Nonlinear Vibration of Fluid-loaded Double-walled Carbon Nanotubes Subjected To A Moving Load Based on Stochastic FEM

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Abstract - This paper adopts stochastic FEM to study the statistical dynamic behaviors of nonlinear vibration of the fluid-conveying double-walled carbon nanotubes (DWCNTs) under a moving load by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. The Young’s modulus of elasticity of the DWCNTs is considered as stochastic with respect to the position to actually characterize the random material properties of the DWCNTs. Besides, the small scale effects of the nonlinear vibration of the DWCNTs are studied by using the theory of nonlocal elasticity. Based on the Hamilton’s principle, the nonlinear governing equations of the fluid-conveying double-walled carbon nanotubes under a moving load are formulated. The stochastic finite element method along with the perturbation technique is adopted to study the statistical dynamic response of the DWCNTs. Some statistical dynamic response of the DWCNTs such as the mean values and standard deviations of the non-dimensional dynamic deflections are computed and checked by the Monte Carlo Simulation, meanwhile the effects of the nonlocal parameter and aspect ratio on the statistical dynamic response of the DWCNTs are investigated. It can be concluded that the nonlocal solutions of the dynamic deflections get larger with the increase of the nonlocal parameters due to the small scale effect, and as the aspect ratio increases, the small scale effect has less effect on the maxima non-dimensional dynamic deflections of the DWCNTs.

Keywords: Nonlinear vibration; Double-walled carbon nanotubes; Stochastic FEM; Perturbation technique; Small scale effect.

1 Introduction

Since the landmark paper published by Iijima [1], carbon nanotubes (CNTs) have attracted worldwide attention due to their potential use in the fields of chemistry, physics, nano-engineering, electrical engineering, materials science, reinforced composite structures and construction engineering. Carbon nanotubes (CNTs) are used for a variety of technological and biomedical applications including nanocounters for gas storage and nanopipes conveying fluids [2-8]. Some important applications of carbon nanotubes (CNTs) are such as nanotubes conveying fluids [3,7-8], different types of fluid flows like water [9], dynamic flow of methane, ethane and ethylene molecules [10] and the diffusive transport of light gases [11] had been reported, and the effects of these fluids on the mechanical properties of CNTs had been investigated. Natsuki et al. [12] adopted a simplified Flügge shell model to investigate the wave propagation of single- and double-walled CNTs conveying fluid. The single-elastic beam model [13-14] and the multiple-elastic beam model [15-19] were also broadly adopted to study the dynamic behaviors of fluid-conveying single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs). The vibration frequencies of the linear system and the system’s stability related to the internal moving fluid were investigated. Moreover, the nonlocal elasticity theory was incorporated into the elastic beam model to study the small scale effect on the dynamics of SWCNT conveying fluid [20]. Chang and Liu [21-22] studied small scale effects on the flow-induced instability of double-walled carbon nanotubes (DWCNTs) by using the nonlocal elasticity theory. More recently, Chang [23-24] investigated the thermal-mechanical vibration and instability of fluid-conveying single-walled carbon nanotubes (SWCNTs) based on nonlocal elasticity theory. Generally speaking, the beam models mentioned above are linear; however, the vdW forces in the interlay space of MWCNTs are essentially nonlinear. Furthermore, the slender ratios are normally large if the beam models are adopted, that is, the large deformation will occur. Therefore, it is quite essential to consider two types of nonlinear factors, namely, the geometric nonlinearity and the nonlinearity of vdW force in investigating the dynamic behaviors of fluid-conveying MWCNTs. Kuang et al. [25] investigated the dynamic behaviors of double-walled carbon nanotubes (DWCNTs) conveying fluid by considering two types of nonlinearities mentioned above. Due to the rapid process of nanotechnology, the motion of neutral atoms and nanoparticles in nanotubes has been of remarkable interest [26]. Carbon nanotubes are utilized as molecular channels for the transportation of nanoparticles, such as water and protons [27]. In the process of these applications, carbon nanotubes might be subjected to moving load, and this causes the transverse vibration of carbon nanotubes. Therefore, it is quite necessary to
investigate the dynamic behavior of carbon nanotubes under moving loads. So far, most researchers have studied static, buckling or free vibration analysis of nanotubes or nanobeams based on the local or nonlocal elasticity theory, forced vibration of DWCNTs under moving loads is rarely investigated. Until recently, Simsek [28] performed the vibration analysis of a SWCNT under action of a moving harmonic load based on nonlocal elasticity theory. Kiani and Wang [29] adopted nonlocal elasticity theory to investigate the interaction of a single-walled carbon nanotube with a moving nanoparticle.

Salvetat et al. [30] measured the flexural Young’s modulus and shear modulus using AFM test on clamped–clamped nanoropes, getting values with 50% of error. Information related to statistical distributions of experimental data is also rare, and the important study from Krishnan et al. [31] provides one of the few examples available of histogram distribution of the flexural Young’s modulus derived from 27 CNTs. The Young’s modulus was estimated observing free-standing vibrations at room temperature using transmission electron-microscope (TEM), with a mean value of 1.3 TPa - 0.4 TPa/±0.6 TPa Pronouncedly, in [32], stochastically averaged probability amplitude for the vibration modes is computed to obtain the root-mean-square vibration profile along the length of the tubes. Uncertainty is also associated to the equivalent atomistic-continuum models adopted extensively in particular by the engineering and materials science communities. Hence, to be realistic, the Young’s modulus of elasticity of carbon nanotube (CNTs) should be considered as stochastic with respect to the position to actually describe the random property of the CNTs under certain conditions. In the present study, we investigate the statistical dynamic behaviors of the fluid-conveying double-walled carbon nanotubes (DWCNTs) under a moving load by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. The Young’s modulus of elasticity of the DWCNTs is modeled as a double-tube pipe which is composed of the inner tube of radius $R_1$ and the outer tube of radius $R_2$. The thickness of each tube is $h$, the length is $L$, and Young’s modulus of elasticity is $E$. It is noted that the Young’s modulus of elasticity $E$ is assumed as stochastic with respect to the position to actually describe the random material property of the DWCNTs. The internal fluid is assumed to flow steadily through the inner tube with a constant velocity $U$. Besides, the boundary conditions of the DWCNTs are assumed as simply-supported at both ends. Based on the theory of Euler–Bernoulli beam and a nonlinear strain–displacement relationship of Von Karman type, the displacement field and strain–displacement relation can be written as follows:

$$\dddot{u}_i(x, z, t) = u_i(x, t) - z \dddot{\omega}_i(x, t)$$  \hspace{1cm} (1)

$$\dddot{w}_i(x, t) = w_i(x, t)$$

$$\varepsilon_i = \frac{\dddot{u}_i}{c \dot{x}} + \frac{1}{2} \left( \frac{\dddot{w}_i}{c \dot{x}} \right)^2$$

where $x$ is the axial coordinate, $t$ is time, $u_i$ and $w_i$ denote the total displacements of the $i$th tube along the $x$ coordinate directions, $u_i$ and $w_i$ define the axial and transverse displacements of the $i$th tube on the neutral axis, $\varepsilon_i$ the corresponding total strain, and the subscript $i = 1$ and $i = 2$. Notice that tube 1 is the inner tube while tube 2 is the outer tube.

Based on Eq. (1), the potential energy $V$ stored in a DWCNTs and the virtual kinetic energy $T$ in the DWCNTs as
well as the fluid inside the DWCNTs can be individually determined. Based on Hamilton’s principle, the variational form of the equations of motion for the DWCNTs can be given by

$$\int_{t_0}^{t_f} (\delta V - \delta T - \delta \Psi) \, dt = 0$$  \hspace{1cm} (2)$$

where $\delta \Psi$ is the virtual work due to the vdW interaction and the interaction between tube 1 and the flowing fluid. Based on Eq. (2) and the formulations derived by Chang [22, 24], the coupled nonlinear governing equations for the vibration of DWCNTs conveying fluid based on nonlocal elasticity theory are given as follows:

$$\frac{\partial^2}{\partial x^2} \left[ E(x) I_2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right] - 2(1-v_a) \frac{M U}{\partial x^2} \frac{\partial^2 w}{\partial x^2} (e_a) \left[ (M + m_i) \frac{\partial^2 w}{\partial x^2} \right] + \left[ 1-(e_a) \frac{\partial}{\partial x} \right] \left[ \left( 1-(e_a) \frac{\partial}{\partial x} \right) \right] = 0$$  \hspace{1cm} (3)$$

$$\frac{\partial^2}{\partial x^2} \left[ E(x) I_2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right] + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \left( \frac{\partial w}{\partial x} \right)^2 - \left( \frac{\partial w}{\partial x} \right)^2 \frac{E(x) A_4}{\partial x^2} \frac{\partial}{\partial x} \frac{\partial^2 w}{\partial x^2} - \left( \frac{\partial w}{\partial x} \right)^2 \frac{E(x) A_4}{\partial x^2} \frac{\partial}{\partial x} \frac{\partial^2 w}{\partial x^2} - \left( \frac{\partial w}{\partial x} \right)^2 \frac{E(x) A_4}{\partial x^2} \frac{\partial}{\partial x} \frac{\partial^2 w}{\partial x^2} - \left( \frac{\partial w}{\partial x} \right)^2 \frac{E(x) A_4}{\partial x^2} \frac{\partial}{\partial x} \frac{\partial^2 w}{\partial x^2} = 0$$  \hspace{1cm} (4)$$

where $[M]$ is the global consistent mass matrix of the structure, $[C]$ is the global damping matrix of the structure, $\dot{W}$ is the global velocity vector of the structure, $\ddot{W}$ is the global acceleration vector of the structure, $P$ is the global external force vector of the structure and $R(W)$ is the global vector of restoring forces of the structure that depends on the displacement field. Based on equation (5), the governing equation of the structure at time $t + \Delta t$ is given by

$$[M] \ddot{W}^{t+\Delta t} + [C] \dot{W}^{t+\Delta t} + [K] \Delta W = P^{t+\Delta t} - R(W')$$  \hspace{1cm} (6)$$

The above equation can be solved by any direct time integration method even it is nonlinear. In order to improve the solution accuracy, it is necessary to carry out the equilibrium iteration in each time step. In this study, the Newton-Raphson method in conjunction with Newmark scheme is adopted to perform the numerical analysis.

### 4 Perturbation technique

In this study, only the Young’s modulus of elasticity $E$ is assumed to be stochastic in position, the geometric shapes and sizes of the structure and the moving load and the fluid load are assumed to be deterministic. Applying the perturbation technique, the randomly fluctuating Young’s modulus of elasticity $E$ can be assumed as:

$$E(x) = E^{(0)} \left[ 1 + \alpha(x) \right] = E^{(0)} + E^{(0)} \alpha(x)$$  \hspace{1cm} (7)$$

where $E^{(0)}$ is the mean value of the Young’s modulus of elasticity, $\alpha(x)$ is random variable with zero mean, and $E^{(0)} \alpha(x)$ is homogeneous stochastic field representing the fluctuation of the Young’s modulus of elasticity around its mean value. Assuming the random variable $\alpha$ is uniform within the element, then the stochastic nodal displacement vector can be expanded about $\alpha$ by using Taylor series as:

$$W^{i\times\Delta t} = W^{(0)\times\Delta t} + \sum_{i=1}^{NE} \sum_{j=1}^{NE} W^{(i\times\Delta t)} a_i a_j + \frac{1}{2} \sum_{i=1}^{NE} \sum_{j=1}^{NE} \sum_{k=1}^{NE} \sum_{l=1}^{NE} W^{(i\times\Delta t)} a_i a_j a_k a_l + \ldots$$  \hspace{1cm} (8)$$

$$\Delta W = \Delta W^{(0)} + \sum_{i=1}^{NE} \sum_{j=1}^{NE} \Delta W^{(i)} a_i a_j + \frac{1}{2} \sum_{i=1}^{NE} \sum_{j=1}^{NE} \sum_{k=1}^{NE} \sum_{l=1}^{NE} \Delta W^{(2)} a_i a_j a_k a_l + \ldots$$  \hspace{1cm} (9)$$

where the superscript $(0)$ represents the mean value term, both $i$ and $j$ denote the element numbers, $NE$ is the total number of the element and $\Sigma$ means the merging with respect to element. Similarly, the restoring force vectors and the tangent
stiffness matrix can be written in similar fashion. Then applying the perturbation technique to equation (6), the higher order terms are truncated, and comparing equal order terms for the random variable $\alpha$, the zero, first, and second order equations for the problem are obtained, respectively. The solutions of these equations are achieved successively by using the procedures described in the previous section. The statistical dynamic responses of DWCNTs can be obtained after calculating the zero, first and second order equations. For example, at any fixed time, both expected value of deflection and autocorrelation of the deflection between two different points $p$ and $q$ can be obtained based on the first order approximation by neglecting the third term in equation (8) as follows:

$$E[W] = W^{(0)}$$

$$R_w(x_p,x_q) = E[(W - E[W])(W - E[W])^T]$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} W^{(i)}_p W^{(i)}_q E[\alpha \alpha_j]$$

$$E[\alpha_i \alpha_j] = R_\alpha(x_i - x_j) = R_\alpha(\Delta x)$$

where $E[\bullet]$ is the expectation and the $R_\alpha(\Delta x)$ is the autocorrelation function of random variable $\alpha$ assuming that the Gaussian stochastic process of the Young’s modulus of elasticity $E$ is homogeneous with respect to the position, $x_p$ and $x_q$ are the coordinate at the center of the element $p$ and $q$. Based on equation (11), the stochastic process of deflection is assumed to be homogeneous with respect to position as well, $R_w(x_i,x_j)$ can be replaced by $R_w(\Delta x)$. Therefore, the autocorrelation $R_w(\Delta x)$ can be computed readily provided that the spectra density of the Young’s modulus of elasticity is given.

5 Numerical examples and discussion

In the numerical computations, the simply supported boundary condition is considered for the DWCNTs conveying fluid. The inner and the outer tubes are assumed to have the same Young’s modulus, the same thickness and the same mass density. The numerical values of the parameters are adopted as follows: Mean value of Young’s modulus $E=1$ Tpa, tube thickness $h=0.34$ nm, mass density $\rho = 2300 Kg/m^3$, the mass density of water flow is $\rho_f = 1000 Kg/m^3$, the inner radius $R_i = 0.7 nm$ and the outer radius $R_e = 1.04 nm$, the standard deviation of random variable $\alpha$ is assumed as $\sigma_\alpha = 0.1$. The length of the DWCNTs is considered as a variable for the different values of the aspect ratio $L/d$. In the present study, the nonlocal parameter is chosen as $0 \leq \epsilon_0 a \leq 2.0 nm$ to investigate the small scale effects on the dynamic responses. For a constant velocity of the moving load, the non-dimensional dynamic deflection is normalized as the ratio between the dynamic deflection and the static deflection, which is $D = \frac{F_\alpha L^3}{48E^{(0)}I}$, of a beam under a point load $F_0$ at the middle point of the beam. In the following numerical computations, the internal fluid velocity of the DWCNTs is assumed as $U = 400 m/sec$, the non-dimensional velocity $\overline{V} = 0.2$ is assumed for the moving load and the aspect ratio $L/d = 10$ is considered, unless they are specified otherwise. In Figs. 2-3, the mean values and standard deviations of the non-dimensional dynamic deflections of the DWCNTs are depicted. Fig. 2 presents the mean value of the non-dimensional dynamic deflections $w_2(L/2,t)/D$ versus the non-dimensional time $T$ for various values of the nonlocal parameter $\epsilon_0 a$. As it can be seen from Fig. 2, the numerical results based on the present study are checked by Monte Carlo Simulation, they are in excellent agreements. Fig. 3 presents the standard deviation of the non-dimensional dynamic deflections $w_2(L/2,t)/D$ versus the normalized dimensional time $T$ for various values of the nonlocal parameter $\epsilon_0 a$. Once again, the numerical results based on the present study are in good agreements with those estimated by Monte Carlo Simulation except that the results from Monte Carlo Simulation are slightly larger than those from the present study. Fig. 4 presents the mean values of the maximum non-dimensional dynamic deflections $w_2(x,t)/D$ versus the aspect ratio $L/d$ for various values of the nonlocal parameter $\epsilon_0 a$ at the constant moving load velocity $\overline{V} = 0.2$. As it can be detected from the figure, the maxima non-dimensional dynamic deflections computed using the nonlocal model are larger than those of the local (classical) model thanks to the small scale effect. Based on the results in Fig. 5, the maxima non-dimensional deflections get larger as the nonlocal parameter increases, and the effect of the nonlocal parameter depends on the aspect ratio.
6 Conclusions

This paper investigates the statistical dynamic behaviors of nonlinear vibration of the fluid-conveying double-walled carbon nanotubes (DWCNTs) under a moving load by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. The Young's modulus of elasticity of the DWCNTs is considered as stochastic with respect to the position to actually characterize the random material properties of the DWCNTs. In addition, the small scale effects of the nonlinear vibration of the DWCNTs are studied by using the theory of nonlocal elasticity. Based on the Hamilton's principle, the nonlinear governing equations of the fluid-conveying double-walled carbon nanotubes under a moving load are formulated. The
stochastic finite element method along with the perturbation technique is adopted to study the statistical response of the DWCNTs; in particular, the Newton-Raphson iteration procedure in conjunction with Newmark scheme is utilized to solve the nonlinearity of the dynamic governing equation of the DWCNTs. Some statistical results obtained by the perturbation technique and those from the Monte Carlo simulation approach show good agreements. Some statistical dynamic response of the DWCNTs such as the mean values and standard deviations of the non-dimensional dynamic deflections are calculated, meanwhile the effects of the nonlocal parameter and aspect ratio on the statistical dynamic response of the DWCNTs are investigated. It can be concluded that the nonlocal solutions of the dynamic deflections get larger with the increase of the nonlocal parameters due to the small scale effect. It is noted that the computed stochastic dynamic response plays an important role in evaluating the structural reliability of the DWCNTs.

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7 References


