Joint-Design of PHY/MAC Layers for Throughput Optimization of Opportunistic Relay Networks

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Abstract - This article develops a unified analytical framework for maximizing the user throughput of an opportunistic multi-relay network over generalized wireless channels using the symbol rate, packet length and constellation size of M-ary PSK/QAM digital modulation schemes as optimization variables. Optimization equations for each of the above degrees of freedom are derived in closed-form. A low-complexity discrete optimization algorithm for finding the “optimal” parameter-triplet is developed to solve the resulting non-linear joint-optimization problem. Numerical results reveal that the opportunistic routing protocols outperform the traditional cooperative system. Results also showed that increasing the number of opportunistic relays increases the throughput contrary to the case for the ‘traditional’ cooperative relays. Our results also reveal that it is sufficient to adapt the symbol rate and number of opportunistic relays in the low SNR regime while a joint-optimization of the packet length, constellation size and the number of opportunistic relays is desirable in the high SNR regime for maximum throughput.

Keywords: cross-layer optimization, adaptive PHY/MAC, generalized fading channels

1 Introduction

It is well-known that adaptive transmission techniques can dramatically improve the spectrum utilization efficiency over time-varying channels [1]. The underlying premise is to “match” the transmission design parameters (e.g. signaling rate, power, coding rate, constellation size, packet length, etc.) to the prevailing channel conditions (i.e., based on the feedback of channel side information from the receiver to the sender). While the literature on adaptive link layer techniques are quite extensive that span over four decades, the art of adaptive link layer has received a renewed research interest in recent years, but in a cross-layer design framework, since the publications of [2]-[3]. For instance, [4]-[5] examined the throughput maximization problem via cross-layer adaptive designs following the approach in [2]-[3] for OFDM and MIMO systems, respectively. In [6]-[7], the efficacy of adaptive asymmetric (multi-resolution) modulation is investigated for multimedia data transmission. More recently, [8] has generalized the analytical framework for single user throughput optimization at the MAC layer in [2] (restricted to M-QAM and AWGN/Rayleigh environment) to different fading environments and modulation schemes.

Yet all of the above articles are restricted to point-to-point non-cooperative wireless network. Since cooperative diversity strategies can enhance the throughput in the low SNR regime. Reference [9]-[10]. However, their analyses are limited to a Rayleigh fading environment, and more critically the source rate adaptation was not considered. However, as pointed out in [2], [3] and [8], this parameter adaptation is extremely critical in the low SNR regime (e.g., tactical-edge or cell boundaries). Moreover, cooperative diversity is also most attractive to enhance the throughput in the low SNR regime. Reference [15] compares different cooperative diversity and opportunistic routing protocols. In this article we also extend the framework in [15] (restricted to Nakagami independent and identically distributed (i.i.d) environment) to different fading i.e. Rayleigh and Rice environments with independent and non-identically distributed (i.n.d) fading statistics.

The key objectives of this paper is to achieve the following: (i) develop a unified framework for throughput maximization of relay networks using opportunistic routing protocols with cross-layer adaptive designs at the PHY/MAC layers; and (ii) derive optimization equations in closed-form for optimal joint-adaptation of design parameters over generalized fading channels. To the best of our knowledge, these are still open research problems. To get the maximum throughput, we optimize the symbol rate $R$, and packet length $L$ (which are MAC layer parameters) and the constellation size $M = 2^b$ (which is a PHY layer parameter) over a myriad of fading environments (Rayleigh, Rice, etc.) in a unified manner. Hence, this problem bears the characteristics of a cross-layer optimization problem across PHY and MAC layers. Our new framework allows us to gain insights on the impact of fade distribution on the optimal choice of design parameters and the optimized throughput performance for cooperative relay networks as well as comparison of different routing protocols.

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The remainder of the article is organized as follows. In Section II, we discuss the system model and the optimization framework while Section III derives the optimization equations in closed-form for a myriad of fading channel models and M-PSK modulation scheme. Selected computational results and concluding remarks are provided in Section IV and V respectively.

2 System Model & Optimization Framework

Consider a cross-layer design with the different protocol stack being considered in the proposed cross-layer design and optimization. The application layer generates the information at the rate \( R_s \) required by the data link layer. The network layer handles the packet routing and utilizes the preferred routing protocol for the application. Here, we assume the opportunistic routing is done at the network layer. Whichever case, the transmission route is either the combination of the entire \( N \) relays (maximum ratio combining) or selection of the relay in the best route. This relay selection can take place at the source in the route discovery process or can be distributed among the relays (through channel state dependent back off time). Also, the relay selection can take place at the destination and in this case it resembles traditional selection diversity combining. Each of these network layer protocols has different effect on the overall achievable spectral efficiency of the system and different channel side information requirement. The medium access control (MAC) and data link layers handle the packetization and determine the payload (i.e. packet length \( L \)) of the physical system. Lastly, the physical layer maps the bits in the packet to symbols based on the chosen modulation scheme. The number of bits per symbols \( b \), determines the efficiency of the physical layer. In short, the adaptation of these four layers enables spectrally efficient and robust data transmission over time-varying channels i.e. ‘match’ transmission design parameters to the prevailing channel or network conditions. Therefore, efficient resource utilization strategies for handling multimedia data over wireless networks can benefit substantially from this cross-layer paradigm.

Consider a source node \( S \) with application/signaling rate \( R_s \), trying to deliver a packet of length \( L \) using modulation rate \( b \), over a cooperative multi-relay network with \( N \) relays to the destination node \( D \). If we assume that the packet length \( L \) bits, includes \( C \) CRC bits (to ensure bits received in error are detected at the receiver) and \( K = L - C \) information bits (payload). We further assume that the CRC decoder only exist at the destination node and is able to detect all errors in the received packet (since the probability of undetected errors is negligible for reasonable values of \( C \)). Hence, we assume here that the relays in this system are passive and do not decode the information but only amplify the received signal and forward it towards the destination. The additional assumptions made here is that the transmission parameters such as the signaling rate, modulation scheme, constellation size, packet/payload length are identical at both the source and the relays, irrespective of their wireless link conditions to the destination node, to reduce the implementation complexity. Similar to the assumptions in [3] and [8] and for ease of analysis, we also assume that both positive acknowledgements (ACK) and negative acknowledgments (NACK) from the destination node \( D \) are sent through a separate control channel and arrives at the transmitter \( S \) instantaneously and without error. If the mean channel statistics are assumed to be the same during first transmission and retransmissions of a packet, then the throughput of a selective repeat automatic repeat request (SR-ARQ) protocol has been shown in [8] to be independent of the number of retransmissions per packet. Thus, the normalized throughput (average spectral efficiency) equation for cooperative diversity/opportunistic routing protocols with orthogonal transmissions is given by

\[
\frac{\eta}{W} = \frac{L-C}{L_f(N)} b R_s (1 - P_e) = \frac{L-C}{L_f(N)} b \frac{R_s}{W} (1 - \overline{P}_r(b, \Omega_s))^{1/b} \tag{1}
\]

where \( \overline{P}_r(b, \Omega_s) \) denotes the average symbol error rate (ASER) of a specified \( M \)-ary modulation over fading channels, \( \Omega_s \) is the mean received end-to-end SNR, \( W \) is the system bandwidth, \( f(N) \) is the number of time slot or channels usage per each source transmission. This function is dependent on the choice of opportunistic routing protocol as will be shown later. Eq. (1) allows us to investigate the throughput optimization problem for cooperative relay networks without delving into the underlying protocol implementation as long as the end-to-end symbol error rate can be evaluated or known. The additional assumptions made here is that the transmission parameters such as the source rate, modulation scheme, constellation size, packet/payload length are identical at both the source and the relays, irrespective of their wireless link conditions to the destination node, to reduce the implementation complexity.

In order to obtain the optimum throughput performance, the three design parameters \( (b, R_s, L) \) will be jointly-adapted. Before exploring the various combinations of design parameter optimization, we will first derive the moment generating function (MGF) of the relayed path that will later be utilized in evaluating the end-to-end average symbol error rate (ASER) of opportunistic relay networks over generalized fading channels and with any digital modulation scheme.

2.1 MGF of Opportunistic Routing Networks

2.1.1 AF Relaying with Opportunistic Route Selection and SDC (ORS-SDC) at the Source

In this protocol implementation, the best route is selected at the source based on the end-to-end relay SNR. The selection process can be done during the route discovery or in a distributed fashion similar to the proposition in [16]. Therefore the statistics of the best route selection is the same as the selection diversity combining at the destination as the best of \( N+1 \) links is being selected. However, the channel usage per source transmission in the case is \( f(N) = 2 \). Therefore, the spectral efficiency here does not reduce with
increasing number of relay as in the case of traditional cooperative diversity. Also the amount of channel side information and implementation complexity is highly reduced. The effective SNR is given by

\[ \gamma^{\text{ORS-SDC}} = \max(\gamma_{s,d}, \gamma_1, \ldots, \gamma_N) \]

\[ \approx \max \left( \gamma_{s,d}, \min(\gamma_{s,d}, \gamma), \ldots, \min(\gamma_{s,d}, \gamma) \right) \] (2)

The cumulative distribution function (CDF) of the end-to-end SNR is then given by

\[ F_{y,s,d}^{\text{ORS-SDC}}(\gamma) = F_{y,s,d}(\gamma) \prod_{r=1}^{N} F_{y,s}(\gamma) \]

\[ \approx F_{y,s,d}(\gamma) \prod_{r=1}^{N} \left(1 - [1 - F_{y,s}(\gamma)][1 - F_{y,s,d}(\gamma)]\right) \] (3)

The effective MGF can then be evaluated using the differentiation property of the Laplace transform via a single integral expression

\[ \phi_{y,s,d}^{\text{ORS-SDC}}(s) = \int_{0}^{s} e^{-sy} F_{y,s,d}(\gamma) \prod_{r=1}^{N} F_{y,s}(\gamma) d\gamma \]

\[ \approx \int_{0}^{s} e^{-sy} F_{y,s,d}(\gamma) \prod_{r=1}^{N} \left(1 - [1 - F_{y,s}(\gamma)][1 - F_{y,s,d}(\gamma)]\right) d\gamma \] (4)

For special case of independent and identically distributed (i.i.d.) Rayleigh channel, the MGF can be reduced to

\[ \phi_{y,s,d}^{\text{ORS-SDC}}(s) \approx s \int_{0}^{s} e^{-sy} \left(1 - e^{-s\gamma_{s,d}}\right) \prod_{r=1}^{N} \left(1 - e^{-s\gamma_{s,d}, r}\right) d\gamma \] (5)

2.1.2 AF Relaying with Opportunistic Relay Selection and MRC (ORS-MRC) at Destination

This protocol implementation takes advantage of the half duplex nature of relay transmission to achieve better performance than the ORS-SDC protocol. Here, since the source transmits in the first transmission phase, and due to the broadcast nature of wireless channel, the destination can be close enough to receive this signal before receiving the signal from the relay. This is particularly true in the distributed ORS protocol implementation proposed in [16]. Therefore, if the channels side information of both links is available, the received signal can be combined with maximum ratio combining scheme at the destination. Note that the transmission channel usage \( f(N) = 2 \) per each source transmission but the statistics is slightly different from the ORS-SDC protocol. The effective end-to-end SNR of ORS-MRC protocol can be expressed as

\[ \gamma^{\text{ORS-MRC}} = \gamma_{s,d} + \max(\gamma_1, \gamma_2, \ldots, \gamma_N) \]

\[ \approx \gamma_{s,d} + \max \left( \min(\gamma_1, \gamma_2), \ldots, \min(\gamma_N, \gamma_N) \right) \] (6)

The effective MGF can then be evaluated using the addition and differentiation property of the Laplace transform via a single integral expression

\[ \phi_{y,s,d}^{\text{ORS-MRC}}(s) = s \phi_{y,s,d}(s) \int_{0}^{s} e^{-sy} \prod_{r=1}^{N} F_{y,s}(\gamma) d\gamma \]

\[ \approx s \phi_{y,s,d}(s) \int_{0}^{s} e^{-sy} \prod_{r=1}^{N} \left(1 - [1 - F_{y,s}(\gamma)][1 - F_{y,s,d}(\gamma)]\right) d\gamma \] (7)

For special case of independent and identically distributed (i.i.d.) Rayleigh channel, the MGF can be reduced to

\[ \phi_{y,s,d}^{\text{ORS-MRC}}(s) \approx \frac{s}{1+s\Omega_{s,d}} \int_{0}^{s} e^{-sy} \prod_{r=1}^{N} \left(1 - e^{-s\Omega_{s,d}, r}\right) d\gamma \] (8)

The respective CDF and MGF expressions for different fade distributions are given in Table 2.

2.2 Symbol Rate Optimization

In order to determine the optimum value for symbol rate \( R \), (that maximizes the throughput) in a generalized fading channel, we need to differentiate (1) with respect to \( \Omega \), set the resulting expression to zero, and then solve it to obtain the optimum SNR/symbol \( \Omega^* \), viz.,

\[ \Omega^* = \frac{d\tilde{P}_b(b, \Omega)}{d\Omega} \mid_{\Omega = \Omega^*} = -\frac{1 - \tilde{P}_b(b, \Omega^*)}{L \beta} \] (9)

Once \( \Omega^* \) is determined (from (9)), we can then find \( R \) using relation in (10), viz,

\[ R^* = \frac{P_r}{\Omega N_0} = \frac{\Omega W}{\Omega} \] (10)

It is important to note that in the low SNR regime (i.e., when the received SNR/symbol \( \Omega \) is lower than the “optimum” SNR/symbol \( \Omega^* \)), we should select \( R^* \) that satisfies (10) to maximize the throughput. However, in the high SNR regime (i.e., when \( \Omega > \Omega^* \)), the bandwidth constraint prevents us from increasing the symbol rate beyond a certain limit \( (R^* \leq W) \). In this case, we set \( R^* = W \) and pack more bits per digital modulation symbol (i.e., to maximize the spectral efficiency at the prevailing channel condition).

2.3 Packet Length Optimization

In order to find an analytic solution for the optimal packet length \( L^* \), we need to assume that \( L \) takes continuous values. Differentiating (1) with respect to \( L \) and solving the resulting expression leads to a closed-form formula for \( L^* \), viz., [2]

\[ L^* = \frac{1}{2} \sqrt{\frac{C^2 - \frac{4 \beta C}{\ln(1 - \tilde{P}_b(b, \Omega))}}{2}} \] (11)

From (11), it can be seen that the optimal packet length \( L^* \) depends on the constellation size and ASER (which in turn depends on the SNR per symbol \( \Omega_{s,d} \)). In practice, the packet lengths are generally in the form of integer multiples of the number of bits per symbol (i.e., \( L = b, 2b, 3b, \ldots \)), and so on.

2.4 Constellation Size Optimization

Optimum value of \( b \) for which the throughput is maximized leads to the optimum constellation size. In order to derive an expression for finding the optimum \( b \), we assumed that \( b \) is a continuous variable. Hence differentiating (1) with respect to \( b \) and solving the resulting to find its saddle-point, we obtain
\[ \frac{d \tilde{P}(b, \Omega_s)}{db} \bigg|_{b^*} = \frac{1 - \tilde{P}(b^*, \Omega_s)}{L} \]  

In a practical discrete-rate adaptive modulation, \( b \) should assume a positive integer value. Thus the value \( b \) obtained from solving the transcendental equation (12) should be rounded to the nearest positive integer value.

2.5 Joint Parameter-Triplet Optimization

Joint optimization can be done by jointly varying the three optimization parameters \((b, \Omega_s, L)\). Alternatively, we can find an optimal triplet set (for every channel realization) that simultaneously satisfies equations (9), (11) and (12). From our discussions in the preceding subsections, it can be shown that the optimal throughput is given by

\[
\eta^* = \begin{cases} 
\frac{\tilde{L} - C}{E} b^* \tilde{R}_b (1 - \tilde{P}(b^*, \Omega_s))^{L^*/b} & \Omega_s \leq \Omega_s^* \\
\frac{\tilde{L} - C}{E} b^* \tilde{W} (1 - \tilde{P}(b^*, \Omega_s))^{L^*/b} & \Omega_s > \Omega_s^* 
\end{cases}
\]  

(13)

From (9) and (12), it can be observed that (1) is strictly concave for all range of \( \Omega_s \) and \( b \) and therefore there is a singular maximum point for both \( \Omega_s^* \) and \( b^* \). However, since (9), (11) and (12) are non-linear equations but \( b \) and \( L \) are integers for practical purposes, it might be difficult to solve the combinatorial programming problem. Here, we propose a very simple routine that can yield highly accurate optimal points. The routine steps are given as follows.

<table>
<thead>
<tr>
<th>A Simple Algorithm for Joint Optimization of Parameter Triplet ((b, \Omega_s, L))</th>
</tr>
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</table>
| **Step 1:** Specify a range for \( b \). The choice of \( b \) should be constrained to integer values for practical discrete modulation schemes. In this work, we assume \( b = \{1, 2, 3, ..., 5\} \).
| **Step 2:** Substitute (10) (i.e., \( R_s = (\xi P_s/N_0) / (\xi) \)) and (11) into (10) where \( \xi \) denotes the mean received SNR for the S-D link while \( \xi \) is a constant that relates \( \Omega_s \) to \( \Omega_s' \). For any fixed \( P_s/N_0 \), the resulting auxiliary expression is a concave function with respect to \( \Omega_s \). For each \( b \) specified in Step 1, sweep and find \( \Omega_s^* \) that maximizes this auxiliary function.
| **Step 3:** Obtain \( R_s^* \) from \( R_s = W/\Omega_s' \) using \( \Omega_s^* \) from Step 2.
| **Step 4:** Substitute \( R_s^* \) from Step 3 and (11) into (13) with the condition that \( \tilde{L} = L_{\text{max}} \) if \( \tilde{L} > L_{\text{max}} \) to obtain \( b^* \) (that maximizes the throughput).
| **Step 5:** Compute \( L^* \) using (13) (with the condition that \( \tilde{L} = L_{\text{max}} \) if \( \tilde{L} > L_{\text{max}} \)) with \( b^* \) from Step 4.

The above algorithm greatly simplifies the combinatorial programming problem for the parameter-triplet optimization and can be readily applied to a number of other cases of interest including the shadowed fading environments.

3 Optimization Equations in Closed-Form

In order to derive a closed-form expression for end-to-end ASER of opportunistic routing networks, we exploit a desirable exponential form for the conditional error probability (CEP) for various digital modulation schemes. For instance, the CEP for the MPSK in AWGN is given by [13, Table II]

\[ P_s = ae^{-b_s \sin^2(\pi/2)} + ce^{-2b_s \sin^2(\pi/2)}, \quad b \geq 2 \]  

(14)

where coefficients \( a \), \( b_0 \) and \( c \) are tabulated in [13, Table I]. This form is very attractive since the averaging of the CEP shown in (14) over the probability density function (PDF) of the fading SNR can be simplified dramatically (i.e., the ASER is simply a function of the Laplace transform of the PDF). Therefore, the end-to-end ASER can be expressed as

\[ P_{\text{e}} = a\phi(s, \Omega) + c\phi(2s, \Omega) \]  

(15)

where \( s = b_0 \sin^2(\pi/M) = b_0 \sin^2(\pi/2^b) \) and \( \phi(.,.) \) is the MGF of end-to-end SNR. Substituting (4) and (7) into (15), we obtain a unified expression for the ASER in a myriad of fading environments (including mixed-fading).

4 Numerical Results

For the purpose of illustration, we have considered a triple opportunistic relay system in our numerical analysis although the developed framework is applicable to any arbitrary number of relays. Unless otherwise stated, the parameters in Table 1 will be used to generate the plots in this section.

<table>
<thead>
<tr>
<th>TABLE 1. SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>CRC, C</td>
</tr>
<tr>
<td>Packet Length ( L_{\text{max}} )</td>
</tr>
<tr>
<td>Bandwidth ( W )</td>
</tr>
<tr>
<td>Constellation size ( b_{\text{max}} )</td>
</tr>
<tr>
<td>Power factor, ( \delta_r = \delta_w = 2 )</td>
</tr>
</tbody>
</table>

Fig. 1 shows the plot of \( L^* \) as a function of average SNR for different values of \( b \). It can be observed that the optimum \( L^* \) increases as the average SNR increases. However, in practical implementation, the value \( L^* \) cannot be more than \( L_{\text{max}} \) as this can lead to more delay and large memory requirement. Also, we see \( L^* \) is maximum when \( b = 1 \) although the throughput is not maximum, as will be shown in Fig. 2.
Fig. 2 shows the throughput vs. average SNR using the $L^*$ values obtained from (11). It can be observed that lower $b$ values give the best performance at low SNR and a higher $b$ gives better performance as the SNR increases. This is an example of two parameter optimization at fixed $R_s$.

In Fig. 3 we illustrate the auxiliary function (in step 2 of the algorithm for joint triplet parameter optimization) vs. average SNR at $L^*$ and variable $b$. This plot shows the existence of a preferred mean SNR $\Omega^*_{s}$ for each $b$ and can be used to determine the optimum $R'_s$. Note that $\Omega^*_{s}$ for each of the constellation size is obtained from the peak throughput. For instance, $\Omega^*_{s} = 6$ dB for $b = 2$. Therefore, when $\Omega_s \geq \Omega^*_{s}$, $R'_s = W = 1024\text{kbps}$, and when $\Omega_s < \Omega^*_{s}$, we obtain $R'_s$ using (10). In this example, $R'_s = 646\text{kbps}$ at 4 dB.

Fig. 4 shows the throughput against the average SNR with $R'_s$ and $L^*$ values for different values of $b$. The optimum curve shows the optimum values of $b$ across the range of average SNR and therefore shows the maximum achievable throughput when all the triplet parameters have been optimized.
Fig. 5 compares the efficiency of different network layer protocols on the cross-layer design optimization with Rayleigh channel, and \( N=\{1, 3\} \).

In this article, we have examined the impact of opportunistic route selection and the joint-optimization of symbol rate, packet length and constellation size on the user throughput of cooperative relay networks. We concluded that the opportunistic routing is the more advantageous than the cooperative diversity at high SNR regime and better channel condition, and with large number of participating relays. Thus the number of cooperating relays and the source rate can be jointly optimized for maximizing the throughput at the tactical-edge or cell boundaries, while packet length, constellation size and number of cooperating relays should be jointly optimized to maximize the throughput in better channel conditions.

6 References


TABLE 2
MGF of SNR FOR SEVERAL COMMON STOCHASTIC CHANNEL MODELS

<table>
<thead>
<tr>
<th>Channel Model</th>
<th>MGF of SNR</th>
<th>CDF of SNR</th>
</tr>
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<tbody>
<tr>
<td>Rayleigh</td>
<td>$(1 + s \Omega_s)^{-1}$</td>
<td>$1 - e^{-\gamma/\Omega}$</td>
</tr>
<tr>
<td>Nakagami-n (Rice: $K=\alpha^2$)</td>
<td>$\frac{1 + K}{1 + \Omega_s} \exp\left(\frac{-Ks\Omega_s}{1 + \Omega_s}\right)$</td>
<td>$1 - Q_1\left(\sqrt{2K}, \sqrt{2(1 + K)} / \Omega_s\right)$</td>
</tr>
<tr>
<td>Nakagami-m</td>
<td>$\left(1 + \frac{s\Omega_s}{m}\right)^m$</td>
<td>$\frac{\Gamma\left(m, \frac{m\gamma}{\Omega_s}\right)}{\Gamma(m)}$</td>
</tr>
<tr>
<td>G-distribution</td>
<td>$m \sum_{r=0}^{m+k-1} \binom{m+k-1}{r} (-1)^{r} s^{r} \sum_{p=0}^{r} \binom{r}{p} \frac{2^{r-p}}{\sqrt{r-p+1}} \Gamma(r+p+1) H_{r+p+1} \frac{b^{r+1}}{2 \sqrt{\pi} \Gamma(r+1)} \frac{\exp(\mu)}{2 \sinh(\mu)} \theta = \exp(\mu + \mu^2) \beta = 2m \theta \cdot \alpha = \eta \Omega_s \cdot \beta = \frac{1}{\theta} \sqrt{\eta \Omega_s}$</td>
<td>$-A \sum_{k=0}^{m} \frac{2^{2k} \Gamma^{m-k}(\sqrt{\alpha + \beta^2})}{\Gamma(m) \left(\sqrt{\alpha + \beta^2}\right)^{m-k+1}} \frac{K_{2k}(\sqrt{\alpha + \beta^2})}{\Gamma(m) \left(\sqrt{\alpha + \beta^2}\right)^{m-k+1}} \exp\left(\frac{\eta}{\theta} \left(\frac{m}{\Omega_s}\right)^m\right)$</td>
</tr>
</tbody>
</table>

where $K$ and $m$ are the fading indices for rice and nakagami channels. $\mu$ (dB) and $\sigma$ (dB) are the mean and the standard deviation of $10 \log_{10} \Omega_s$ respectively.