Image De-noising Using an Improved Bivariate Threshold Function in Tetrolet Domain

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Abstract - The paper presents a new image denoising method based on an improved Bivariate Model (BM) in Tetrolet domain. This model fits the joint distribution of parent-child tetrolet coefficients with a Scale Variable Parameter Bivariate Model (SVPBM). Corresponding nonlinear threshold shrinkage functions are derived from SVPBM by using maximum a posteriori (MAP) estimator. To evaluate the performance of the proposed method, the algorithm is applied to images that are corrupted with additive Gaussian noise over a wide range of noise variance. Experimental results are compared with different denoising schemes. The experimental results indicate that the proposed method provides promising results and is advantageous both in terms of PSNR and in visual quality.

Keywords: Image denoising; Tetrolet transform; Bivariate shrinkage; Noise variance;

1 Introduction

Image is an important way of access to information for people. But noises largely reduce the perceptual quality of images and may result in fatal errors [1]. Image denoising has been a fundamental problem in image processing. The wavelet transform is one of the most popular tools in image denosing due to its promising properties for singularity analysis and efficient computational complexity [2].

In the past decades, a fair amount ways and methods for image denoising are proposed by international and domestic academicians [3-8]. VisuShrink [3] is the most popular approaches by setting all the coefficients smaller than the universal threshold and preserving or shrinking the rest, which was proposed by Donoho in 1994. But it may lose many of the details in over-smoothed image. SureShrink [4] are proposed to overcome the weakness in 1995. Since then, wavelet denoising got very quick development in theory and technology area. Because of ability in preserving more useful image edge and details while removing noise, statistical approaches have drawn more and more attention of academics. According to the distribution features of wavelet coefficients, researchers exploit different types of dependencies between the wavelet coefficients to improve denoising further, a General Gauss Model (GGM) of wavelet coefficients is introduced by Grace Chang [5] in 2000. But the model only considers the intrascale dependencies between wavelet coefficients and ignores the interscale dependencies between wavelet coefficients. BM based on interscale dependencies between wavelet coefficients is introduced by Sendur [6,7] in 2002. The corresponding shrinkage functions are derived from the models using Bayesian estimation theory [8]. The denoising result of BM is superior to that of GGM. But in fact, the joint probability distribution function (PDF) of parent-child wavelet coefficient pairs in different scales is different. It is inaccurate to represent the PDF with the same BM. Some improved bivariate models is proposed by reference [9-18] to solve the problem. Applications of wavelets have been widely used in scientific and engineering fields, traditional wavelets perform well for representing point singularity. Recently, some researchers extend the ideas of bivariate shrinkage to the geometric wavelets shrinkage methods, e.g., curvelets [19], contourlets[20,21], directionlet[22] and shrinkage in high dimensional space [23]. Tetrolet Transform is a new adaptive Haar wavelet transform introduced by Jens Krommweh [24,25] in 2009. It has been applied to image processing [24]. But the image must be divided into blocks before Tetrolet Transform and the blocks are transformed separately. There are blocking artifacts and non-smoothness in the denoised image due to the non-smoothness of basic block functions. The blocking artifacts in denoised image increase with the noise variance. The de-noise capability of Tetrolete Tansform need to be improved and developed further. The paper focus on the Sendur’s BM. The main idea is to extend the idea of BM to tetrolet domain and improve the denoising ability of Tetrolete Transform.

The main contribution of this paper is that a new joint shrinkage function is given and a new method based on it in Tetrolet domain is proposed for image denoising. This function can estimate the present coefficient according to the dependencies between Tetrolet coefficients and their parents in detail. The basic idea was inspired by Sendur’s Bivariate Shrinkage Theory (BST) and Jens Krommweh’s Tetrolet Transform Theory.

The rest of the paper is organized as follows. In section 2, the basic idea of Tetrolet Transform will be briefly described. Section 3 will analyse the lack of BM and propose SVPBM function. Then, a novel method based on Tetrolet transform is presented for image denoising. In section 4, some computer
simulations will be performed to evaluate the performance of the proposed method. Several experimental results will be presented and discussed. Finally, some concluding remark and future work are given in section 5.

2 Tetrolet Transform

As a Multiscale geometric analysis method, Tetrolet Transform can sparse represent image due to the non-redundance of basis functions. The image is divided into 4×4 blocks before Tetrolet Transform, then a tetromino partition in each block which is adapted to the image geometry in the block. Originally, tetrominoes were introduced by Golomb [26] in 1994, and they became popular through the famous computer game classic 'Tetris'. During Tetrolet Transform, tetrominoes are shapes made by connecting four equal-sized squares, each joined together with at least one other square along an edge. Disregarding rotations and reflections there are five different shapes, the so called free tetrominoes, see Fig.1. Larsson [27] verified that there are 117 solutions for disjoint covering of a 4×4 board with four tetrominoes in 1937. The tetrolet transform will choose the most appropriate one to fit the local structure in a 4×4 block of an image, and then apply the above template to the elements in the four tetrominoes.

Input image \( a_0 = (a[i, j])_{i,j=0}^{N-1}, N = 2^r, J \in \mathbb{N} \) is decomposed into \( r \) levels. The detailed steps are as follows:

(1) Divide the low-pass image \( a^r-1 \) into \( 4 \times 4 \) blocks \( Q_{ij}, i, j = 0, \ldots, N/4 - 1. \)

(2) Consider 117 admissible tetromino coverings \( c = 1, \ldots, 117 \) for each block \( Q_{ij} \). A Haar Wavelet transform is performed on tetromino subsets \( I_s \), \( s = 0, 1, 2, 3 \). For each tilling, four low-pass coefficients and 12 high pass tetrolet coefficients can be obtained. Then, the covering \( c^* \) such that the \( l_1 \)-norm of 12 tetrolet coefficients is minimal. The covering \( c^* \) can be chosen.

\[
c^* = \arg \min_c \sum_{i=1}^3 \| w_i f_c \| = \arg \min_c \sum_{i=1}^3 \sum_{s=0}^3 \| w_i f_c \| [s] \] \tag{1}
\]

(3) For further scales of tetrolet decomposition, the low-pass coefficients and high-pass coefficients need to be rearranged of each block into a \( 2 \times 2 \) block.

(4) Store the high-pass coefficients.

Repeat these steps, the input image can be decomposed into \( r \) levels.

3 Image Denoising Using BST

The BM proposed by Sendur is approximately able to produce similar plots as shown in Fig. 2. The distribution of Tetrolet coefficients is similar to the distribution of Wavelet coefficients. According to Tetrolet Transform’s superiority in representing for geometric properties of directed structures in image, the paper extend the idea of BST from wavelet to Tetrolet Transform domain with some improvements. Fig. 2 shows the joint parent-child histogram of wavelet coefficients with different noise variance. Noise deviation in Fig. 2 (a) is 10 and in Fig. 2 (b) is 20. It is easy to see that the smaller noise variance, the narrower joint parent-child histogram of wavelet coefficients.

From Fig.3, it is easy to see that the joint parent-child histogram of tetrolet coefficients between scale 4 and 5 is different from that between 5 and 6. The BM assumes the variances of wavelet coefficients are the same for all scales, which conflicts with the fact that the variances of tetrolet coefficients of noisy images are quite different from scale to scale. The variances of tetrolet coefficients of natural images are quite different from subband to subband. So, it is inaccurate to model the tetrolet coefficients with BM. It should consider the influence of scale and subband on noise variance. In order to exactly describe joint distribution of tetrolet coefficients, the paper proposed SVPBM. In order to eliminate the influence of different scale and subband on joint distribution of tetrolet coefficients, the paper introduced scale parameters and control parameters.

![Fig. 1: The five free tetrominoes. O-I-T-S-L tetromino.](image-url)
SVPBM models the joint distribution of tetrolet coefficients with a circularly symmetric PDF and are uncorrelated but not independent:

\[
p_{w}(\omega) = \frac{(\varepsilon J)^{2}}{2\pi\sigma^{2}} \exp \left[ -\frac{\varepsilon J}{\sigma} \sqrt{\omega_{1}^{2} + \omega_{2}^{2}} \right]
\]

(2)

Let \( f(\omega) = \log p_{w}(\omega) \), then

\[
\hat{\omega}(y) = \arg \max_{\omega} \left[ -\frac{(y_{1} - \omega_{1})^{2}}{2\sigma_{n}^{2}} - \frac{(y_{2} - \omega_{2})^{2}}{2\sigma_{n}^{2}} + f(\omega) \right]
\]

(3)

This is equivalent to solving the following equations if \( p_{w}(\omega) \) is assumed to be strictly convex and differentiable:

\[
\frac{y_{1} - \hat{\omega}_{1}}{\sigma_{n}^{2}} + f_{1}(\hat{\omega}) = 0
\]

(4)

\[
\frac{y_{2} - \hat{\omega}_{2}}{\sigma_{n}^{2}} + f_{2}(\hat{\omega}) = 0
\]

(5)

where \( f_{1} \) and \( f_{2} \) represent the derivative of \( f(\omega) \) with respect to \( \omega_{1} \) and \( \omega_{2} \).

Solving the equation (4) and (5) by using .

\[
f(\omega) = \log \left( \frac{(\varepsilon J)^{2}}{2\pi\sigma^{2}} \right) - \frac{\varepsilon J}{\sigma} \sqrt{\omega_{1}^{2} + \omega_{2}^{2}}
\]

(6)

The joint bivariate shrinkage function can be written as:

\[
\hat{\omega}_{i} = \frac{(\sqrt{y_{1}^{2} + y_{2}^{2}} - \frac{\varepsilon J\sigma_{n}^{2}}{\sigma})}{\sqrt{y_{1}^{2} + y_{2}^{2}}} \cdot y_{1},
\]

(7)

where \( \sigma_{n}^{2} = \text{median}(y_{i}) / 0.6745 \) is noise variance, the estimator of standard deviation of true coefficient is

\[
\sigma = \sqrt{\max(\tilde{\sigma}_{y_{1}}^{2} - \tilde{\sigma}_{n}^{2}, 0)} = \sqrt{(\tilde{\sigma}_{y_{1}}^{2} - \tilde{\sigma}_{n}^{2})},
\]

(8)

where \( \tilde{\sigma}_{n}^{2} = \frac{1}{N} \sum_{y_{i} \in \mathbb{N}(k)} y_{u}^{2} \).

From derivation above, it can be seen that joint bivariate shrinkage function doesn’t consider the estimator of tetrolet coefficients at final scale. Image denoising using bivariate shrinkage function should consider the threshold processing on tetrolet coefficients at final scale. The paper proposes the image denoising method based on SVPBM function.

Specific steps are follows:

1. Decompose the noisy image with Tetrolet Transform.
2. Construct coefficient vector \( y = (y_{1}, y_{2}) \) with high-pass coefficients at scale \( J \) and parent scale \( J+1 \). \( y_{1} \) is the coefficient at the same position as \( y_{2} \).
3. Threshold process present coefficient \( y_{1} \) with equation (7).
4. Threshold process the high-pass coefficients at final scale.
5. Tetrolet Transform are are inversed to get the denoised image.

4 Experimental Results

4.1 Experimental Setup

In order to evaluate the performance of the proposed method, the experiment is performed on a representative set of standard 8-bit grayscale images extracted from CVG-UGR database[28], such as Lena, Mandrill, Indians and Boat, each of size 512 × 512, corrupted by simulated additive Gaussian white noise with a standard deviation equal to 20, 30, 40, 50, 60. Several methods were used to filter the noisy image. The paper evaluated the performance of proposed method using the quality measure PSNR which is calculated as follows:

\[
PSNR = 10 \times \log \left( \frac{255^{2}}{\text{MSE}} \right)
\]

(9)

Here, the performance of proposed method is compared with different de-noising schemes that include Wavelet, Contourlet, Tetrolet.

4.2 Experimental Results

The comparison of PSNR obtained with these four denoising methods can be seen in Table 1 and Table 2. In Table 1, Tetrolet Transform’s Tetromino coverings \( c = 32 \) and control parameters of SVPBM \( \varepsilon = 2 \). In Table 2, Tetrolet Transform’s Tetromino coverings \( c = 16 \) and control parameters of SVPBM \( \varepsilon = \sqrt{6} \).

As shown in Tables 1-2, the PSNR of the image denoised by the proposed method is obviously outperforms Wavelet, Contourlet and Tetrolet. It can be seen that PSNR obtained with the proposed method is enhanced 1.35 dB on average more than wavelet denoising methods. Compared with Contourlet method, PSNR obtained with the proposed method is still enhanced 1.3 dB on average. With the increase of noise variance, PSNR obtained with all these methods have a decrease trend, but PSNR obtained with the new method is still more than other PSNRs and proposed method has the best performance for all noise levels. A comparison of PSNR
between proposed method and denoising method in tetrolet domain, is also made here. The proposed method gains over Tetrolet by about 1.0 dB on average. The paper attributes the better performance of proposed method to the better ability of the BM in matching the underlying distribution of the tetrolet coefficients. Further more, the better ability of Tetrolet Transform in image approximation and representing for geometric properties of directed structures in image is also very important.

Table 1 PSNR values (dB) obtained with “Man” and “Mandrill”.

<table>
<thead>
<tr>
<th>σ</th>
<th>Wavelet</th>
<th>Contourlet</th>
<th>Tetrolet</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>25.7546</td>
<td>25.5769</td>
<td>25.7030</td>
<td>27.4844</td>
</tr>
<tr>
<td>30</td>
<td>24.1934</td>
<td>24.0817</td>
<td>24.4327</td>
<td>25.6884</td>
</tr>
<tr>
<td>40</td>
<td>22.9875</td>
<td>23.0236</td>
<td>23.5060</td>
<td>24.3261</td>
</tr>
<tr>
<td>50</td>
<td>22.0687</td>
<td>22.2481</td>
<td>22.7213</td>
<td>23.2261</td>
</tr>
<tr>
<td>60</td>
<td>21.2985</td>
<td>21.6143</td>
<td>22.0638</td>
<td>22.3498</td>
</tr>
</tbody>
</table>

Table 2 PSNR values (dB) obtained with “Lena” and “Boat”

<table>
<thead>
<tr>
<th>σ</th>
<th>Wavelet</th>
<th>Contourlet</th>
<th>Tetrolet</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>28.6716</td>
<td>28.5539</td>
<td>28.1448</td>
<td>29.6102</td>
</tr>
<tr>
<td>30</td>
<td>26.6407</td>
<td>26.8369</td>
<td>26.5903</td>
<td>27.5168</td>
</tr>
<tr>
<td>40</td>
<td>25.1928</td>
<td>25.5977</td>
<td>25.4855</td>
<td>26.1235</td>
</tr>
<tr>
<td>50</td>
<td>24.0550</td>
<td>24.5974</td>
<td>24.5626</td>
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<tr>
<td>60</td>
<td>23.0555</td>
<td>23.7738</td>
<td>23.7807</td>
<td>24.1137</td>
</tr>
</tbody>
</table>

From Table 1 and Table 2, it can be seen that the PSNR gap between proposed method and Wavelet descends from 1.35 to 0.95 dB. With the increase of noise variance, the PSNR gap between proposed method and Contourlet descends from 1.3 to 0.21 dB. The PSNR gap between proposed method and Tetrolet descends from 1.0 to 0.29 dB. It should be mentioned that the test images in Table 1 have strong localized linear singularity and the images in Table 2 have strong point singularities. So it can be seen that the proposed method has advantage in image with linear singularity over in image with point singularities.

For visual evaluation, two examples using standard “Man” and “Lena” are given in Fig.4 (a) and Fig.4 (b). As can be seen from Fig.4, the denoised image of the proposed has fewer artifacts and is clear than that of Wavelet, Contourlet and Tetrolet. There are still a little blocking artifacts in the denoised image when the noise deviation is large.

Fig.4.Denoising results using different methods. Noisy image (a) noisy deviation $\sigma = 30$, tetrolet coverings $c = 32$, control parameters $\varepsilon = 2$; (b) noisy deviation $\sigma = 40$, tetrolet coverings $c = 16$, control parameters $\varepsilon = \sqrt{6}$.

5 Conclusion and future work

To improve the performance of Tetrolet in image denoising, the paper extends the ideas of bivariate shrinkage to Tetrolet Transform. The paper models the distribution of tetrolet coefficients with SVPBM and derives the scale bivariate shrinkage function according to the fact that the variances of tetrolet coefficients of noisy images are quite different from scale to scale. The SVPBM has better ability in fitting the distribution of tetrolet coefficients. A new improved image denoising method based on SVPBM is proposed. The paper compared the method with several other denoising schemes and the results showed that the proposed method is superior to Wavelet, Contourlet and Tetrolet both visually and in terms of PSNR.

It should be noted that the proposed method has advantage in image with linear singularity over in image with point singularities.
singularities. But the image must be divided into $4 \times 4$ blocks before Tetrolet Transform and the blocks are transformed separately. It is not enough to improve the denoising performance only depend on improvement in tetrolet domain. It also should be considered combining the image denoising method both in frequency domain and space domain.

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**References**


