Hardware-Software Cosimulation of Feedback Controller for Synchronization of Inferior Olive Neurons

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Abstract—The design of a control system for the synchronization of Inferior Olive Neurons (IONs) and hardware-software cosimulation of the closed-loop system are presented in this paper. Each ION is described by a set of four nonlinear differential equations. These IONs exhibit limit cycle oscillations (LCO), but are not necessarily in phase. The objective is to control one of the IONs so that both oscillate in unison (with zero relative phase). A simple linear feedback control law for the synchronization of the IONs using an output variable is developed. In the closed-loop system, asymptotic convergence of the state vectors of the IONs is accomplished. Then the hardware-software cosimulation of the complete closed-loop system is considered. Hardware synthesis and HW-SW cosimulation tests are performed to examine the performance of the controller. The oscillatory properties of the Inferior Olive Neurons (IONs) can be used to provide timing signals for motors to mimic vertebral-like movements. These vertebral-like movements can be used in bio-inspired underwater and unmanned vehicles with oscillating fins.

Keywords: Inferior Olive Neuron, ION synchrony, HW-SW simulation

1. Introduction

The synchronous activity of olivo-cerebellar system, is one of the key neuronal circuits in the brain, provides motor control signals for the movement execution. This neuronal network is organized around clusters of inferior olive neurons (IONs) \([6], [16], [15]\). The inferior olive neurons (IONs) have various features including, the subthreshold activity in which the membrane potential has sustained fluctuations. This rhythmic activity has been termed as spontaneous subthreshold oscillations. The orbits of the IONs may have different shapes (sinusoidal, quasi-periodic, periodic waveform with spikes, and irregular) \([12]\).

The dynamical behavior and functional significance of interconnection has been explored by Armstrong \([1]\). Bernard and Foster \([3]\) have examined the oscillatory pattern of IONs. A structural study of inferior olivary nucleus of cat has been performed by Sotelo et al. \([19]\). For low amplitude oscillations, a model based on electrical coupling of neurons with heterogeneous channel densities has been considered by Manor et al. \([17]\). A variety of mathematical models capturing the important characteristics of IONs have been proposed in literature. Velarde et al. \([20]\) and Linas et al. \([16]\) developed ION models using Vander Pol Oscillator and FitzHugh-Nagumo (FN) systems. A relatively simple ION model has been also proposed by Kazantsev et al. \([11]\), \([10]\). The dynamical behavior of ION depends on its parameters. The structure of the orbit of an ION undergoes drastic changes when its parameters vary. The study of qualitative changes in the orbit structure termed as, bifurcation of neurons has been performed by Guckenheimer and Labouriau \([5]\) and Izhikevich \([7]\), \([8]\). For inferior olive neurons, a bifurcation analysis has been treated by Katori et al. \([9]\); Lee and Singh \([14]\). Authors have also studied the synchronous activity of interconnected neurons. Neurons are connected via gap junction to form electrical coupling. Katori et al. \([9]\) have studied the spatio-temporal dynamics of IONs using conductance-based model. Neuronal synchronization in the mammalian brain have been examined by Bennett and Zukin \([4]\).

In view of the important role of the IONs in motor control, researchers have shown interest in the models of these IONs for the control of autonomous air and underwater bio-robotic vehicles. Robust oscillatory patterns of a variety of shapes and sizes of the IONs are suitable for executing different kinds of maneuvers. For the control of robotic vehicles, it is essential to control the relative phase angles of the cluster of IONs. As such the problem of synchronization of IONs is important \([22]\). In literature, synchronization of IONs has been explored and linear or nonlinear or adaptive control systems have been developed by Bandyopadhy et al. \([2]\) and Lee and Singh \([13]\), \([14]\). Bennett and Zukin \([4]\) have shown that a model of gap junction in conjunction with postsynaptic capacitance as a low pass filter achieves neuronal synchronization. In view of the work of Bennett and Zukin \([4]\), it is of interest to develop control systems which are simple in form because nonlinear and adaptive control systems developed for synchronization of IONs are not attractive from the viewpoint of implementation.

Contribution of this paper lies in the development of a simple robust control system for the synchronization of IONs. It is assumed that a single output of each ION is measured for feedback. One of the IONs is treated as a
reference ION, and the other ION is controlled by the application of an extracellular stimulus. A control law is developed so that the trajectories of the controlled ION asymptotically follows the trajectories of the reference ION. The control input is the product of a suitably chosen gain and the scalar output error of the IONS. It is shown that synchronization is accomplished for a set of values for the feedback gain. The control system is robust to variations in the key parameters of the IONS and the controller gain. Then the hardware implementation of the complete closed-loop system, including a pair of IONS and the feedback control system, is developed. Laboratory tests as well as numerical simulation results are obtained. It is seen that the hardware circuit produces the waveforms observed in digital simulation closely. The hardware circuit is especially useful for real-time control of air and underwater vehicles using flapping wings and fins.

The organization of the paper is as follows: Section 2 presents the mathematical model of the IONS. A control law for synchronization is developed in Section 3. The results of digital simulation is given in Section 4. Section 5 considers the HW-SW implementation and simulation of the closed-loop system, and laboratory test results and comparison of these with the simulated results are presented in Section 6.

2. IO neuron model

In this paper, the ION model described in Kazantsev et al. [11], [10] are considered for synchronization. Let the state vector of the $i$th neuron be $(u_i, v_i, z_i, w_i)^T$, $i = 1, 2$. ($T$ denotes matrix transposition.) The model has polynomial nonlinearities in variables $u_i$ and $z_i$ of degree three. For simplicity in notation, often the arguments of various functions will be suppressed. The nonlinear equations describing the ION1 (ION1) are

$$
\begin{align*}
\dot{u}_1 &= k \epsilon_{Na}^{-1} [u_1^2 - u_1^3 + (u_2^2 - u_1)a - v_1] \\
\dot{v}_1 &= k (u_1 - z_1 + I_{Ca} - I_{Na}) \\
\dot{z}_1 &= [z_2^2 - z_2^3 + (z_1^2 - z_1)a - w_1] \\
\dot{w}_1 &= \epsilon_{Ca} (z_1 - I_{Ca} - \mu^*-I_{ext1})
\end{align*}
$$

The variables $z_1$ and $w_1$ are responsible for subthreshold oscillations and low-threshold ($Ca^{2+}$-dependent) spiking, and the variables $u_1$ and $v_1$ describe the higher-threshold ($Na^{+}$-dependent) spiking. The oscillation time scales are controlled by the parameters $\epsilon_{Ca}$ and $\epsilon_{Na}$; and $I_{Ca}$ and $I_{Na}$ drive the depolarization level (equilibrium point) of the system.

The parameter $k$ sets the relative time scale of the two systems. The parameters $\alpha_i$’s (appearing in the nonlinear functions) play an important role in shaping the trajectories of the IONS. $I_{ext1}$ denotes the extracellular excitation used here as the control input. The bias term $\mu^*$ provides flexibility in getting different kinds of waveforms. The ION1 is treated as the slave ION.

The reference ION is described by

$$
\begin{align*}
\dot{u}_2 &= k \epsilon_{Na}^{-1} [u_2^2 - u_2^3 + (u_2^2 - u_1)a - v_2] \\
\dot{v}_2 &= k (u_2 - z_2 + I_{Ca} - I_{Na}) \\
\dot{z}_2 &= [z_2^2 - z_2^3 + (z_2^2 - z_2)a - w_2] \\
\dot{w}_2 &= \epsilon_{Ca} (z_2 - I_{Ca} - \mu^*)
\end{align*}
$$

Note that ION2 has no input. These IONS (with $I_{ext1}$ $=0$) exhibit limit cycle oscillations as well as bursting phenomenon for a set of values of $\mu^*$ and $\alpha$ [13]. However these oscillations are not necessarily in phase. We are interested in designing a simple linear feedback control law such that in the closed-loop system, the state vector of ION1 asymptotically tracks the state vector of the reference ION. Furthermore, it is assumed that only the output error signal $(z_1 - z_2)$ is measured for feedback.

3. Synchronizing Control System

In this section, for the synchronization of the IONS, a linear feedback control law is designed. Define state vectors $x_1 = (u_1, v_1, z_1, w_1)^T \in \mathbb{R}^4$ and $x_2 = (u_2, v_2, z_2, w_2)^T \in \mathbb{R}^4$. Then (1) and (2), can be compactly written as

$$
\begin{align*}
\dot{x}_1 &= f(x_1) + BI_{ext1} \\
\dot{x}_2 &= f(x_2)
\end{align*}
$$

where the nonlinear vector function $f(x_1) \in \mathbb{R}^4$ and $f(x_2) \in \mathbb{R}^4$ are easily obtained from (1) and (2), and one has $B = [0, 0, 0, -\epsilon_{Ca}]^T$.

Let $e = x_1 - x_2$ be the state vector error of the two IONS. Then using (3), the dynamics of the error are given by

$$
\dot{e} = f(x_1) - f(x_2) + BI_{ext1}
$$

Expanding $f(x_1) = f(x_2 + \epsilon)$ about $x_2$ gives

$$
\dot{e} = f(x_2) + \frac{\partial f(x_2)}{\partial x_1} \epsilon + BI_{ext1} - f(x_2) + h.o.t
$$

where h.o.t denotes higher-order terms in $\epsilon$. For small $\epsilon$, (5) can be approximated by the variational equation of the form

$$
\dot{e} = A(t) \epsilon + BI_{ext1}
$$

where

$$
A(t) = \frac{\partial f(x_2(t))}{\partial x_1}
$$

is the Jacobian matrix, evaluated along the trajectory $x_2(t)$ of ION2. It easily follows that the matrix $A(t)$ is

$$
\begin{bmatrix}
\alpha & -k \\
0 & -k & 0 \\
0 & 0 & \beta & -1 \\
0 & 0 & 0 & \epsilon_{Ca}
\end{bmatrix}
$$
where \( \alpha = \frac{6}{5} [2u_2 - 3u_2^2 + (2u_2 - 1)a] \) and \( \beta = 2z_2 - 3z_2^2 + (2z_2 - 1)a \).

Let us assume that the reference ION is undergoing a limit cycle oscillation. As such \( x_2(t) \) is a periodic trajectory. Suppose that the period of \( x_2(t) \) is \( T_p \). Then the matrix \( A(t) \) is also periodic and its period is \( T_p \). For the synchrony of the two IONs, it is essential to design a control system such that the state vector error \( e(t) \) converges to zero. Let us select a control signal of the form
\[
I_{ext1} = g(z_1 - z_2)
\]
where \( g \) is a feedback gain (yet to be determined). The closed-loop error system is
\[
\dot{e} = f(x_1) - f(x_2) + gBC(x_1 - x_2) = f_e(e, t)
\]
where \( e = [0, 0, 1, 0] \), and the argument \( t \) indicates the dependence of \( f_e \) on \( x_2(t) \). Substituting the control input \( (8) \) in \( (6) \), gives the variational equation
\[
\dot{e} = (A(t) + gBe)e = \hat{A}_c(t)e
\]
The matrix \( \hat{A}_c(t) \) is also periodic.

First consider stability of the equilibrium point \( e = 0 \) of the variational equation \( (10) \). For this purpose, let us compute the transition matrix of \( \hat{A}_c(t) \) by solving the matrix differential equation
\[
\Phi(t, t_0) = \hat{A}_c(t)\Phi(t, t_0); \Phi(t_0, t_0) = I_{4 \times 4}
\]
where \( I \) denotes an identity matrix of indicated dimension. The growth property of \( \Phi(t, t_0) \) depends on the characteristic multipliers (eigenvalues of \( \Phi(T_p, 0) \)). For the asymptotic stability of the origin of \( (10) \), the characteristic multipliers must be strictly within the unit disk \([18]\). Note that \( \hat{A}_c(t) \) is a function of the gain \( g \), and its characteristic multipliers depend on it. It will be seen in the next section that there exists a set of values of the gain for which asymptotic stability of \( (10) \) is assured. Of course, asymptotic stability of the variational equation implies asymptotic stability of \( e = 0 \) of the nonlinear time varying system \( (9) \). In the next section, a set of values of \( g \) are obtained and the performance of the controller is examined.

5. Hardware Implementation

This work also presents implementation of the proposed ION synchronization and the ION (reference ION) in hardware using a Hardware-Software (HW-SW) co-simulation methodology. The Xilinx System Generator (XSG) for DSP is an add-on module for the Mathworks Simulink software. The ION model and the proposed loopback control are designed and implemented using Xilinx System Generator for DSP using predefined device optimized DSP blocksets and custom HDL codes. The tool generates synthesizable HDL code that is mapped in the Xilinx reconfigurable chip (FPGA). The developed system represents a bit-accurate and cycle-accurate model of the software simulation model. The advantages of using System Generator for DSP can be summarized as follows: 1) Rapid prototyping of complex and high-performance DSP systems from high-level abstraction, 2) Bit and cycle accurate floating and fixed point implementation of the DSP algorithm, 3) Automatic generation of HDL code for synthesis to reconfigurable devices, 4) Support for hardware-software cosimulation, and
5) Support of Xilinx tools to explore power, latency and device utilization of the developed implementation.

In our implementation the Xilinx System Generator for DSP 12.1 was used to generate HDL codes that were synthesized using Xilinx ISE 12.1 Design Suite. A XSG implementation of the ION and the top-level implementation of the reference ION and the controller are shown in Figures 6 and 7 respectively. The design was implemented on the XtremeDSP development platform that contains a Spartan-3A DSP 3400A Xilinx FPGA. The Spartan-3A DSP Development Platform provides a great environment for developing signal processing designs. The Xilinx partan-3A DSP 3400A FPGA is based on the XtremeDSP DSP48A Slice. The 250 MHz DSP48A Slice provides an 18-bit x 18-bit multiplier, 18-bit pre-adder, 48-bit post-adder/accumulator, and cascade capabilities for various DSP applications. The DSP48A slice also support a wide math functions, DSP filters, and complex arithmetic without the use of general FPGA fabric.

6. Results

The paper accomplishes the following research objectives:
1) An ION model has been discussed, simulated and implemented in reconfigurable hardware. The Matlab/Simulink simulation results and HW-SW cosimulation results are shown in Figures 1 and 4, 2) A linear feedback control law to synchronize two IONs has been developed, simulated and implemented in reconfigurable hardware. The Matlab/Simulink simulation results and HW-SW co-simulation results are shown in Figures 3 and 5, 3) Table 1, shows a detailed hardware resource usage for our implementations, and 4) Table 1, also shows the maximum latency of the hardware implementation and the maximum frequency of operation of the implemented hardware.

<table>
<thead>
<tr>
<th></th>
<th>ION</th>
<th>ION Synchronizer</th>
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<tr>
<td>Slices</td>
<td>734</td>
<td>1467</td>
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<tr>
<td>Flip-Flops</td>
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<td>990</td>
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<tr>
<td>LUTs</td>
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<td>1885</td>
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<tr>
<td>IOB</td>
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<td>145</td>
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<tr>
<td>DSP Slices</td>
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<td>20</td>
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<td>1</td>
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<tr>
<td>L_max</td>
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<td>41.739 ns</td>
</tr>
<tr>
<td>f_max</td>
<td>25.524 MHz</td>
<td>23.958 MHz</td>
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</table>

7. Conclusions

This paper implements a hardware closed-loop control system for synchronization of inferior olive neurons (IONs). Each ION is modeled by a set of four non-linear differential equation. The model of the ION has been simulated and implemented in reconfigurable hardware and the results verified using HW-SW cosimulation. The software simulation and HW-SW co-simulation results are identical. The closed-loop control system containing two IONs and a simple linear feedback control law has been implemented. The object of controlling one ION so that both oscillate in unison has been achieved. The closed-loop control system has been implemented in reconfigurable hardware and the software simulation and HW-SW cosimulation results are identical. The performance of the control system for hardware latency and device utilization has been evaluated and discussed. Typically an array or network of such IONs are required and there exists a need to develop controllers for ION synchronicity. In future work, the problem of synchronizing an array of IONs and Hardware in Loop (HIL) testing of the ION with motors will be addressed.
Fig. 3: Trajectories of both the IONs and the time history of the eigenvalues of $\Phi(T_p,0)$ - Simulation

Fig. 4: Trajectories of the reference ION - HW-SW Co-simulation

Fig. 5: Trajectories of both the IONs - HW-SW Co-simulation

Fig. 6: XSG implementation of the reference ION

Fig. 7: XSG implementation of the synchronous ION system

References

[15] Llinas RR (2009) Inferior olive oscillation as the temporal basis for motricity and oscillatory reset as the basis for motor error correction. Neuroscience 162(3) : 797-804