Abstract—This article studies iterative refinement algorithms for PDE-based image restoration models. In order to restore fine structures in the image, iterative refinement procedures employing an original idea by Bregman have been introduced in image restoration. However, the Bregman iterative procedure first recovers fine scales of the image and then restores the noise to converge to the observed noisy image; it must be stopped manually when the quality of the obtained image appears satisfactory. This article introduces an effective refinement procedure called the smooth curvature correction (SCC) model to overcome the drawback of the Bregman iteration. By incorporating the smoothed curvature of the previous iterate as a source term, the new model can successfully produce a convergent sequence of images having a better restoration quality than the best result of the Bregman procedure. Various numerical examples are given to confirm the claim and to show effectiveness of the SCC model.

Keywords: Smooth curvature correction (SCC) model, iterative refinement, partial differential equation (PDE), image restoration, Bregman iteration.

1. Introduction

Mathematical techniques have become important components of image processing, as the field requires higher reliability and efficiency. During the last two decades or so, mathematical tools of partial differential equations (PDEs) and functional analysis have been successfully applied for various image processing tasks, particularly for image denoising and restoration [1], [2], [3], [4], [5], [6]. Those PDE-based models have allowed researchers and practitioners not only to introduce new, effective computational procedures but also to improve traditional algorithms in image restoration.

However, these PDE-based models tend to either converge to a piecewise constant image or introduce image blur (undesired dissipation), partially because the models are derived by minimizing a functional of the image gradient. As a consequence, the conventional PDE-based models may lose interesting image fine structures. In order to reduce the artifact, researchers have studied various mathematical and numerical techniques which either incorporate more effective constraint terms and iterative refinement [7], [3], [8] or minimize a functional of second derivatives of the image [9], [10]. These new mathematical models may preserve fine structures better than conventional ones; however, more advanced models and appropriate numerical procedures are yet to be developed.

Iterative refinement procedures have been introduced in image restoration [11], [8], employing an original idea of Bregman [12], in order to recover fine structures in the image. The Bregman iterative procedure tries to produce a sequence of images for the signal adjusted by all of the previous residuals. Thus, it recovers not only fine scales of the image but also the noise and reveals a strong tendency to converge to the observed noisy signal. For this reason the Bregman iterative procedure must be stopped manually when the quality of the obtained image appears satisfactory.

This article suggests an effective refinement procedure called the smooth curvature correction (SCC) model to overcome the drawback of the Bregman iteration. The SCC model computes new iterates incorporating the smoothed curvature of the last iterate. It has been numerically verified that the new model can successfully produce a convergent sequence of images having a better restoration quality than the best result of the Bregman procedure.

An outline of the paper is as follows. In the next section, we begin with a brief review of PDE-based image denoising models and then present the Bregman iterative refinement procedure, as preliminaries. Section 3 introduces the SCC model, a new iterative refinement procedure, as an alternative to the Bregman procedure. A numerical strategy is also considered in the same section in order to choose effective residual-driven constraint parameters. In Section 4, the new SCC model is compared with the Bregman procedure, with
and without the residual-driven constraint parameters. Various numerical examples are presented to show effectiveness of the SCC model. Section 5 summarizes and concludes our experiments.

2. Preliminaries

In this section, we present a brief review of PDE-based image denoising models, followed by the Bregman iterative refinement.

2.1 PDE-based denoising models

Given an observed (noisy) image \( f : \Omega \rightarrow \mathbb{R} \), where \( \Omega \) is the image domain which is an open subset in \( \mathbb{R}^2 \), we consider the noise model of the form

\[
 f = u + g(u)\eta,
\]

where \( u \) is the desired image and \( g(u)\eta \) denotes the noise with \( \eta \) having a zero mean. For example, \( g(u) = 1 \) for Gaussian noise and \( g(u) = \sqrt{u} \) for speckle noise in ultrasound images [13]. Then a common denoising technique is to minimize a functional of gradient:

\[
 u = \arg\min_u \left\{ \int_{\Omega} \rho(|\nabla u|) \, dx + \frac{\lambda}{2} \int_{\Omega} \left( \frac{f - u}{g(u)} \right)^2 \, dx \right\}, \tag{2}
\]

where \( \rho \) is an increasing function (often, convex) and \( \lambda \geq 0 \) denotes the constraint parameter. It is often convenient to transform the minimization problem (2) into a differential equation, called the Euler-Lagrange equation, by applying the variational calculus [14]:

\[
 -\psi(u) \nabla \cdot \left( \rho'(|\nabla u|) \frac{\nabla u}{|\nabla u|} \right) = \lambda (f - u), \tag{3}
\]

where

\[
 \psi(u) = \frac{g(u)^3}{g(u) + (f - u)g'(u)}. \]

For an edge-adaptive image denoising, it is required to hold \( \rho'(s)/s \rightarrow 0 \) as \( s \rightarrow \infty \). For the speckle noise in ultrasound images, Krissian et al. [13] set \( g(u) = \sqrt{u} \), which implies \( \psi(u) = 2u^2/(f + u) \approx u; \) the diffusion term becomes large at largely perturbed pixels (speckles) and therefore the resulting model can suppress speckles more effectively.

When \( \rho(s) = s \) and \( g(u) \equiv 1 \), the model (3) becomes the total variation (TV) model [6]:

\[
 \kappa(u) = \lambda (f - u), \tag{TV}
\]

where \( \kappa(u) \) is the negation of the mean curvature defined as

\[
 \kappa(u) = -\nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right). \tag{5}
\]

It is often the case that the constraint parameter \( \lambda \) is set as a constant, as suggested by Rudin-Osher-Fatemi [6]. In order to find the parameter, the authors merely multiplied (4) by \((f - u)\) and averaged the resulting equation over the whole image domain \( \Omega \):

\[
 \lambda = \frac{1}{\sigma^2} \frac{1}{|\Omega|} \int_{\Omega} (f - u) \kappa(u) \, dx, \tag{6}
\]

where \( \sigma^2 \) is the noise variance

\[
 \sigma^2 = \frac{1}{|\Omega|} \int_{\Omega} (f - u)^2 \, dx. \tag{7}
\]

(In [6], \( \lambda \) was evaluated after applying integration by parts on the right-side of (6), which could avoid approximations of second-derivatives.)

As another example of (3), the Perona-Malik (PM) model [5] can be obtained by setting \( \rho(s) = \frac{1}{2} K^2 \ln(1 + s^2/K^2) \), for some \( K > 0 \), and \( \lambda = 0 \):

\[
 -\nabla \cdot (c(|\nabla u|) \nabla u) = 0, \tag{PM}
\]

where \( c(s) = \rho'(s)/s = (1 + s^2/K^2)^{-1} \). Note that for the PM model, the function \( \rho \) is strictly convex for \( s < K \) and strictly concave for \( s > K \). (\( K \) is a threshold.) Thus the model can enhance image content of large gradient magnitudes such as edges and speckles; however, it will flatten regions of slow transitions.

Most of conventional PDE-based restoration models have shown either to converge to a piecewise constant image or to lose fine structures of the given image. Although these results are important for understanding of the current diffusion-like models, the resultant signals may not be desired in applications where the preservation of both slow transitions and fine structures is important.

The TV model tends to converge to a piecewise constant image. Such a phenomenon is called the staircasing effect. In order to suppress it, Marquina and Osher [3] suggested to multiply the stationary TV model by a factor of \(|\nabla u|\):

\[
 |\nabla u| \kappa(u) = \lambda |\nabla u| (f - u). \tag{ITV}
\]

Since \(|\nabla u|\) vanishes only on flat regions, its steady state is analytically the same as that of the TV model (4). We will call (9) the improved TV (ITV) model. Such a non-variational reformulation turns out to reduce the staircasing effect successfully; however, it is yet to be improved for a better preservation of fine structures.

The conventional PDE-based denoising models, including ones presented in this section, can be written in the following general form

\[
 \mathcal{L}(u) = \mathcal{C}(f - u), \tag{10}
\]

where \( \mathcal{L} \) is a diffusion (smoothing) operator and \( \mathcal{C} \) denotes the constraint parameter.

2.2 Iterative refinement: Bregman iteration

In order to recover fine structures in the image, iterative refinement procedures employing an original idea by Bregman [12] have been introduced in image restoration and image zooming [11], [8]. The Bregman iterative refinement applied
to the general denoising model (10) reads as follows: if $u_1$ is the solution of (10)

$$\mathcal{L}(u_1) = \mathcal{C}(f - u_1),$$

we denote the corresponding residual by $r_1$, i.e.,

$$r_1 = f - u_1.$$

Then we again solve the TV model with the signal replaced by $f + r_1$; the solution $u_2$ will satisfy

$$\mathcal{L}(u_2) = \mathcal{C}(f + r_1 - u_2),$$

and the new residual is defined as

$$r_2 = f + r_1 - u_2.$$

In general, the $m$-th iterate of Bregman iteration, $u_m$, is computed as the restoration for the signal $f + r_{m-1}$, i.e.,

$$\mathcal{L}(u_m) = \mathcal{C}(f + r_{m-1} - u_m), \quad m \geq 1,$$

where $r_0 = 0$, and the new residual is defined as

$$r_m = f + r_{m-1} - u_m.$$

As for other conventional PDE-based denoising models, each step of the Bregman iteration (13) may be parameterized by an artificial time step of the Bregman iteration (13) may be parameterized by $\Delta t$, such that the corresponding evolutionary equation can be obtained by adding $\partial u_m/\partial t$ on the left side of (13).

$$\frac{\partial u_m}{\partial t} + \mathcal{L}(u_m) = \mathcal{C}(f + r_{m-1} - u_m), \quad m \geq 1. \quad (15)$$

The explicit temporal discretization of (15) can be formulated as

$$u_m^n = u_{m-1}^n + \Delta t \left[ -\mathcal{L}(u_{m-1}^n) + \mathcal{C}(f + r_{m-1} - u_{m-1}^n) \right], \quad (16)$$

where $u_{m-1}^n = u_{m-1}$ and $\Delta t$ denotes the temporal stepsize.

As indicated in [11], the Bregman iterative procedure first recovers fine scales of the image and then recovers the noise to converge to the observed noisy image $f$. For this reason the Bregman iterative procedure must be stopped manually when the quality of the obtained image appears satisfactory. Notice that the residuals $r_m$ in (14) read

$$r_m = (f - u_m) + r_{m-1}$$

$$= (f - u_m) + (f - u_{m-1}) + r_{m-2}$$

$$= \cdots = \sum_{i=1}^{m} (f - u_i). \quad (17)$$

Thus the $m$-th iterate of Bregman iteration, $u_m$, is computed for the signal

$$f + r_{m-1} = f + \sum_{i=1}^{m-1} (f - u_i), \quad (18)$$

which is the original image $f$ added by differences between each of the previous iterates and $f$. The additive amendment makes the constraint term accentuated, which in return forces the new iterate $u_m$ become closer to $f$. One may try to modify the last term in (18) or the whole right side, by either normalizing or smoothing, in order to prevent the iterates $u_m$ from recovering the noise from $f$. However, every trial has failed to improve image quality.

An research objective in this article is to develop PDE-based, iterative refinement denoising models which can restore images effectively and stop automatically satisfying the user-defined stopping criterion.

### 3. The New Iterative Refinement Model

#### 3.1 The smooth curvature correction model

As an iterative refinement model for the basic restoration model of the form (10), we suggest the following. Given a noisy image $f$, set $v_0 = 0$ and find $v_m$ by recursively solving

$$\mathcal{L}(v_m) = \mathcal{C}(f - v_m) + \mathcal{L}(v_{m-1}), \quad m \geq 1. \quad (19)$$

The new model deserves the following remarks.

1) The Bregman procedure (13) can be rewritten as

$$\mathcal{L}(u_m) = \mathcal{C}(f - u_m) + \mathcal{C} r_{m-1}.$$

When the problem is solved exactly in each iteration, we have $\mathcal{L}(u_m) = \mathcal{C} r_{m}$ for all $m \geq 1$. Thus the last term of the above equation, $\mathcal{C} r_{m-1}$, can be replaced by $\mathcal{L}(v_{m-1})$:

$$\mathcal{L}(u_m) = \mathcal{C}(f - u_m) + \mathcal{L}(v_{m-1}). \quad (20)$$

Since the first iterates of (13) and (19) are the same each other, i.e., $v_1 = u_1$, it follows from (19) and (20) that $v_m = u_m$ for all $m \geq 1$. In practice, however, it is often the case that each step in (13) is solved approximately, employing an iterative linearized solver (inner iteration) as in (16). Thus $\mathcal{L}(u_{m-1})$ becomes different from $\mathcal{L} r_{m-1}$ and therefore $v_m$ differs from $u_m$.

2) It has been numerically verified that as $m$ grows, the iterates of the new algorithm $v_m$ shows a tendency of gaining the noise from the observed image $f$, but much weaker than the Bregman iterates. We have been able to stop the tendency of converging to $f$ by slightly smoothing $\mathcal{L}(v_{m-1})$, the last term in (19); see numerical results shown in Section 4 below, more specifically, Table 2. In this article the iterative refinement (19) will be called the smooth curvature correction (SCC) model, when $\mathcal{L}(v_{m-1})$ is smoothed slightly by a smoothing method.

3) As for the Bregman iterative refinement, each step of the new denoising model (19) can be parameterized...
by an artificial time $t$ for a convenient numerical simulation.

$$\frac{\partial v_m}{\partial t} + \mathcal{L}(v_m) = \mathcal{C}(f - v_m) + \mathcal{L}(v_{m-1}), \quad m \geq 1. \quad (21)$$

Its equilibrium solution is a smooth, restored image of $f$ with $\mathcal{L}(v_{m-1})$ incorporated as a source/correction term. It has been numerically verified that the curvature correction term allows the new iterate $v_m$ to restore fine features more effectively. The explicit temporal discretization of (21) reads

$$v^n_m = v^{n-1}_m + \Delta t \left[ -\mathcal{L}(v^{n-1}_m) + \mathcal{C}(f - v^{n-1}_m) + \mathcal{L}(v_{m-1}) \right], \quad (22)$$

where $v^0_m = v_{m-1}$ and $\Delta t$ is the temporal stepsize.

### 3.2 Residual-driven variable constraint coefficients

The determination of the constraint parameter has been an interesting problem for PDE-based denoising models, of which the basic mechanism is diffusion. Thus the parameter $\mathcal{C}$ cannot be too large; it must be small enough to introduce a sufficient amount of diffusion. On the other hand, it should be large enough to keep the details in the image. However, in the literature the parameter has been chosen constant for most cases so that the resulting models can either smear out fine structures excessively or maintain an objectionable amount of noise into the restored image.

In order to overcome the difficulty, the parameter must be set variable, more precisely, edge-adaptive. Our strategy toward the objective is to

(a) initialize the parameter to be small, and

(b) allow the parameter grow wherever undesired dissipation is excessive, keeping it small elsewhere.

Note that the parameter would better be initialized small so that in the early stage of computation, the model (10) can remove the noise effectively and equally everywhere. Then, by letting the parameter grow, the model can return structural components (lost in the residual) back to the image.

An automatic and effective numerical method for the determination of the constraint coefficient $\mathcal{C}$, as a function of $(x, t)$, can be formulated as follows.

1. Select a desirable interval $I_c = [c_0, c_1]$ for which $\mathcal{C}(x, t) \in I_c$, where $c_0 \geq 0$ is sufficiently small.
2. Initialize $\mathcal{C}$ as a constant:

   $$\mathcal{C}^0 = \mathcal{C}(x, t = 0) = c_0. \quad (23)$$

3. Set $\mathcal{C}^1 = \mathcal{C}^0$ and for $n = 2, 3, \cdots$

   (3a) Compute the absolute residual $\mathcal{R}^{n-1}$ and the correction vector $\mathcal{H}^{n-1}$:

   $$\mathcal{R}^{n-1} = |f - u^{n-1}_m|,$$

   $$\mathcal{H}^{n-1} = \max \left( 0, G_k(\mathcal{R}^{n-1}) - A_v(\mathcal{R}^{n-1}) \right), \quad (24)$$

   where $G_k$ is a localized Gaussian smoothing of radius $k$ and $A_v(\mathcal{R}^{n-1})$ denotes the $L^2$-average of $\mathcal{R}^{n-1}$.

   (3b) Update:

   $$\mathcal{C}^n = \mathcal{C}^{n-1} + \xi^n \mathcal{H}^{n-1}, \quad (25)$$

   where $\xi^n$ is a scaling factor. For example, when the constraint coefficient is to be limited in a prescribed interval $[c_0, c_1]$, i.e., $\mathcal{C}(x, t) \in [c_0, c_1]$ for all $(x, t)$, the scaling factor $\xi^n$ can be chosen as

   $$\xi^n = \frac{1}{2^{n-1}} \cdot \frac{c_1 - c_0}{\|\mathcal{H}^{n-1}\|_{\infty}}, \quad n = 2, 3, \cdots. \quad (26)$$

Remark. The $L^2$-average of $\mathcal{R}^{n-1}$ is the standard deviation (SD) of the residual, i.e.,

$$A_v(\mathcal{R}^{n-1}) = \left( \frac{1}{|\Omega|} \int_{\Omega} (f - u^{n-1}_m)^2 \, dx \right)^{1/2} =: \sigma^{n-1}.$$  

The above procedure has been motivated from the observation that PDE-based denoising models tend to introduce a large numerical dissipation near fine structures such as edges and textures and the tendency in turn makes the residual have structural components there. Such structural components can be viewed as an indicator for an undesired dissipation. By adding the components to the constraint coefficient $\mathcal{C}$, we may reduce the undesired dissipation from the resulting image. We call the procedure the residual-driven constraint (RDC) parameterization.

### 4. Numerical Experiments

For comparison purposes, we consider the following four PDE-based restoration models, of which the last three are new models suggested to reduce drawbacks of the model $\mathcal{M}_1$.

(\mathcal{M}_1) The Bregman iterative refinement (13), with $\mathcal{C}_m^n = \lambda |\nabla u^{n-1}_m|$ for a constant $\lambda$  

(\mathcal{M}_2) The Bregman iterative refinement (13), with RDC in (23)–(26)

(\mathcal{M}_3) The SCC model (19), with $\mathcal{C}_m^n = \lambda |\nabla u^{n-1}_m|$ for a constant $\lambda$

(\mathcal{M}_4) The SCC model (19), with RDC in (23)–(26)

Here the ITv model (9) is selected for the basic PDE-based restoration model, i.e.,

$$\mathcal{L}(u) = -|\nabla u| \kappa(u) = -|\nabla u| \nabla \left( \frac{\nabla u}{|\nabla u|} \right),$$

and $\mathcal{C}_m^n$ denotes the constraint parameter for the $m$-th refinement (outer) iteration and the $n$-th inner iteration. For the SCC model (19), the last term $\mathcal{L}(v_{m-1})$ is smoothed using the weighted box filter

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$
The models $M_1$ and $M_3$ perform differently depending somewhat strongly on the choice of the constant $\lambda$. We have found from various experiments that the constant $\lambda$ can be chosen to make the maximum of $C_n^m$ a constant for all $n \geq 1$, i.e.,

$$\|C_n^m\|_\infty = \lambda \|\nabla u_n^m\|_\infty = \hat{c}, \quad n \geq 1,$$

for some $\hat{c} > 0$. Thus the constraint parameter for $M_1$ and $M_3$ can be found as follows: compute $|\nabla u_n^m|$ and scale it to make its maximum $\hat{c}$. We select $\hat{c} = 3$ for all examples presented in this article. (During the processing, all images are considered as discrete functions having real-values between 0 and 1, by scaling by a factor of 1/255. After processing, they are scaled back for the 8-bit display.)

Public domain images are downloaded, as shown in Figure 1, and then deteriorated by Gaussian noise. For the numerical schemes in (16) and (22), we choose $\Delta t = 0.2$ and the inner iteration is stopped when

$$\|u_n^m - u_{n-1}^m\|_\infty < 0.01$$

is satisfied or the maximum 50 iterations are performed. The outer iteration runs till the 1%-tolerance is satisfied:

$$\|u_n^m - u_{n-1}^m\|_\infty < 0.01.$$

Table 1 shows a PSNR analysis for the four models applied for the restoration of the sample images contaminated by Gaussian noise in two different levels. By PSNR, we mean the peak signal-to-noise ratio (PSNR) defined as

$$\text{PSNR} = 10 \log_{10} \left( \frac{\sum_{ij} 255^2}{\sum_{ij} (g_{ij} - u_{ij})^2} \right) \text{dB},$$

where $g$ denotes the original image and $u$ is the restored image from a noisy image, $f$, which is a contamination of $g$ by Gaussian noise. The floating point numbers in the table indicate the best PSNR values that the models can reach, the

<table>
<thead>
<tr>
<th>$f$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$M_4$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>26.81</td>
<td>34.26(2)</td>
<td>33.45(1)</td>
<td>34.41(3)</td>
</tr>
<tr>
<td>Dog</td>
<td>26.83</td>
<td>31.74(2)</td>
<td>31.78(2)</td>
<td>31.79(2)</td>
</tr>
<tr>
<td>Elaine</td>
<td>26.49</td>
<td>31.27(2)</td>
<td>31.31(2)</td>
<td>31.29(3)</td>
</tr>
<tr>
<td>House</td>
<td>26.63</td>
<td>33.41(2)</td>
<td>33.05(2)</td>
<td>33.37(4)</td>
</tr>
<tr>
<td>Lena</td>
<td>27.11</td>
<td>32.50(2)</td>
<td>32.39(2)</td>
<td>32.62(4)</td>
</tr>
<tr>
<td>Swan</td>
<td>25.98</td>
<td>32.09(2)</td>
<td>32.02(2)</td>
<td>32.22(3)</td>
</tr>
<tr>
<td>Balloons</td>
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<td>30.94(1)</td>
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<tr>
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<td>30.14(1)</td>
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<td>30.26(1)</td>
<td>30.89(3)</td>
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<tr>
<td>Lena</td>
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<td>29.59(1)</td>
<td>29.39(1)</td>
<td>29.80(2)</td>
</tr>
<tr>
<td>Swan</td>
<td>23.31</td>
<td>30.78(1)</td>
<td>30.56(1)</td>
<td>30.91(2)</td>
</tr>
</tbody>
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Table 2: Convergence analysis for the Lena image.

<table>
<thead>
<tr>
<th></th>
<th>f</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁</td>
<td>27.11</td>
<td>30.62</td>
<td><strong>32.50</strong></td>
<td>32.30</td>
<td>31.73</td>
<td>30.96</td>
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<td>27.11</td>
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<td><strong>32.58</strong></td>
<td>31.73</td>
<td>31.00</td>
<td>30.22</td>
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<tr>
<td>M₃</td>
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<td>30.62</td>
<td>32.10</td>
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<td><strong>32.62</strong></td>
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<tr>
<td>M₄</td>
<td>27.11</td>
<td>31.26</td>
<td><strong>32.58</strong></td>
<td>32.57</td>
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<td>M₅</td>
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<td>28.50</td>
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<tr>
<td>M₈</td>
<td>22.47</td>
<td><strong>29.39</strong></td>
<td>29.64</td>
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</table>

Integers in parentheses denote the number of outer iterations for the models to obtain the best PSNR. The Bregman iterative refinement models (M₁ and M₂) have reached at the best image in one or two outer iterations for all images. Particularly, for the heavier noise (presented in the bottom part of the table), the Bregman refinement models show the best image after one outer iteration for all images except the House image. This implies that the Bregman models are hardly able to improve image quality through iterative refinement when the noise is relatively heavy. On the other hand, the SCC models (M₃ and M₄) have improved the image quality through iterative refinement. It is easy to see that the models M₁ and M₄ have the same first iterate and so do M₂ and M₄.

The incorporation of RDC (Section 3.2) has promoted the PSNR values for only the Bregman model applied for the restoration of images from relatively low noise levels.

Table 2 exhibits a convergence analysis for the four denoising models. The Bregman refinement models (M₁ and M₂) are stopped in six outer iterations manually, while the SCC models (M₃ and M₄) have converged in three or four outer iterations. As one can see from the table, when the noise PSNR is 27.11, the Bregman refinement models get the highest PSNR for the second iterate; the PSNR decreases rapidly for later iterations, gaining the noise from the noisy image f. When the noise PSNR is 22.47, the Bregman models have failed to refine the resulting image; the PSNR decreases continuously from the beginning. On the other hand, the SCC models have converged in three or four iterations, although they reveal a weak tendency of catching up the noise from the given image. Model M₃ has resulted in best images for most cases.

Figure 2 depicts the noisy image of Lena, having the noise PSNR=22.47, and the restored images by M₁ and M₃. As one can see from the figure, the SCC model has performed superior to the Bregman model. The resulting image obtained from the SCC model shows sharper and clearer edges than the best image of the Bregman procedure.

5. Conclusions

Partial differential equation (PDE)-based denoising models often lose important fine structures due to an excessive dissipation. In order to minimize such undesired dissipation, we have considered a new iterative refinement procedure called the smooth curvature correction (SCC) model, as an alternative to the Bregman iterative procedure. By incorporating the smoothed curvature of the previous iterate as a source/correction term, the new model has been able to produce a convergent sequence of images; the resulting image has preserved fine structures successfully and has shown a better restoration quality than the best image of the Bregman procedure. Various numerical examples have been presented to confirm the claim and to show effectiveness of the SCC model.

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References