Grouped-Subcarrier Based Null-Data Switching for PAPR Reduction of OFDM with Low Computational Complexity

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Abstract—Being a multicarrier modulation scheme, Orthogonal Frequency Division Multiplexing (OFDM) systems generally produce transmit signals with high Peak to Average Power Ratio or PAPR. Amongst numerous PAPR reduction strategies found in the literature, a relatively new but promising technique is the null and data subcarrier switching method. In this paper, we propose a new approach of null-data subcarrier switching that can achieve acceptable PAPR reduction with significantly less computational overhead. Apart from offering advantageous features like side-information less detection, distortion-free signal transmission and compatibility with other existing PAPR reduction techniques, our method also overcomes the problem of high computational requirement especially when higher number of null subcarriers are used for switching. This is achieved by applying incremental searching within preformed subcarrier groups. We demonstrate the effectiveness of our method by presenting PAPR and Bit Error Ratio (BER) related simulation results.

Keywords: OFDM, Null and data subcarrier switching, PAPR.

1. Introduction

As a multicarrier modulation scheme, Orthogonal Frequency Division Multiplexing or OFDM offers many attractive features, e.g., high data rate, robust performance in inter-symbol interference (ISI) channels, good spectrum efficiency and so on. Considering these advantages with many others, it has been selected as the physical layer standard for different contemporary communications systems [1]-[3]. However, OFDM system often contains occasional very high peaks in its transmit signal. This high peaks are generally quantified by a parameter called the Peak to Average Power Ratio or PAPR. High PAPR is a problem when the issue of amplification comes into play. High PAPR demands a highly linear amplifier so as not to cause non-linear distortion. Because, if peak power of the signal crosses the operating range of the amplifier, it may get driven well into the saturation region causing severe Bit Error Ratio (BER) degradation. On the other hand, amplifier are generally operated near the saturation point for attaining better efficiency. This is imperative especially for power constrained hand-held mobile devices. These two scenarios combined makes PAPR reduction a crucial issue for any system that employs OFDM.

In consideration of its effect on the overall system performance, PAPR reduction remains a major research interest for quite some time. And as outcome, quite a significant number of PAPR reduction techniques can be traced in literature. Amongst them, some very well known techniques are selected mapping (SLM), partial transmit sequences (PTS) signal clipping, dummy sequence insertion etc. [4]-[7]. Signal clipping is a very simple and effective PAPR reduction technique. But it can give birth to spectral regrowth of the transmitted signal and there-by cause inter-channel interference (ICI) [8]. Filtering can solve this issue, but only at the cost of potential peak regrowth. On the other hand, SLM and PTS are distortion less techniques and hence do not suffer from the above mentioned problems. However, the transmitter must send additional information known as side-information to the receiver without which the later can not reconstruct the data sequences. This means the effective data-rate gets reduced or valuable bandwidth is lost. Moreover, the associated computational costs for both these schemes can be very high. Dummy sequence insertion method does not require side-information but data throughput needs to be sacrificed for accommodating dummy bits.

A survey of recent PAPR related research work reveals
that moving from the earlier approach of considering generalized architecture, current investigations are considering systems based on standard specifications, e.g., WiMAX (IEEE 802.16) or WLAN (IEEE 802.11a) [9]-[11]. For example, the tone reservation with null subcarriers (TRNS) scheme designed for WiMAX systems shows good PAPR reduction capability. But finding the best set of subcarriers as reserved tones within acceptable computation cost is still an open problem.

A relatively more recent scheme called the null and data subcarrier switching method [11] also shows significant PAPR reduction by switching null subcarriers with data subcarriers specified in the IEEE802.11a standard specifications [1]. Unlike signal clipping, it does not distort the transmission signal and also there is no need for side-information transmission as required by SLM or PTS techniques. But the problem of this method is that the computational complexity can be extremely high when higher number of null subcarriers for switching are used or systems with higher number of subcarriers, e.g., WiMAX is considered.

In this paper, we present new methods of subcarrier switching to reduce computational complexity of the original null and data subcarrier switching scheme without sacrificing PAPR reduction capability. For this, we at first propose incremental searching of data subcarriers for switching with null subcarriers. We show that this approach achieves similar level of PAPR reduction with significantly low computational overhead. But we also report that it may need to sacrifice the no side-information advantage especially when the number of null subcarriers for switching is increased. We overcome this problem by proposing further modification where we partition the data subcarriers into groups and search within them. We demonstrate that this method requires much less computations compared to the original method, achieves almost same level of PAPR reduction and also does not compromise on the no side-information advantage even when higher number of null subcarriers are used for switching. We argue that to achieve similar PAPR reduction, the original method with no side-information requirement becomes almost impractical due to its very high level of computational overhead.

2. System Architecture

In Fig. 1, we show the system model of our study. In this figure, blocks drawn with dashed lines represent the components required for PAPR reduction functionality. Here, at first binary random input data is baseband modulated that generates input symbols given by

$$x[i] = x[0], x[1], ..., x[N - 1].$$

They are then converted from serial to parallel and fed into the IFFT module. The IFFT module performs the task of multicharacter modulation. Output of the IFFT is parallel to serial converted and the resultant time domain signal, $X[n]$, is given by

$$X[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} x[i] e^{2\pi j i n / N}, 0 \leq n \leq N - 1 \quad (1)$$

After this, the time domain signal is amplified by a High Power Amplifier (HPA) and transmitted to the noisy wireless channel. The receiver contains blocks that perform the operations of serial to parallel conversion, FFT, parallel to serial conversion and finally baseband demodulation.

Now, for PAPR reduction purpose a subcarrier switching module along with PAPR estimator block are inserted in the transmitter whereas a subcarrier de-switching module is placed in the receiver. The subcarrier switching module performs the operation of switching between null and data subcarriers and the de-switching block in the receiver executes the opposite operation. The PAPR estimator block determines and stores the PAPR value of all the switching combinations and transmits the signal with the lowest PAPR. PAPR in dB is expressed as below,

$$PAPR(dB) = 10 \log_{10} \frac{\max \left| X[n] \right|^2}{E[\left| X[n] \right|^2]} \quad (2)$$

where $E[\cdot]$ denotes expectation.

Finally, for simulating the HPA, we consider Solid State Power Amplifier (SSPA) model given in [14]. The AM-to-AM conversion characteristics of this model is depicted in Fig. 2 and the corresponding mathematical expression is given by

$$F[x] = \frac{x}{1 + (x/A)^{2r}}$$

Here, $x$ is the amplitude of the input signal, $A$ is the saturated output level and $r$ is the non-linearity level. This model only considers AM-to-AM non-linearity. The parameter $r$ can be used to tune the level of non-linearity. A large value of $r$ turns this amplifier into a linear one where as very small values make it behave as a simple clipping amplifier.

![Fig. 2: Input-Output characteristics of non-linear amplifier.](image-url)
null subcarriers. Among them, 6 null subcarriers work as the guard-band at the low-frequency edge of the spectral band while 5 null subcarriers serve as the guard-band at the high-frequency edge. The remaining one is placed at the middle to avoid direct current energy. The IFFT layout is shown in Fig. 3 [1]. The idea behind null-data subcarrier switching is to swap some of these null subcarriers with data subcarriers for PAPR reduction. [11].

Let us consider an OFDM system where the IFFT size and the total number of null subcarrier are \( N \) and \( L \) respectively. For this system we explain the null-data switching concept with the help of the following notations.

- Null subcarrier set: \( \mathcal{N} = \{ g_l, l = 1, \ldots, L \} \)
- Data subcarrier set: \( \mathcal{D} = \{ h_{d}, d = 1, \ldots, N - L \} \)
- Number of null subcarriers used for switching: \( P \)
- Indices of switching null subcarriers: \( \{ \hat{g}_p, p = 1, \ldots, P \} \subset \mathcal{N} \)
- Indices of switching data subcarriers: \( \{ \hat{h}_p, p = 1, \ldots, P \} \subset \mathcal{D} \)

The main concept here is to switch \( P \) number of null subcarriers with \( P \) number of data subcarriers such that if \( \hat{h}_p < \hat{h}_{p+1} \) then \( \hat{g}_p < \hat{g}_{p+1} \), all indices in ascending order and the task is to identify the \( \hat{h}_p \) from \( h_d \) that results in lowest PAPR. As a result, \( \binom{N-L}{P} = \frac{(N-L)!}{P!(N-L-P)!} \) number of different switching combinations need to be searched by the transmitter in order to choose the combination that yields the least PAPR. On the receiver side, subcarriers with low power levels are detected as null and they are de-switched with corresponding data subcarriers. This de-switching can be done without any side-information because of the constraint if \( \hat{h}_p < \hat{h}_{p+1} \) then \( \hat{g}_p < \hat{g}_{p+1} \). Hence, this constraint is a must for facilitating side-information free de-modulation by the receiver.

Now, as mentioned above, \( \binom{N-L}{P} \) number of search operations are involved for every input data block, which in turn means that many IFFT operations are required. For example, for a system with \( N = 64 \) and \( L = 12 \), if \( P = 4 \) is considered, \( \binom{52}{4} = 270725 \) number of search operations are required. The time involved in such calculation may make this technique impractical for many delay sensitive applications. Moreover, power consumption for such exhaustive calculations can also be a hindrance to the main objective of reducing PAPR, i.e., achieving power efficiency.

### 3.2 Computational Complexity Reduction by Incremental Search

In order to reduce the computational burden of the original null and data subcarrier switching method without making significant sacrifice in PAPR reduction, we proposed incremental searching by considering one null-data subcarrier switching at a time [12]. For example, for \( P = 2 \), i.e., number of null subcarriers to be switched is two, we start by considering \( \hat{g}_1 \) and search \( \binom{N-L-1}{1} \), i.e., \( N - L \) times to look for the data subcarrier position, say \( \hat{h}_1 \) yielding lowest PAPR when switched with \( \hat{g}_1 \). After this, we apply the same operation for the second null subcarrier to be switched. But in order to keep the order of null subcarriers and switched data subcarriers, we first remove all the data subcarriers positions from the search space, i.e., \( h_d \), \( \{ d = (> \hat{h}_1), \ldots, N - L \} \). Thus the size of the data subcarrier search space for \( \hat{g}_p \) when

<table>
<thead>
<tr>
<th>Algorithm 1 Proposed method-2.</th>
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<tr>
<td><strong>Input:</strong> ( P, \hat{g}<em>p, { p = 1, \ldots, P \subset \mathcal{N} } ), ( \mathcal{D} = { h</em>{d}, d = 1, \ldots, N - L } ).</td>
</tr>
<tr>
<td><strong>Output:</strong> ( \hat{h}_p ).</td>
</tr>
<tr>
<td>1: Divide ( \mathcal{D} ) into ( \mathcal{G}_p ) groups.</td>
</tr>
<tr>
<td>2: for ( p = 1 ) to ( P ) do</td>
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<tr>
<td>3: for ( i = 1 ) to ( sizeaf(G_p) ) do</td>
</tr>
<tr>
<td>4: Switch ( \hat{g}_p ) with ( h_i ).</td>
</tr>
<tr>
<td>5: Apply IFFT and Calculate PAPR(_i).</td>
</tr>
<tr>
<td>6: end for</td>
</tr>
<tr>
<td>7: ( \hat{h}_p = \text{arg min}_i PAPR(_i). )</td>
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<tr>
<td>8: end for</td>
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### Table 1: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>IFFT size</td>
<td>64</td>
</tr>
<tr>
<td>Number of data subcarriers</td>
<td>52</td>
</tr>
<tr>
<td>Number of switched null subcarriers</td>
<td>24</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
<tr>
<td>Oversampling factor</td>
<td>4</td>
</tr>
<tr>
<td>HPA Model</td>
<td>SSPA</td>
</tr>
<tr>
<td>HPA Saturation level</td>
<td>3dB</td>
</tr>
<tr>
<td>HPA level of non-linearity</td>
<td>1</td>
</tr>
<tr>
<td>Channel model</td>
<td>AWGN</td>
</tr>
</tbody>
</table>
$p > 1$ is dependent on the outcome of search with $\hat{g}_{p-1}$. For this approach, since the search space becomes smaller for every increment in $p$, there remains a possibility that the search space would become very limited for $\hat{g}_p$ when $p > 1$, specially when $P > 2$, causing significant negative effect on PAPR reduction. One way of avoiding this problem, is to relax the need for keeping the order of the switched null and data subcarriers same. But it means the receiver now can not perform de-switching unless some extra information, i.e., side-information is transmitted.

### 3.3 Computational Complexity Reduction by Incremental Search within Subcarrier Groups

We propose segregating the data subcarriers into groups to facilitate searching with less computational burden without compromising the no side-information advantage. The algorithm behind this method is depicted in Alg. 1. Here, we at first partition total number of data subcarriers into $G_p$, \{\(p = 1, ..., P\)\} groups and put $\frac{N - L}{p}$ number of subcarriers into each group when $N - L$ is divisible by $P$. Otherwise, in the first group we put slightly higher number of subcarriers compared to the other $P - 1$ groups all holding equal number of subcarriers. Then we start with $G_1$ and search for the data subcarrier $\hat{h}_1$ which when switched with null subcarrier $\hat{g}_1$ yields lowest PAPR. We carry on the same process until $p$ reaches $P$. Unlike [13], here the length of the subcarrier groups remain constant throughout the entire search operation and hence there is no additional overhead for dynamic adjustment of subcarrier bands. This approach of searching reduces the computational burden significantly. For example, with $P = 4$, our method requires 52 number of searchings compared to 270725 of the original method. And as we show in the following section, the PAPR reduction capability remains almost the same.

### 4. Simulation Results and Discussion

For comparative performance analysis, we performed simulation based on the model shown in Fig. 1. We consider BPSK modulation and investigate scenarios primarily for $P = 4$. In addition, we also show results for $P = 2$ and $P = 4$ where we do not implement subcarrier grouping. For $P = 2$ and $P = 4$, we select null subcarriers for switching positioned at $\pm 27$ and $\pm 28$ respectively ([11] page no.12). The rationale behind choosing the null subcarrier positions is to select them in a balanced way on either side of the data subcarrier bands so that any degradation to the guard bands can be kept to a minimum. All the pertinent simulation parameters are listed in Table 1. In Figs. 4 and 5, we show the results where we consider incremental search but do not divide data subcarriers into different groups. Figure 4 shows the comparative cumulative distribution function (CCDF) of PAPR for OFDM without any PAPR reduction scheme, the original null and data subcarrier switching and our method where number of null subcarriers for switching is 2, i.e., $P = 2$. Here, we denote the original null and data subcarrier switching as “Original Method” and our method as “Proposed Method-1”, respectively. As seen here, compared to no PAPR reduction scheme, our method achieves significant PAPR reduction which is slightly inferior to the original method.

Now in Fig. 5, for $P = 4$, we show the results without and with the need of side-information transmission. As was mentioned earlier also, for $P > 2$, we can see from this figure that the no side-information advantage needs to be sacrificed in order to achieve significant PAPR reduction, otherwise PAPR reduction is drastically reduced.

In Fig. 6, we show the PAPR performance when segregation of subcarriers into different groups is considered. We referred to it as “Proposed Method-2” in In Fig. 6. Here, we can see that for PAPR values $\leq 6.5$dB, our proposed
method suffers marginal loss in PAPR reduction compared to the original scheme. But for higher PAPR, it shows almost same performance as the original one. This is accompanied by the fact that our method achieves significant reduction in computational overhead since corresponding number of searchings for the original and our proposed methods are 270725 and 52 respectively.

Finally, in order to show the effects of PAPR on the BER performance of the system, we consider a non-linear power amplifier and an Additive White Gaussian Noise (AWGN) channel. Figure 7 shows the comparative BER results of our proposed method, the original method for $P = 4$ and an OFDM system without any PAPR reduction mechanism applied. The combined effects of noise contamination of the wireless channel and the non-linear amplification due to high PAPR govern the results. As evident from Fig. 7, OFDM without any PAPR reduction mechanism shows considerable amount of error floor even at very high SNR level. And compared to it, our method shows significantly improved BER performance which is almost similar to that of the original method.

5. Conclusions

A new approach of null and data subcarrier switching scheme for PAPR reduction in OFDM systems is proposed. We show that incremental searching can reduce computational complexity of the original method but may need to sacrifice the no side-information advantage when higher number of null subcarriers are used for switching to achieve greater PAPR reduction. We then propose incremental searching within pre-formed data subcarrier groups and show that besides achieving PAPR reduction with low computational overhead it also retains the no side-information advantage. Through simulation results, we show the PAPR reduction capability and the BER performance of our method.

As a plan for future research, we are interested in investigating the robustness of our method by considering higher level modulation schemes, e.g., QPSK or QAM along with multipath propagation environment. In parallel, we are also interested in exploring the effect on the spectrum containment when switching between null and data subcarriers takes place.

References


