CFO Estimation Schemes Using the Cyclic Prefix for OFDM Systems in Non-Gaussian Noise Environments

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Abstract—In this paper, carrier frequency offset (CFO) estimation schemes robust to the non-Gaussian noise for orthogonal frequency division multiplexing (OFDM) systems are proposed. Applying the probability density function of the cyclic prefix of OFDM symbols to the maximum-likelihood (ML) criterion, we propose the ML and low-complexity ML estimation schemes. Simulation results show that the proposed schemes offer a robustness and a substantial performance improvement over the conventional estimation scheme using cyclic prefix in non-gaussian noise environments.

Keywords: carrier frequency offset; cyclic prefix; maximum-likelihood; non-Gaussian noise; OFDM

1. Introduction

Due to its immunity to multipath fading and high spectral efficiency, orthogonal frequency division multiplexing (OFDM) has been adopted as a modulation format in a wide variety of wireless systems such as digital video broadcasting-terrestrial (DVB-T), wireless local area network (WLAN), and worldwide interoperability for microwave access (WiMAX) [1]-[3]. However, OFDM is very sensitive to the carrier frequency offset (CFO) caused by Doppler shift or oscillator instabilities, and thus, the frequency offset estimation is one of the most important technical issues in OFDM systems [1], [4]. Specifically, we are concerned about the FO estimation based on the blind approach, which uses the cyclic prefix (CP) of OFDM symbols [4].

Conventionally, the CFO estimation schemes have been proposed under the assumption that the ambient noise is a Gaussian process [5], which is generally justified with the central limit theorem. However, it has been observed that the ambient noise often exhibits non-Gaussian nature in wireless channels, mostly due to the impulsive nature originated from various sources such as car ignitions, moving obstacles, lightning in the atmosphere, and reflections from sea waves [6], [7]. The conventional estimation schemes developed under the Gaussian assumption on the ambient noise could suffer from severe performance degradation under such non-Gaussian noise environments.

In this paper, we propose robust blind CFO estimation schemes in non-Gaussian noise environments. Based on the CP structure of OFDM, we first derive a maximum-likelihood (ML) CFO estimation scheme in non-gaussian noise modeled as a complex isotropic Cauchy noise, and then, derive a simpler blind estimation scheme with a lower complexity. From simulation results, the proposed schemes are confirmed to offer a substantial performance improvement over conventional blind estimation scheme in non-Gaussian noise environments.

2. Signal Model

The kth received OFDM sample \( r(k) \) can be expressed as

\[
r(k) = x(k)e^{j2\pi f_c k/N} + n(k)
\]

for \( k = -G, \ldots, -1, 0, 1, \ldots, N-1 \), where \( x(k) \) is the kth sample of the transmitted OFDM symbol generated by the inverse fast Fourier transform (IFFT) of size \( N \), \( G \) is the size of the CP, \( f_c \) is the CFO normalized to the subcarrier spacing \( 1/N \), and \( n(k) \) is the kth sample of additive noise.

In this paper, we adopt the complex isotropic symmetric \( \alpha \) stable (CIS\( \alpha \)S) model for the independent and identically distributed noise samples \( \{n(k)\}_{k=0}^{N-1} \). This model has been widely employed due to its strong agreement with experimental data [8], [9]. The probability density function (pdf) of \( n(k) \) is then given by [8]

\[
f_n(\rho) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\gamma(u^2+v^2)^{\frac{3}{2}}} - j\mathbb{R}\{e^{j(\rho-u-jv)}\} dudv,
\]

where \( \mathbb{R}\{\cdot\} \) denotes the real part, the dispersion \( \gamma > 0 \) is related to the spread of the pdf, and the characteristic exponent \( \alpha \in (0, 2] \) is related to the heaviness of the tails of the pdf: A smaller value of \( \alpha \) indicates a higher degree of impulsiveness, whereas a value closer to 2 indicates a more Gaussian behavior.

A closed-form expression of (2) is not known to exist except for the special cases of \( \alpha = 1 \) (complex isotropic Cauchy) and \( \alpha = 2 \) (complex isotropic Gaussian). In particular, we have

\[
f_n(\rho) = \begin{cases} \frac{1}{2\pi} (|\rho|^2 + \gamma^2)^{-\frac{3}{2}}, & \text{when } \alpha = 1 \\ \frac{1}{4\pi\gamma} \exp\left(-\frac{|\rho|^2}{4\gamma}\right), & \text{when } \alpha = 2. \end{cases}
\]

Due to such a lack of closed-form expressions, we concentrate on the case of \( \alpha = 1 \): We shall see in Section 4 that the
estimation schemes obtained for \( \alpha = 1 \) are not only more robust to the variation of \( \alpha \), but they also provide a better performance for most values of \( \alpha \), than the conventional estimation scheme.

3. Proposed Schemes

3.1 Maximum-likelihood CFO Estimation Scheme

In estimating the CFO, we consider a property of the CP structure of OFDM, i.e., \( x(k) = x(k+N) \) for \( k = -G, -G+1 \cdots, -1 \) as in [5]. Then, from (1), we have

\[
 r(k+N) - r(k)e^{j2\pi\varepsilon} = n(k+N) - n(k)e^{j2\pi\varepsilon} \tag{4}
\]

for \( k = -G, -G+1 \cdots, -1 \). Observing that \( n(k+N) - n(k)e^{j2\pi\varepsilon} \) obeys the complex isotropic Cauchy distribution with dispersion \( 2\gamma \) (since the distribution of \( -n(k)e^{j2\pi\varepsilon} \) is the same as that of \( n(k) \)), we obtain the pdf

\[
f_r(r|\varepsilon) = \prod_{k=-G}^{G} \frac{\gamma}{\pi (r(k+N) - r(k)e^{j2\pi\varepsilon})^2 + 4\gamma^2} \tag{5}
\]

of \( r = \{r(k+N) - r(k)e^{j2\pi\varepsilon}\}_{k=-G}^{G} \) conditioned on \( \varepsilon \). The ML estimation is then to choose \( \hat{\varepsilon} \) such that

\[
\hat{\varepsilon} = \arg \max_{\varepsilon} \log f_r(r|\varepsilon) = \arg \min_{\varepsilon} \Lambda(\varepsilon), \tag{6}
\]

where \( \varepsilon \) denotes the candidate value of \( \varepsilon \) and the log-likelihood function \( \Lambda(\varepsilon) = \sum_{k=-G}^{G} \log \left\{ |r(k+N) - r(k)e^{j2\pi\varepsilon}|^2 + 4\gamma^2 \right\} \) is a periodic function of \( \varepsilon \) with period 1: The minima of \( \Lambda(\varepsilon) \) occur at a distance of 1 from each other, causing an ambiguity in estimation. Assuming that \( \varepsilon \) is distributed equally over positive and negative sides around zero, the valid estimation range of the ML estimation scheme can be set to \(-0.5 < \varepsilon \leq 0.5 \), as in [5]. The estimation scheme (6) will be called the Cauchy ML blind estimation (CMBE) scheme.

3.2 Low-complexity CFO Estimation Scheme

The CMBE scheme is based on the exhaustive search over the whole estimation range \((|\varepsilon| < 0.5)\), which requires high computational complexity. Thus, we propose a low-complexity CFO estimation scheme with the reduced set of the candidate values.

In order to obtain the reduced set of the candidate values, we exploit the fact that \( \varepsilon = \frac{1}{2\pi} \angle \left\{ x^*(k)x(k+N) \right\} = \frac{1}{2\pi} \angle \left\{ r^*(k)r(k+N) \right\} \) for \( k = -G, -G+1 \cdots, -1 \) in the absence of noise. Based on this property, we obtain the set of the candidate values

\[
\varepsilon(k) = \frac{1}{2\pi} \angle \left\{ r^*(k)r(k+N) \right\}, \text{ for } k = -G, -G+1 \cdots, -1. \tag{7}
\]

Exploiting the set of the candidate values in (7), the CFO estimate \( \hat{\varepsilon}_L \) can be obtained as follows

\[
\hat{\varepsilon}_L = \arg \min_{\varepsilon(k)} \Lambda(\varepsilon(k)), \text{ for } k = -G, -G+1 \cdots, -1. \tag{8}
\]

In the following, (8) is denoted as the low-complexity CMBE (L-CMBE) scheme. Using only \( N/2 \) candidate values, the L-CMBE scheme can offer an almost same performance as the CMBE scheme with the exhaustive search, which is verified by simulation results in Section 4.

4. Simulation Results

In this section, the proposed CMBE and L-CMBE schemes are compared with the Gaussian ML blind estima-
tion (GMBE) scheme in [5] in terms of the mean squared error (MSE). We assume the following parameters: The IFFT size \( N = 64 \), CFO \( \varepsilon = 0.25 \), the search spacing of 0.001 for the CMBE scheme, and a multipath Rayleigh fading channel with length \( L = 8 \) and an exponential power delay profile of \( \mathbb{E}[|h_l|^2] = \exp(-l/L)/(\sum_{l=0}^{L-1} \exp(-l/L)) \) for \( l = 0, 1, \ldots, 7 \), where \( h_l \) is the \( l \)th channel coefficient of a multipath channel and \( \mathbb{E}[\cdot] \) denotes the statistical expectation. Since CIS\( \alpha \) noise with \( \alpha < 2 \) has an infinite variance, the standard signal-to-noise ratio (SNR) becomes meaningless for such a noise. Thus, we employ the geometric SNR (GSNR) defined as \( \mathbb{E}[|x(k)|^2]/(\alpha C^{-1+2/\alpha}) \), where \( C = \exp\{\lim_{m \to \infty} (\sum_{i=1}^{m} \frac{1}{i} - \ln m)\} \approx 1.78 \) is the exponential of the Euler constant [10]. The GSNR indicates the relative strength between the information-bearing signal and the CIS\( \alpha \) noise with \( \alpha < 2 \). Clearly, the GSNR becomes the standard SNR when \( \alpha = 2 \). Since \( \gamma \) can be easily and exactly estimated using only the sample mean and variance of the received samples [11], it may be regarded as a known value: Thus, \( \gamma \) is set to 1 without loss of generality.

Figs. 1-3 show the MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of \( \alpha \) when the GSNR is 15 dB.

5. Conclusion

In this paper, we have proposed CFO estimation schemes using CP in non-Gaussian noise environments. We have first obtained the pdf of the CP of OFDM symbols, and subsequently, applied the pdf to the ML criterion to derive the ML CFO estimation scheme in non-gaussian noise modeled as a complex isotropic Cauchy noise. Then, we also have derived a simpler CFO estimation scheme with a lower complexity. From simulation results, it has been confirmed that the proposed schemes offer a robustness and a substantial performance improvement over the conventional estimation scheme in non-gaussian noise environments.

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