpler and more efficient numerical scheme in the implementation than those used for conventional level set formulations.

The edge indicator function \( g \) can move the zero level set to the object boundaries in image segmentation, it defines as:

\[ g = \frac{1}{1 + |G_{\sigma} \ast I|^2} \]  

(6)

Here, \( I \) represents an image. \( G_{\sigma} \), the Gaussian kernel with standard deviation \( \sigma \), is used to smooth the image to reduce noise. The energy functional is defined as:

\[ E = mE_{int} + E_{ext} = mE(\phi) + \lambda L_g(\phi) + \alpha A_g(\phi) \]  

(7)

It's just the co-activation of the internal and external energies that make the zero level set curve \( C \) matching the boundaries well and reach a perfect effect of image segmentation. Where, \( m \) is a parameter controlling the penalization effect of the internal energy. The energy functional \( L_g(\phi) \) computes the line integral of the function \( g \) along the zero level contour of \( \phi \). The energy functional \( A_g(\phi) \) computes a weighted area of the region inside the zero level set. \( \lambda \) is the coefficient of the energy functional \( L_g(\phi) \), and \( \alpha \) is the coefficient of the energy functional \( A_g(\phi) \). For images with weak object boundaries, the value of \( \alpha \) should be chosen small to avoid the active contour passing through the object boundaries. \( L_g(\phi) \) and \( A_g(\phi) \) are defined by

\[ L_g(\phi) = \int \delta |\nabla \phi| dx \]  

(8)

\[ A_g(\phi) = \int gH(0,\phi) dx \]  

(9)

Where, \( \delta \) and \( H \) are the Dirac delta function and the Heaviside function, respectively. In most level set methods, the Heaviside function \( H \) and the Dirac delta function \( \delta \) are approximated by the following functions:

\[ H_\xi(x) = \begin{cases} 
\frac{1}{2} \left( 1 + \frac{x}{\xi} + \frac{x - \xi}{\xi} \right), & |x| \leq \xi \\
1, & x > \xi \\
0, & x < -\xi 
\end{cases} \]  

(10)
\[ \delta_\xi(x) = \begin{cases} \frac{1}{2\xi} & |x| \leq \xi \\ 0 & |x| > \xi \end{cases} \]

\( \delta_\xi \) is the derivative of \( H_\xi \), i.e. \( H_\xi' = \delta_\xi \). By calculus of variations, the Gateaux derivative of the functional \( E \) in can be written as:

\[ \frac{\partial E}{\partial \phi} = \frac{mc}{\partial \phi} \left( \Delta \phi - dI \frac{\nabla \phi}{|\nabla \phi|} \right) + \lambda \delta(\phi) \frac{\partial}{\partial \phi} \left( f \frac{\nabla \phi}{|\nabla \phi|} \right) + \nu \delta(\phi) \]

(12)

3 Improved level set model

In order to deal with intensity inhomogeneities in medical image segmentation, we formulate our method based on the detection of lesions and to locate suspicious regions in medical images for more detail examination by the attending physicians, in which intensity inhomogeneity is attributed to a component of an image. Since, several medical image databases are too big, it costs too much computational time. To take these two factors into account, we make our efforts on proposing new efficient level set method applied to medical images segmentation. Actually, our purpose is to remove blurring and highlight edge from medical images. Therefore, the Laplace operator is used, which could make the bright spot becoming much brighter than the surrounded pixels in the image. As is well known, Laplace operator, one of the edge detection operators, has nothing to do with the direction of an edge. This operator is also one of the simplest sharpening filters, whose response to isolated pixel is stronger than the response to the edge or line. It is noted that the Laplace operator is a second order differential operator in the n dimensional Euclidean space, applied to biomedical image \( I(x,y) \) is defined as:

\[ \Delta I(x,y) = \nabla^2 I(x,y) = \frac{\partial^2 I(x,y)}{\partial x^2} + \frac{\partial^2 I(x,y)}{\partial y^2} \]

(13)

Here, \( \Delta \) is the Laplace operator. The equation (13) can be implemented in discretized form as the following finite difference equation:

\[ \Delta I = (I_{i+1,j} + I_{i-1,j} + I_{i,j+1} + I_{i,j-1}) - 4I_{i,j} \]

(14)

Which illustrates that the gradient of pixel \( I_{i,j} \) only relates to the four adjacent pixels in an image, i.e. \( I_{i+1,j}, I_{i-1,j}, I_{i,j+1}, I_{i,j-1} \), but is independent of \( I_{i+1,j+1}, I_{i+1,j-1}, I_{i-1,j+1}, I_{i-1,j-1} \) which are usually on the edge of an image. We apply quadric-direction Laplace operator defined as follow:

\[ \Delta I(x,y) = I_{xx} + I_{yy} + I_{xy} + I_{yy} \]

(15)

This equation is discretized as follows:

\[ \Delta I = (I_{i+1,j} + I_{i-1,j} + I_{i,j+1} + I_{i,j-1} + I_{i+1,j+1} + I_{i+1,j-1} + I_{i-1,j+1} + I_{i-1,j-1}) - 8I_{i,j} \]

(16)

Fig.1 and Fig.2 express quadric-direction Laplace operator and its template form, respectively. Let \( J=\Delta I \), \( I \) is the true image, which measures an intrinsic physical property of the mammography being imaged, is therefore assumed to be piecewise (approximately) constant. So we redefined the edge indicator function by

\[ f = \frac{1}{1+|\nabla G_\sigma * J|} \]

(17)

Where, \( G_\sigma \) is also a Gaussian kernel with standard deviation \( \sigma \) with same function in equation (6). Because the gray scale of mass region in mammographic image is higher than the region surrounded, this function \( f \) usually takes smaller values at object boundaries than at other locations. As the application of Laplace operator, the bright mass becoming much brighter than the surrounded pixels in the image. So the edge detection function \( f \) is easier to become smaller.

\[ \frac{\partial \phi}{\partial t} = -\frac{\partial E}{\partial \phi} = \frac{mc}{\partial \phi} \left( \Delta \phi - dI \frac{\nabla \phi}{|\nabla \phi|} \right) + \lambda \delta(\phi) \frac{\partial}{\partial \phi} \left( f \frac{\nabla \phi}{|\nabla \phi|} \right) + \nu \delta(\phi) \]

(20)
The function $\phi$ minimizes this functional to satisfy the Euler-Lagrange equation.

4 Update the proposed level set function

The partial differential equation in the continuous domain defined in Eq.(20) can be solved by a finite difference method in numerical scheme. All the spatial partial derivatives are approximated by the central difference and the temporal partial derivative is approximated by the forward difference. Then, the numerical scheme of the gradient flow mentioned above using the forward difference can be simply written as follows:

$$\frac{\partial \phi}{\partial t} = \frac{\phi^{k+1} - \phi^k}{\tau} = L(\phi^k) \tag{21}$$

Where $\tau$ is the time-step, we choose a fixed step size $\tau$. $L(\phi^k)$ is the numerical approximation of the right-hand side in (17). In $L(\phi^k)$, the corresponding curvature $\kappa$ is defined as:

$$\kappa = d\ln \left\{ \frac{\nabla \phi}{|\nabla \phi|} \right\} = \frac{\phi_{x,x}^2 + 2\phi_{x,y}^2 + \phi_{y,y}^2}{(\phi_x^2 + \phi_y^2)^{3/2}} \tag{22}$$

Here, the curvature is discrete using a second-order central differencing scheme. For a sake of clarity, Eq.(20) can be implemented as follows:

$$\frac{\partial \phi}{\partial t} = \frac{\phi^{k+1} - \phi^k}{\tau} = L(\phi^k) = m(\phi_{x,x} + \phi_{x,y} + \phi_{y,y} + \phi_{x,y} - 4\phi_{x,y} - \kappa) + \lambda \delta_i(\phi_{x,x}) + \nu \delta_i(\phi_{y,y}) \tag{23}$$

Where, $\kappa$ and $\delta_i$ are computed according to (19) and (11), respectively.

4.1 Initialization of the proposed level set function

We initialize the level set function as following:

$$\phi_i(x,y) = \begin{cases} -d, & (x,y) \in \Omega_0 - \partial\Omega_0 \\ 0, & (x,y) \in \partial\Omega_0 \\ d, & (x,y) \in \Omega - \Omega_0 \end{cases} \tag{24}$$

It is a binary step function defined above that can be generated efficiently. Where $\Omega$ is an image domain, $\Omega_0$ is a sub-region of the image domain, and $\partial\Omega_0$ is the boundary of $\Omega$, and $(x, y)$ is any pixel of the image. What is needed at the very beginning is a list of the coordinates of all required grid points together with their initial level set values. As previously mentioned in [5], if the regions of interest can be obtained in some way, then we can use these roughly obtained regions as the region $\Omega_0$ to construct the initial level set function $\phi$. Moreover, if the initial subset $\Omega_0$ is close to the region to be segmented thus, not only a small number of iterations are needed to move the zero level set from the boundary of to the desired object boundary, but also the segmentation result is more efficient.

We propose to use a binary step function in (21) as the initial LSF, as it can be generated extremely efficiently.

Thus, only a small number of iterations are needed to move zero level set from the boundary of $\Omega_0$ to the desired object boundary.

The level set function evolves from a binary step function to an approximate signed distance function on a signed distance band (SDB). Because its values vary from $-d$ to $d$ across the band at the rate of $|\nabla \phi|=1$ when the function $\phi$ becomes a signed distance function in the SDB. This means that the width of the SDB is approximately $2d$. Therefore, the width of the SDB is controlled by the constant $d$. In fact, the image domain is discrete grid, and the SDB should have at least one grid point on each side of the zero level set. In this context, the initial value of $d$ is usually set as 2 mentioned in [8], $d$ is chosen from the range $d \geq 1$ unless otherwise specified. In summary, the initial LSF in (22) can be defined again:

$$\phi_i(x,y) = \begin{cases} -2, & (x,y) \in \Omega_0 - \partial\Omega_0 \\ 0, & (x,y) \in \partial\Omega_0 \\ 2, & (x,y) \in \Omega - \Omega_0 \end{cases} \tag{25}$$

Where, $\Omega_0$ can be obtained by thresholding or other efficient methods.

5 Experimental results

There are a variety of parameters, such as $m$, $\lambda$, $\nu$ and the time-step $\tau$. As well-known that the level set function will speed faster if the time-step is a relatively large. Nevertheless, the larger time step leads to steeply evolution. As many experimental results shown, it is better to set the evolution time-step as 5.0. The coefficient $m$ as the weight value of internal energy, makes the level set function close to the signed distance function. Stability can be enforced using the CFL conditions, which means the numerical wave speed must be at least as fast as the physical wave speed [4]. In order to maintain level set evolution, the condition $mt \leq 0.25$ must be satisfied, which is the requirement of CFL conditions. The time step $\tau$ is already 5.0, so $m$ is fixed as 0.04. Additionally, $\lambda$ and $\nu$, the coefficients of external energy, are usually set as 5.0 and 0, respectively.

On the choice of medical ultrasound image, the study samples are the typical gallstone features and non-multiple lesions images. The actual gallbladder ultrasound images selected in this paper come from local hospital that is a subset of ultrasound images with typical characteristics from the hospital of Lanzhou. Gallstone is a high incidence of gallbladder disease, especially in the northwest of China. This is a publicly available and real dataset. The experimental results of gallstones extraction show in the first column of Fig.3. The first column shows multiple gallstones at the bottom of gallbladder and the new moon-shaped gallstone, respectively. The mammographic images segmentation results are shown in second column. The dataset used for evaluation of the proposed approaches for mammograms segmentation are selected from the Mini Mammographic image analysis database (MIAS) in United Kingdom ([Suckling et al., 1994]). The images were stored
Furthermore, the binary images obtained after morphologic processing. Moreover, there are also two efficiency algorithm for iris image location in [11,12]. The last column shows examples of biological images polysaccharides of plant. In terms of different number of polysaccharides in each image, we all can accurately get them with our proposed method. After morphologic processing, we can clearly obtain 12 polysaccharides in the first picture and 15 in the next picture.

In Fig.3, all the experiments with red line are the results with our proposed method. Respectively, the blue line is the results with Li’s model. Compared to Li’s model, our proposed algorithm not only has better performance to detect the edge of each gallstone ultrasound image, mammograms, iris inner location and polysaccharides segmentation, but also accurately segment and extract.

TABLE I shows the detection results of different lesions. $Se^a$, $Se^b$ represents the sensitivity of our method and Li’s method [8], respectively.

<table>
<thead>
<tr>
<th>Class of Lesions</th>
<th>Amount of Images</th>
<th>Our method</th>
<th>Li’s method [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Images</td>
<td>$Se^a$ (%)</td>
<td>Images</td>
</tr>
<tr>
<td>CIRC</td>
<td>23</td>
<td>19</td>
<td>82.6</td>
</tr>
<tr>
<td>SPIC</td>
<td>19</td>
<td>15</td>
<td>78.9</td>
</tr>
<tr>
<td>MISC</td>
<td>14</td>
<td>12</td>
<td>85.7</td>
</tr>
<tr>
<td>ARCH</td>
<td>19</td>
<td>14</td>
<td>73.7</td>
</tr>
<tr>
<td>ASYM</td>
<td>15</td>
<td>12</td>
<td>80</td>
</tr>
</tbody>
</table>

Although Li’s model is much better at separating the main objects in the original images into meaningful regions with more natural shape, all the segmentation results are ineffective due to the low contrast in biomedical images.

6 Conclusions

In this paper, we have presented a novel automatic algorithm based on level set method for medical image segmentation. The main contribution of this work lies in that we introduce the Laplace operator in energy functional of level set method. Comparative experiments on biomedical image segmentation show that our method can achieve accurate segmentation results with same initial contours. In the future, we will apply our model to segment other types of images.

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8 References


