FLOW OF A MICROPOLAR FLUID THROUGH A STENOSED ARTERY WITH RADIALY VARIABLE VISCOSITY

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Abstract

A study of the steady flow of blood through a horizontally symmetric artery with mild stenosis is presented by considering blood as a micropolar fluid. The viscosity of blood is considered as radial coordinate dependent. The non-linear pressure equations governing the flow are solved numerically using finite difference technique. The effects of micropolar fluid parameters and hematocrit on axial velocity, microrotation velocity, wall shear stress, wall couple stress and the volumetric flow rate in the stenotic region and at the maximum height of the stenosis (stenosis throat) have been discussed. The results have been compared with the corresponding flow for a Newtonian fluid.

Introduction

The flow of fluids in pipes or channels is important in many biological and biomedical systems, particularly in the human cardiovascular systems. Researches related to many biological phenomena were undertaken from a fluid dynamical point of view. Some of them include the blood flow in an artery with stenosis (i.e. abnormal and unnatural growth in the lumen of the artery). The presence of constriction (stenosis) in the lumen of an artery disturbs the normal blood flow and causes arterial diseases. Several attempts [1-5] have been made to understand the flow characteristics of blood through arteries by assuming blood as Newtonian. The assumption of Newtonian behavior of blood is acceptable for high shear rate flow that is the case of flow through larger arteries. It has been pointed out that under diseased conditions, blood exhibits non-Newtonian fluid properties.

Young [4], Macdonald [5], Deshpande et al [6], Sankar and Hemalatha [7] etc. have analyzed the flow of blood through an arterial stenosis. Lee and Fung [8] have obtained the numerical results for the streamlines and distribution of velocity, pressure, vorticity and shear stress for different Reynolds number in blood flow through locally constricted tubes. In the above models, the flow of blood is represented as one-layered.

Bugliarello and Hayden [9] and Bugliarello and Sevilla [10] have experimentally observed that when blood flows through narrow tubes, there exists a cell-free plasma layer near the wall. In view of their experiments, instead of a one-layered model, a two-layered model was considered by Shukla et al [11-12] in which the peripheral plasma layer and the core are both Newtonian in character.

Ponalagusamy [13], Ikbal et al [14], Chaturani and Kaloni [15], Chaturani and Ponalagusamy [16] etc. have analyzed the flow characteristics of blood by considering the blood as a two-layered model. The micropolar fluid model for blood flow through stenosed artery has been considered by many investigators [17-21]. In all these studies mentioned above, whether it is one-layered model or two-layered model, the viscosity of blood is treated as constant.

The unsteady flow of blood through artery with mild stenosis has been studied by Venkateswarulu and Rao [22] assuming blood to be suspension of red cells in plasma and the fluid to be Newtonian with variable viscosity. The effect of magnetic field in the transverse direction of blood flow in a stenotic artery was investigated by Bali and Awasthi [23] considering viscosity of blood to be radial coordinate dependent and the fluid to be non-Newtonian.

Blood is a suspension of red cells, white cells and platelets in plasma. The main advantage of using micropolar fluid to study the blood flow in
comparison with other classes of non-Newtonian fluids is that it takes care of the rotation of the fluid particles by means of an independent kinematic vector called microrotation vector. With this view in mind, in the present study an one-layer model is considered for blood flow through stenosed artery, wherein the flowing blood is represented by Eringen’s micropolar fluid[24] considering the viscosity of blood as radial coordinate dependent and the microelements are rigid cells suspended in plasma.

Formulation of the problem

The stenosed artery is considered as a narrow cylindrical tube of length $L$. The blood is represented by an incompressible micropolar fluid of density $\rho$ with radially variable viscosity $\mu(r)$. Let $(r, \theta, z)$ be the coordinate of a material point in the cylindrical polar coordinate system. The origin is located on the vessel (stenosed artery) axis, $z$-axis is taken along the axis of the artery while $r$ and $\theta$ are along the radial and circumferential directions respectively.

Based on the discussions made by Young [1], Ponalagusamy [26] and Philip and Chandra [(27)] the radial velocity is negligibly small and can be neglected for low Reynolds number flow through an artery with mild stenosis. Keeping these in view, the governing equations of motion and viscosity are given by

$$\frac{\partial \rho}{\partial r} = 0$$

$$\frac{\partial \rho}{\partial z} + k \frac{\partial w}{\partial r} + \frac{k}{r} \frac{\partial}{\partial r} (rv_{\theta}) + \frac{1}{r} \frac{\partial}{\partial r} (r \mu(r) \frac{\partial w}{\partial r}) = 0$$

and

$$-2k v_{\theta} - k \frac{\partial w}{\partial r} + \gamma \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (rv_{\theta}) \right] = 0$$

where $\mu(r) = \mu_0 \left[ 1 + \beta h_m \left(1 - \frac{r}{R_0}\right)^n \right]$.

Here $w$ is the axial velocity component, $v_{\theta}$ is the microrotation component in the $rz$-plane, $\rho$ is the pressure, $\mu(r)$ is the viscosity of blood, $\mu_0$ is the viscosity near the wall, $k$ is the rotational viscosity, $\gamma$ is the gyro viscosity, $h_m$ is the maximum hematocrit at the centre of the tube, $\beta$ is a constant, $m_2$ is the power law index involved in viscosity profile and $R_0$ is the radius of the normal tube. Since the flow is axisymmetric, all the variables are independent of $\theta$.

The shear stress and the couple stress are respectively defined as $\tau_{rz} = (\mu + k) \frac{\partial w}{\partial r} + k v_{\theta}$ and

$$M_{r\theta} = \gamma \frac{\partial v_{\theta}}{\partial r}.$$ 

The boundary conditions for the equations (1) to (3) are of the form

$$w = v_{\theta} = 0 \text{ at } r = R(z) ; \quad \frac{\partial w}{\partial r} = 0 \text{ at } r = 0; $$

$w$ and $v_{\theta}$ are finite at $r = 0$. (4)

Using the following non-dimensional parameters,

$$R^* = \frac{R}{R_0}, \quad r^* = \frac{r}{R_0}, \quad z^* = \frac{z}{R_0}, \quad p^* = \frac{pR_0}{w_0 \mu_0},$$

$$w^* = \frac{w}{w_0} \text{ and } v_{\theta}^* = \frac{v_{\theta}R_0}{w_0}$$

and dropping stars, equations (1) to (3) reduce to

$$\frac{\partial p}{\partial r} = 0$$

$$\frac{\partial p}{\partial z} + k \frac{\partial w}{\partial r} + \frac{k}{r} \frac{\partial}{\partial r} (rv_{\theta}) + \frac{1}{r} \frac{\partial}{\partial r} (r \mu(r) \frac{\partial w}{\partial r}) = 0$$

and

$$\frac{\partial w}{\partial r} + 2N \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (rv_{\theta}) \right] = 0$$

Here $w_0$ is the velocity averaged over the section of the tube of radius $R_0$, $N = \frac{k}{\mu + k}$ is the coupling number and $m_2 = \frac{R_0^2 \mu_0}{\gamma}$ is the micropolar fluid parameter. The parameters $N$ and $m_2$ determine the concentration and size of the microelements. For a Newtonian fluid $N = m_2 = 0$.

The boundary conditions (4) in dimensionless form become

$$w = v_{\theta} = 0 \text{ at } r = R(z) ; \quad \frac{\partial w}{\partial r} = 0 \text{ at } r = 0; $$

$w$ and $v_{\theta}$ are finite at $r = 0$. (8)
The geometry of the stenosis is shown in Fig. 1 and can be described as [25, 26]

\[ R(z) = 1 - \beta \left[ L_0 m_1^{-1} (z - d) - (z - d)^n \right] \text{, } d \leq z \leq d + L_0 \]

where \( R(z) \) is the radius of the stenosed artery, \( L_0 \) is the length of the stenosis, \( d \) denotes its location and \( \beta = \frac{\delta_s}{m_1 - 1} \cdot \frac{L_0}{m_1 - 1} \) is the maximum height of the stenosis at \( z = d + \frac{L_0}{m_1} \) such that \( \delta_s / R_0 < 1.0 \). When \( m_1 = 2 \), the geometry of the stenosis becomes symmetric at \( z = d + \frac{L_0}{2} \).

**Solution of the problem**

Applying finite difference scheme, the equations (6) and (7) are solved using the boundary conditions (8). To prove convergence of finite difference scheme, the computation is carried out for lower values of step size. No significant change was observed in the value of \( w \) and \( \psi \).

The volumetric flow rate of the fluid in the stenotic region is given by

\[ Q = \int_0^{R(z)} w(r) r dr \]

The skin friction coefficient \( \tau_w \) at the surface of the stenosis is given by

\[ R_c \tau_w = \left[ 1 + \beta h_m \left( 1 - \frac{r}{R_0} \right)^{m_0} + N \right] \frac{\partial w}{\partial r} \text{ at } r = R(z) \],

where \( R_c \) is the Reynolds number of the fluid.

The couple stress coefficient \( M_w \) at the surface of the stenosis is given by

\[ R_c M_w = \frac{1}{m^2} \left( \frac{\partial \psi}{\partial r} \right)_{r = R(z)} \]

**Discussion and Conclusions**

Axial velocity, microrotation rate component, volumetric flow rate, the wall shear stress and the wall couple stress are computed at the stenotic region for various values of the micropolar fluid parameters \( N \) and \( m^2 \), and the maximum hematocrit value \( h_m \).

The axial velocity profile \( w \) has been plotted for different values of \( N \) in Fig. 2 for a fixed value of \( h_m \) and for different values of \( h_m \) in Fig. 3 for a fixed value of \( N \). The effect of \( h_m \) on the axial velocity for Newtonian fluid is shown separately in Fig. 4.
We observe that an increase in $h_m$ or $N$ decreases the axial velocity $W$. The same effect of $h_m$ on Newtonian fluid velocity field was also observed. The microrotation rate component $v_\theta$ has been plotted in Fig. 5 for different values of $N$ and for a fixed value of $h_m$, and vice versa in Fig. 6. It is observed that $v_\theta$ decreases with the increase of $N$.

The wall shear stress $\tau_w$ and the wall couple stress $M_w$ at the stenotic wall are plotted respectively in Figs. 7 and 8. It is observed that the wall shear stress increases with the increase of $h_m$ but decreases with $N$, whereas the wall couple stress increases with the increase of $h_m$. The same type of behavior of $h_m$ on the wall shear stress for a Newtonian fluid was noticed. The variation of flow rate $Q$ with respect to the parameters $h_m$ and $N$ is tabulated in Table 1. It is observed that with the increase in values of $h_m$ and $N$ the flow rate decreases.
Table-1. Variation of volumetric flow rate \((Q)\) at the mid-point of the stenotic region

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References


