Power Spectra of Ionospheric Scintillation

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Abstract **-** *Peculiarities of the spatial power spectrum (SPS) of scattered radiation in magnetized turbulent anisotropic plasma are investigated by smooth perturbation method taking into account diffraction effects. Second order statistical moments: broadening, wave structure functions, angle-of-arrivals, scintillation index are calculated numerically for both anisotropic and power-law correlation functions of electron density fluctuations using experimental data. New features of the evaluation of a double-humped shape of the SPS for different parameter of anisotropy and angle of inclination of prolate irregularities with respect to the external magnetic field taking into account geometry of the task are revealed for the first time. The gap increases in proportion to the anisotropy factor. The power spectra and scintillation level of scattered ordinary and extraordinary waves have been calculated taking into account movement of ionospheric irregularities.*

Keywords: spatial power spectrum, double-humped effect, scintillation, magnetized plasma.

1 Introduction

Statistical characteristics of scattered electromagnetic waves in randomly statistically isotropic media have been intensively studied [1,2]. However in many cases irregularities are anisotropic and are oriented along a certain direction. In the upper atmosphere ionization becomes significantly anisotropic because electrons can move easily along the magnetic field lines than across them. There is therefore a tendency for all irregularities to become aligned along geomagnetic field [3]. Anisotropic irregularities in the ionosphere lead to the damping and amplification of the amplitude of radio waves, fluctuations of the phase and variations of the angle-of-arrival; ionospheric scintillation is a result of complex action of all these effects. For relatively small irregularities, diffraction effects are important. The features of the SPS of scattered radiation in magnetized anisotropic plasma in the complex geometrical optics approximation using the perturbation method have been investigated in [4-7]. The "Double-humped effect" in turbulent anisotropic magnetized plasma has been discovered recently using the smooth perturbation method taking into account diffraction effects [8]; some peculiarities of statistical characteristics of scattered radiation have been reported in [9,10].

This paper is devoted to the investigation of second order statistical moments of the SPS of scattered electromagnetic waves in turbulent collisionless magnetized plasma with electron density fluctuations. New peculiarities of the "Double-humped effect" are revealed analytically in the SPS of a multiple scattered radiation at oblique illumination of magnetized plasma with prolate irregularities by mono-directed incident radiation using the smooth perturbation method taking into account diffraction effects. Numerical calculations are carried out for both anisotropic Gaussian and power-law correlation functions of electron density fluctuations applying experimental data. It was shown that the SPS has a double-peaked shape for the power-law correlation function, the gap increases in proportion to the anisotropy factor, location of its maximum weakly varies and width substantially broadens with increasing distance travelling by electromagnetic waves in randomly-inhomogeneous magnetized plasma.

2 Formulation of the problem

Consider a plane wave propagating in the z direction and the unit vector $\boldsymbol{\tau}$ of an external magnetic field lies in the YZ coordinate plane ($\mathbf{k_0} \parallel z$, $\mathbf{H_0} \in YZ$ - principle plane), $k_0 = 2\pi/\lambda$, λ is the radio wavelength in free space. Absorption in the layer is negligible for high frequency incident wave and components of second-rank tensor ε_{ij} of the collisionless magnetized plasma are [11]:

$$\varepsilon_{xx} = 1 - \mathrm{v}(1-u)^{-1}, \ \varepsilon_{yy} = 1 - \mathrm{v}(1 - u\sin^2\alpha)(1-u)^{-1},$$

$$\varepsilon_{zz} = 1 - \mathrm{v}(1 - u\cos^2\alpha)(1-u)^{-1},$$

$$\tilde{\varepsilon}_{xy} = -\tilde{\varepsilon}_{yx} = \mathrm{v}\sqrt{u}\,\cos\alpha\,(1-u)^{-1},$$

$$\tilde{\varepsilon}_{xz} = -\tilde{\varepsilon}_{zx} = -\mathrm{v}\sqrt{u}\,\sin\alpha\,(1-u)^{-1};$$

where α is the angle between $\mathbf{k_0}$ and $\mathbf{H_0}$ vectors; $\omega_p(\mathbf{r}) = [4\pi N(\mathbf{r})e^2/m]^{1/2}$ is the plasma frequency, $u(\mathbf{r}) = [eH_0(\mathbf{r})/mc\omega]^2$ and $\mathrm{v}(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ are the magneto-ionic parameters, $\Omega_H(\mathbf{r}) = eH_0(\mathbf{r})/mc$ is the electron gyrofrequency. Dielectric permittivity of turbulent magnetized plasma is $\varepsilon_{ij}(\mathbf{r}) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})| << 1$. The first term is the regular (unperturbed) component of the dielectric permittivity connecting with the ionization distribution in the upper atmosphere at different altitudes above the Earth surface; the second term is a random function of the spatial coordinates caused by electron density fluctuations in the ionosphere; $\mathrm{v}(\mathbf{r}) = \mathrm{v}_0\,[1 + n_1(\mathbf{r})]$.

The electric field $\mathbf{E}$ satisfying wave equation

$$\left(\frac{\partial^2}{\partial x_i\,\partial x_j} - \Delta\delta_{ij} - k_0^2\,\varepsilon_{ij}(\mathbf{r})\right)\mathbf{E_j}(\mathbf{r}) = 0\ , \qquad (1)$$

introduce as $E_j(\mathbf{r}) = E_{0j}\exp(\varphi_0 + \varphi_1 + \varphi_2 + ...)$, $\varphi_0 = i k_\perp y + i k_0 z$ ($k_\perp << k_0$); complex phase fluctuations are of the order $\varphi_1 \sim \varepsilon_{ij}^{(1)}$, $\varphi_2 \sim \varepsilon_{ij}^{(1)\,2}$. Wave field Parameter $\mu = k_\perp / k_0$ describing diffraction effects is calculated in zero-order approximation [8]. The irregular plasma structure in the scattering medium imposes a random phase on the transmitted radio wave.

Phase fluctuation in the second order approximation satisfies differential equation

$$\frac{\partial^2\varphi_2}{\partial z\,\partial x} + i\,\mathrm{P}_j\frac{\partial^2\varphi_2}{\partial z\,\partial y} + i k_0\frac{\partial\varphi_2}{\partial x} + (2\Gamma_j k_\perp - \mathrm{P}_j k_0)\frac{\partial\varphi_2}{\partial y}$$

$$-\mathrm{P}_j\,k_\perp\frac{\partial\varphi_2}{\partial z} - i\,\Gamma_j\left(\frac{\partial^2\varphi_2}{\partial x^2} + \frac{\partial^2\varphi_2}{\partial y^2}\right) =$$

$$= i\,\Gamma_j\left[\left(\frac{\partial\varphi_1}{\partial x}\right)^2 + \left(\frac{\partial\varphi_1}{\partial y}\right)^2\right], \qquad (2)$$

where polarization coefficients are [11]:

$$\mathrm{P}_j = \frac{2\sqrt{u_0}\,(1-\mathrm{v}_0)\cos\alpha}{u_0\sin^2\alpha \pm \sqrt{u_0^2\sin^4\alpha + 4u_0(1-\mathrm{v}_0)^2\cos^2\alpha}},$$

$$\Gamma_j = -\frac{\mathrm{v}_0\sqrt{u_0}\,\sin\alpha + \mathrm{P}_j u_0\mathrm{v}_0\sin\alpha\cos\alpha}{1 - u_0 - \mathrm{v}_0 + u_0\mathrm{v}_0\cos^2\alpha}\ . \qquad (3)$$

The upper sign (index $j=1$) corresponds to the extraordinary wave and the lower sign (index $j=2$) corresponds to the ordinary wave. Ordinary and extraordinary waves in magnetized plasma generally are elliptically polarized.

The knowledge of the solution of two-dimensional spectral component of the phase fluctuation satisfying stochastic differential equation (for $j=z$ component) allows us to calculate variance of the phase fluctuations for arbitrary correlation function of electron density fluctuations:

$$<\varphi_1^2(\mathbf{r})> = 2\pi k_0^4\,L\Omega_5\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\,\frac{1}{G_1}\Big[-k_x^2 +$$

$$+\mathrm{P}_j^2\,(k_y^2 - k_\perp^2) + 2i\mathrm{P}_j\,k_x\,k_y\Big]\,V_n\!\left(k_x,\,k_y,\frac{iG_2 - G_3}{G_1}\right), \qquad (4)$$

where: $\Omega_5 = \dfrac{\mathrm{v}_0^2\,u_0}{(1-u_0)^2}(\sin^2\alpha + 2\mathrm{P}_j\sqrt{u_0}\sin^2\alpha\cos\alpha -$

$-2\,\Gamma_j\sqrt{u_0}\sin\alpha\cos^2\alpha - 2\mathrm{P}_j\,\Gamma_j u_0\sin\alpha\cos^3\alpha +$

$+\mathrm{P}_j^2\,u_0\sin^2\alpha\cos^2\alpha + \Gamma_j^2\,u_0\cos^4\alpha)$, k_x, k_y are the spatial wavenumber components in directions X and Y, respectively; L is the physical path through the inhomogeneous plasma, G_i are complicated functions of magnetized plasma parameters and

The knowledge of the correlation function of the phase taking into account that the observation points are spaced apart at a very small distance $\boldsymbol{\rho} = \{\rho_x, \rho_y\}$ perpendicular to the principle plane, allows us to calculate the width of the spatial power spectrum in the XZ and YZ planes respectively:

$$\frac{<\Delta k_x^2>}{k_0^2} = -\frac{\partial^2\tilde{W}_\varphi}{\partial\xi^2}\Bigg|_{\xi=\eta=0},\quad \frac{<\Delta k_y^2>}{k_0^2} = -\frac{\partial^2\tilde{W}_\varphi}{\partial\eta^2}\Bigg|_{\xi=\eta=0} \qquad (5)$$

wave structure functions of the amplitude, phase and mutual correlation functions [1,2]:

$$D_1(\mathbf{r_1},\mathbf{r_2}) = <\big(\varphi_1(\mathbf{r_1}) - \varphi_1(\mathbf{r_2})\big)\big(\varphi_1^*(\mathbf{r_1}) - \varphi_1^*(\mathbf{r_2})\big)>,$$

$$D_2(\mathbf{r_1},\mathbf{r_2}) = <\big(\varphi_1(\mathbf{r_1}) - \varphi_1(\mathbf{r_2})\big)^2>,$$

$$D_\chi(\mathbf{r}_1,\mathbf{r_2}) = \frac{1}{2}(D_1 + \mathrm{Re}\,D_2),\quad D_S(\mathbf{r}_1,\mathbf{r_2}) = \frac{1}{2}(D_1 - \mathrm{Re}\,D_2),$$

$$D_{\chi S} = \frac{1}{2}\,\mathrm{Im}\,D_2\ . \qquad (6)$$

where: $\xi = k_0\,\rho_x$, $\eta = k_0\,\rho_y$.

Transverse correlation function of a scattered field $W_{EE^*}(\boldsymbol{\rho}) = < E(\mathbf{r})\, E^*(\mathbf{r}+\boldsymbol{\rho}) >$ is [8]:

$$W_{EE^*}(\boldsymbol{\rho}, k_\perp) = E_0^2 \exp\left\{ \mathrm{Re}\left[\frac{1}{2}\Big(< \varphi_1^2(\mathbf{r}) > + < \varphi_1^{2*}(\mathbf{r}+\boldsymbol{\rho}) > \Big) + < \varphi_1(\mathbf{r})\, \varphi_1^*(\mathbf{r}+\boldsymbol{\rho}) > + 2 < \varphi_2 > \right] \right\} \exp(-i\, \rho_y\, k_\perp), \qquad (7)$$

where E_0^2 is the intensity of incident radiation.

SPS of scattered field in case of incident plane wave $W(k, k_\perp)$ is easily calculated by Fourier transformation from the transversal correlation function of scattered field [2]:

$$W(k, k_\perp) = \int_{-\infty}^{\infty} d\rho_y\, W_{EE^*}(\rho_y, k_\perp) \exp(i\, k \rho_y). \qquad (8)$$

When the angular spectrum of an incident wave has a finite width and its maximum coincide with z axis, SPS of scattered radiation is given:

$$I(k) = \int_{-\infty}^{\infty} dk_\perp\, W(k, k_\perp) \exp(-k_\perp^2 \beta^2), \qquad (9)$$

where β characterizes the dispersal of an incident radiation (disorder of an incident radiation), k is a transverse component of the wavevector of scattered field [1,2].

Intensity of scintillation is determined by index S_4 characterizing power of a receiving signal. The spatial relationship between the power spectrum of the two-dimensional fluctuating received power $P_S(k_x, k_y, L)$ and the observed power spectrum of the phase fluctuation for a weak scattering medium is given by [12]

$$\tilde{P}_S(k_x, k_y, L) = 4\, \tilde{W}_\varphi(k_x, k_y, L) \sin^2\left(\frac{k_x^2 + k_y^2}{k_f^2} \right), \qquad (10)$$

where $k_f = (4\pi / \lambda L)^{1/2}$ is the Fresnel wavenumber. The sinusoidal term in (10) is responsible for oscillations in the scintillation spectrum. For the scintillation level S_4 (zeroth moment), we have:

$$S_4^2 = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \tilde{P}_S(k_x, k_y, L). \qquad (11)$$

A scintillation spectrum with Fresnel oscillations is interpreted for both Gaussian and power-law wavenumber models. Equations (10) and (11) describe two dimensional diffraction patterns at the ground and also illustrate the strong attenuation of the interference pattern.

If rigid irregularities are moving past the ray path along the Y direction with apparent velocity V_y, the resultant temporal spectrum is given by a strip scan integration along the X axis,

$$P_\varphi(\nu, L) = \frac{2\pi}{V_y} \int_0^{\infty} dk_x\, \tilde{W}_\varphi\left(k_x, k_y = \frac{2\pi\nu}{V_y}, L \right),$$

$$P_S(\nu, L) = 4\, P_\varphi(\nu, L) \sin^2\left(\frac{\nu}{\nu_f} \right)^2. \qquad (12)$$

The Fresnel frequency $\nu_f = V_y / (\pi \lambda z)^{1/2}$ is directly proportional to the drift velocity V_y transverse to the radio path and inversely proportional to the Fresnel radius $(\lambda z)^{1/2}$, z is the mean distance between the observer and the irregularities. The intensity fluctuations are severely attenuated for irregularities larger than this radius or frequencies less than the Fresnel frequency. The oscillation minimums appear with ratios $1{:}\sqrt{2}$, $1{:}\sqrt{3}$, $1{:}\sqrt{4}$, … Observable power spectra $P_S(\nu, L)$ allows us to calculate the spectral width (1st and square root 2nd moments) which is a measure of the scintillation rate.

3 Numerical calculations

Numerical calculations are carried out for 40 MHz ($k_0 = 840\ \mathrm{km}^{-1}$) incident electromagnetic wave; Fresnel radius at the altitude 300 km is equal to 1.5 km. Plasma parameters are: $u_0 = 0.0012$, $\mathrm{v}_0 = 0.0133$. We use both Gaussian and power-law spectra. Anisotropic Gaussian correlation function in the principle *YZ* plane is [5]

$$\tilde{V}_n(k_x, k_y, k_z) = \sigma_n^2 \frac{l_\perp^2\, l_\parallel}{8\pi^{3/2}} \cdot$$

$$\cdot \exp\left(-\frac{k_x^2\, l_\perp^2}{4} - p_1 \frac{k_y^2\, l_\parallel^2}{4} - p_2 \frac{k_z^2\, l_\parallel^2}{4} - p_3\, k_y\, k_z\, l_\parallel^2 \right), \qquad (13)$$

where: $p_1 = (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0)^{-1} \left[1 + (1-\chi^2)^2 \cdot \sin^2\gamma_0 \cos^2\gamma_0 / \chi^2 \right]$, $p_2 = (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0) / \chi^2$,

$p_3 = (1-\chi^2) \sin\gamma_0 \cos\gamma_0 / 2\chi^2$, σ_n^2 is variance of electron density fluctuations. This function contains anisotropy factor of irregularities $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities) and inclination angle γ_0 of prolate irregularities with respect to the external magnetic field.

Measurements of satellite's signal parameters passing through ionospheric layer and measurements aboard of satellite show that in *F*-region of the ionosphere irregularities has the power-law spectrum with different spatial scales. We utilize 3D anisotropic power-law spectrum of electron density irregularities. The corresponding spectral function spatial power-law spectrum for $p > 3$ has the following form [5]

$$\tilde{V}_n(k_x, k_y, k_z) = \frac{\sigma_N^2}{\pi^{5/2}} \Gamma\left(\frac{p}{2}\right) \Gamma\left(\frac{5-p}{2}\right) \sin\left[\frac{(p-3)\pi}{2}\right] \cdot \frac{l_{\parallel}^3}{\chi^2 \left[1 + l_{\perp}^2 (k_x^2 + k_y^2) + l_{\parallel}^2 k_z^2\right]^{p/2}}, \qquad (14)$$

The value of p depends on the specific conditions of development of the turbulence, in particular the instability process involved, the latitude, the altitude, and so on. As observed by measurements with satellite ETS-2, the spectral index is usually between 2 and 4 [13].

Correlation function of the phase fluctuations caused by electron density fluctuations for anisotropic Gaussian correlation function (13) has the following form

$$\tilde{W}_{\varphi}(\xi, \eta, L) = \tilde{\Omega}_5 \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dx \; \frac{x^2 + \mathrm{P}_j^2 (\mu + s)^2}{(\delta_4 + x^2)^2} \cdot \left\{ -T^2 \left[\frac{x^2}{4\chi^2} + \frac{p_1 s^2}{4} + p_2 \frac{(\delta_3 + \delta_2 x^2)^2}{4(\delta_4 + x^2)^2} + 2 p_3 s \frac{\delta_3 + \delta_2 x^2}{\delta_4 + x^2} \right] \right\} \exp(-i\xi x - i\eta s), \qquad (15)$$

where: $\tilde{\Omega}_5 = \dfrac{1}{\pi} B_0 \dfrac{\Omega_5 T^2}{\chi}$, $B_0 = \sigma_n^2 \dfrac{\sqrt{\pi}}{4} \dfrac{T k_0 L}{\chi}$,

$x = \dfrac{k_x}{k_0}$, $s = \dfrac{k_y}{k_0}$, $T = k_0 l_{\parallel}$, $\delta_2 = 1 - \mathrm{P}_j \Gamma_j \mu - \mathrm{P}_j \Gamma_j s$,

$\delta_3 = \mathrm{P}_j s (\mathrm{P}_j \mu - 2\Gamma_j \mu^2 + \mathrm{P}_j s - 3\Gamma_j \mu s - \Gamma_j s^2)$,

$\delta_4 = \mathrm{P}_j^2 (\mu + s)^2$.

Knowledge of these functions allows us to calculate wave structure function and angle-of-arrival in the *XZ* plane using (6):

$$D_s(\xi, \eta, L) = \frac{2}{\sqrt{\pi}} \tilde{\Omega}_1 T \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dx \left[1 + \cos(\xi x + \eta s)\right] \exp\left[-\frac{T^2}{4}(p_2 m_5^2 x^4 + b_6 x^2 + b_7)\right] \qquad (16)$$

where: $\Omega_1 = \dfrac{\mathrm{v}_0^2}{(1-u_0)^2} \Big[1 + u_0 - 2\sqrt{u_0} (\sin\alpha - \cos\alpha + \sqrt{u_0} \sin\alpha \cos\alpha)\Big]$, $\tilde{\Omega}_1 = \dfrac{1}{4\sqrt{\pi}} B_0 \dfrac{\Omega_1 T}{\chi}$,

$m_5 = \dfrac{1}{4}[(s + \mu)\mathrm{P}_j + \Gamma_j]\Gamma_j$, $m_6 = \dfrac{1}{2}(s^2 + 2s\mu)$,

$b_7 = \dfrac{1}{4} p_2 s^4 + (p_2 \mu + 2p_3)s^3 + (p_1 + p_2 \mu^2 + 4p_3 \mu)s^2$,

$b_6 = \dfrac{1}{\chi^2} + 2 p_2 m_5 m_6 + 4 p_3 m_5 s$.

Figure 1 shows the broadening of the SPS versus anisotropy factor χ (left figure) for prolate electron density irregularities; characteristic longitudinal linear scale $l_{\parallel} = 200$ m, electromagnetic wave propagates with respect to the external magnetic field at the angle $\alpha = 20^0$. Numerical calculations show that maxima of the SPS are at $\chi = 5$ and $\chi = 16$ in the *XZ* plane for $\gamma_0 = 0^0$ and $\gamma_0 = 30^0$, respectively. Increasing angle $\alpha = 40^0$ maximum of the SPS displaced to the right $\chi \approx 18$ (for $\gamma_0 = 30^0$) and not displaced for $\gamma_0 = 0^0$. The broadening of the SPS of scattered ordinary and extraordinary waves approximately is the same for $L = 100$ km. External magnetic field has a substantial influence on the broadening of the SPS and narrows in the principle plane, which is in agreement with [8]. Deformation of the SPS depends on the angle α and the distance travelling by electromagnetic wave in turbulent magnetized plasma; except the case $\gamma_0 = 0^0$ (prolate irregularities are stretched along the external magnetic field).

Plots of the phase wave structure function D_s of scattered ordinary wave as a function of η for $\gamma_0 = 0^0 \div 15^0$ are presented on the right figure in the direction perpendicular to the principle plane. Numerical calculations show that maxima of the D_s function is at $\eta = 31$ if $\gamma_0 = 0^0$; and at $\eta = 59$ if $\gamma_0 = 5^0$; phase wave structure function tends to saturation at $\eta = 450$.

Analysis show that the angle-of-arrival in the *XZ* plane is in the interval $0.5'' \div 2'$.

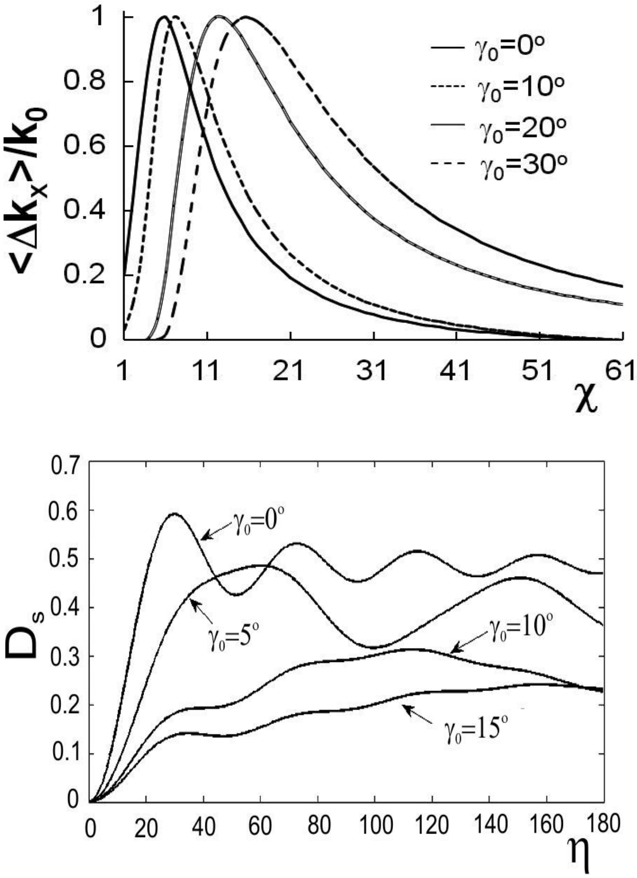


Figure 1. Broadening of the SPS of scattered radiation in the *XZ* plane as a function of the parameter χ at $\alpha = 20^0$ and different angles of inclination of prolate irregularities with respect to the external magnetic field $\gamma_0 = 0^0 \div 30^0$ (top figure). Phase structure function as a function of the distance between observation points η at $k_\perp = 0.114$, $\chi = 10$, $\alpha = 20^0$ for $P_S(\nu, L)$ (bottom figure).

Scintillation index S_4 for power-law correlation function in the *YZ* plane is given by

$$S_4^2 = \frac{8\,\tilde{\Omega}_5\,\tilde{Q}}{\pi} \int\limits_{-\infty}^{\infty} dx \int\limits_{-\infty}^{\infty} ds\, \frac{x^2 + \mathrm{P}_j^2\,(\mu + s)^2}{(x^2 + \delta_4)^2} \cdot$$

$$\cdot \left\{ 1 + T^2 \left[\frac{x^2 + s^2}{\chi^2} + \frac{(\delta_2\, x^2 + \delta_3)^2}{(x^2 + \delta_4)^2} \right] \right\}^{-p/2} \cdot$$

$$\sin^2\left[k_0 L\, (x^2 + s^2) \right], \tag{17}$$

where: $\tilde{Q} = \Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{5-p}{2}\right)\sin\left[\frac{(p-3)\,\pi}{2}\right]$. Square of the scintillation index is proportional to the thickness of a layer and the root-mean-square deviation of small-scale electron density irregularities σ_n^2 which are responsible for signal fading.

From equations (2), (7) and (14) for the SPS of power-law correlation functions of electron density fluctuations in turbulent non-magnetized plasma we obtain:

$$\frac{W_{EE^*}(\xi, \eta, L)}{E_0^2} = \exp(-i\eta\,\mu)\, \exp\left(\pi^{3/2} \frac{T^2\, \mathrm{v}_0^2\, Q_0}{\chi\, \Gamma\left(\frac{p}{2}\right)}\, k_0\, L \cdot \right.$$

$$\cdot \left\{ -\frac{1}{2}\,\Gamma\left(\frac{p-1}{2}\right) \int\limits_{-\infty}^{\infty} ds\, \frac{1}{(1 + C_1\, s^2)^{p-1}} + \left(\frac{\chi\,\xi}{2\,T}\right)^{(p-1)/2} \cdot \right. \tag{18}$$

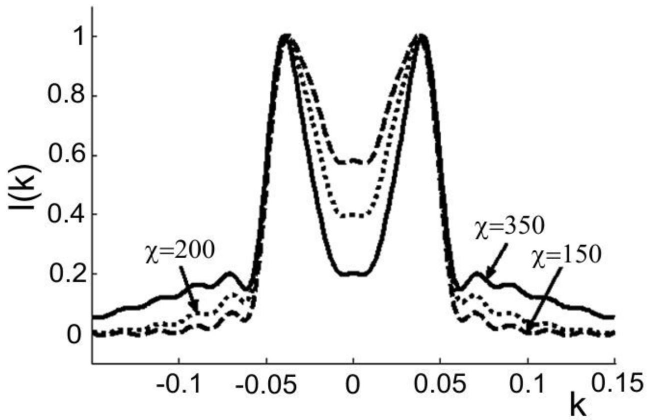
$$\left. \left. \cdot \int\limits_{-\infty}^{\infty} ds\, \frac{\exp(-i\eta\, s)}{\left[1 + \Phi(s)\right]^{(p-1)/2}}\; K_{\frac{p-1}{2}}\left(\frac{\chi\,\xi}{T}\left[1 + \Phi(s)\right] \right) \right\} \right),$$

where: $\Phi(s) = \frac{T^2}{4}\left[s^4 + 4\,\mu\, s^3 + 4\left(\frac{1}{\chi^2} + \mu^2\right) s^2 \right]$,

$Q_0 = \frac{\sigma_n^2}{\pi^{5/2}}\,\tilde{Q}$, $C_1 = \frac{T^2}{\chi^2}(1 + \chi^2\,\mu^2)$, $\Gamma(x)$ is the gamma function, $K_\nu(x)$ is the McDonald function.

Figure 2 illustrates the evaluation of a gap in the SPS of scattered ordinary wave in turbulent anisotropic collisionless non-magnetized plasma with prolate irregularities of electron density fluctuations having characteristic longitudinal linear scale $l_{\|} = 10$ km; spectral index is equal to $p = 3.2$. Numerical calculations show that electromagnetic wave having parameters $\mu = 0.05$, $\alpha = 20^0$, $\beta = 10$ travelling distance 2000 km in the ionosphere broadens on 56% increasing anisotropy factor of prolate irregularities from $\chi = 150$ up to 350. When the scattering is weak, oscillations are observed in the power spectra. These oscillations are attributed to a Fresnel filtering effect. These oscillations are used to calculate the velocity, estimate the irregularity scale size, anisotropy factor and angle of inclination of prolate irregularities with respect to the external magnetic field. The rate of oscillations is dependent on the irregularity velocity. The power spectra

$P_S(\nu, L)$ for the ordinary and extraordinary waves have different minima and allow us to calculate corresponding k_f and ν_f .



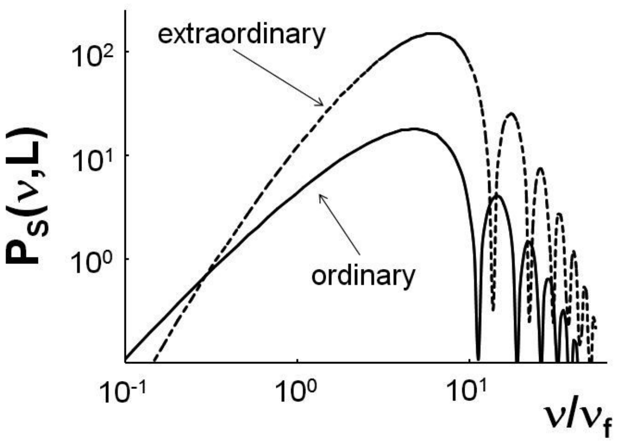


Figure 2 Depicts SPS of scattered ordinary wave versus k for $\mu = 0.05$, $\alpha = 20^0$, $\beta = 10$, at fixed $B_0 = 3300$, plasma parameters: $p = 3.2$, $l_{\parallel} = 10$ km and different anisotropy factor $\chi = 150$, 200, 350 (top figure); The power spectra $P_S(\nu, L)$ for the ordinary and extraordinary waves as a function of non-dimensional frequency parameter ν / ν_f at $\chi = 5$, $T = 250$, $\alpha = 40^0$, $\gamma_0 = 10^0$ are plotted on the bottom figure.

4 Conclusion

Second order statistical moments of scattered radiation: correlation function of the phase fluctuation, broadening of the SPS, angle-of-arrivals and the scintillation index caused by electron density variations are obtained for arbitrary correlation function of electron density fluctuations. Numerical calculations are carried out for both anisotropic Gaussian and power-law correlation functions. New peculiarities of the "Double-humped effect" have been revealed first time analytically in the SPS of multiple scattered high frequency electromagnetic waves at oblique illumination of magnetized plasma with prolate inhomogeneities by mono-directed incident radiation using the smooth perturbation method. It was established that the SPS has a double-peaked shape, the location of its maximum weakly varies and width substantially broadens with increasing distance travelling by electromagnetic waves in turbulent anisotropic magnetized plasma. Anisotropy factor and the angle of inclination of prolate irregularities have a substantial influence on the gap of the SPS. The power spectra for the ordinary and extraordinary waves have different minima allowing determining the Fresnel frequency. The power spectral analysis of scintillation observations offers a new and important method of analyzing the small-scale structure in the ionosphere.

The obtained results will have applications in communication, acoustics, at observations of electromagnetic waves propagation in the upper atmosphere and remote sensing.

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