Approximate Modeling of Wireless Channel Based on Service Process Burstiness

Shuguang Fang\textsuperscript{1,2}, Yuning Dong\textsuperscript{1}, and Haixian Shi\textsuperscript{3}

\textsuperscript{1}College of Telecommunications & Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing, China
\textsuperscript{2}Electronic Engineering Department, Wuxi Institute of Commerce, Wuxi, China
\textsuperscript{3}School of Horticulture, Nanjing Agricultural University, Nanjing, China

Abstract - For link packet loss rate, delay and throughput etc. QoS performance analysis in wireless communication systems, queuing analysis based on measuring or estimating models for traffic flow and wireless channel is one of critical technologies. However, the complicated modern wireless communication systems and unreliable wireless channels usually cause accurate wireless channel models very complicated, which makes queuing analysis be of high computation complexity. Burstiness, induced by the effect of a fading channel process and modulation and coding technology in physical layer, is one of critical factors affecting QoS performance of wireless communication systems. To this end, this paper proposes a novel simpler modeling method, by which one can get a relatively simpler wireless channel model whose burstiness behavior is equivalent to that of the original channel model. Numerical examples are given to show the effectiveness of the proposed model.

Keywords: Wireless Channel; QoS Performance; Burstiness; Approximate Modeling

1 Introduction

Queuing analysis technology is one of critical technologies for link packet loss rate, delay and throughput etc. QoS metrics analysis in wireless communication systems. The accuracy of link QoS metrics to a great extent depends on the accuracy of wireless channel model in queuing analysis technology, therefore it is necessary to model wireless channel accurately. However, varying wireless channel conditions due to mobility and changing environment \cite{1} cause accurate channel modeling to be very challenging.

Wireless channel process modeling through first-order Markovian chains \cite{2, 3, 4, 5} is simplicity and analytical tractability, but later results in \cite{6} show that such methods can be very erroneous in several cases of interests \cite{7}. In ref.\cite{8} a physical-layer 2-D Markov model using both the amplitude and the rate of change of the fading envelope is presented, and based on this multidimensional physical-layer Markov model, the quality-of-service (QoS) at the data-link layer is invested relying on an analytical framework based on a discrete-time Markov chain in multi-rate adaptive modulating and coding(AMC) wireless networks, this model provides a valuable radio-link-level performance measure, but its complexity exponentially increases with the number of states of the arrival process, the number of states of the PHY Markov model, or the maximum queue length of the system.

In ref. \cite{9} a cross-layer analytical framework is presented for analyzing the QoS performance of the decode-and-forward (DF) relaying wireless networks, where the AMC is employed at the physical layer under the conditions of unsaturated traffic and finite-length queue at the data link layer. Considering the characteristic of DF relaying protocol at the physical layer, authors model a two-hop DF relaying wireless channel with AMC as an equivalent Finite State Markov Chain (FSMC) in queuing analysis, its complexity similarly increases with the number of states of the PHY Markov model, or the maximum queue length of the system.

Advanced hidden Markov models(HMM) would accurately model the wireless channel \cite{10}, note that if the HMM of channel is available, certain queuing theoretic results can be applied for more exact analysis for some specific cases\cite{11}, however this method is similarly complex in complicated modern wireless communication system.

Thus there is the need for simple modeling and approximation approaches that capture the most effect of wireless channel process characteristic on link QoS performance analysis in queuing analysis technology. The effect of wireless channel fading process and modulating and coding technology in physical-layer make wireless channel service process be burstiness, which critically affects the wireless link QoS performance \cite{7, 12}. Based on this, we propose a novel simple approximate wireless channel modeling method based on Peakness in discrete time\cite{13}, by which the simple approximate model Peakness equal to the original channel model Peakness, and verify its effective by numerical examples.

The rest of this paper organized as follows. The generalized Peakness of process burstiness and the Peakness in discrete time of Markov modulated batch Bernoulli process (MMBBP) are introduced in Section 2; In Section 3 we propose a novel wireless simple model approximate method based on Peakness in discrete time,
and verify its effectiveness by numerical examples in Section 4; Finally this paper concludes in Section 5 including the main results and future works.

2 The Process Burstiness and Description

Burstiness is a term often used in traffic analysis to represent time correlation in arrival statistics [7]. System complexity of modern wireless networks causes the arrival streams at link and channel service process are highly positively correlated in time, therefore they are all bursty process, in other words, they are all burstiness, which is one of critical factors affecting link QoS performance.

The simplest burstiness measures take only the first-order properties of the process into account, in practice the peak to mean ratio and the squared coefficient of variation are the most frequently used; the burstiness measures expressing second-order properties of the process are more complex, including the autocorrelation function, the indices of dispersion and generalized peakedness [13]. Comparing with the squared coefficient of variation, the autocorrelation function and the indices of dispersion, generalized peakedness taking the first-order and second-order properties of process and the system servicing the process into account is more effective and efficient process burstiness measure.

Peakedness, defined based on the Infinite-Server Effect Principle [14], was originally developed by teletraffic engineers to characterize call arrival streams modeled as stationary point process for approximating block probability at trunk groups [15, 16], in which the service time distribution of the fictitious infinite server group usually is exponential service time distribution. In ref. [17], Eckberg defined the generalized Peakedness for any service distribution \( B(t) \), and whose mean is \( \frac{1}{\mu} \) ( \( \mu \) is the fictitious infinite server group service rate), then the generalized Peakedness \( z \{ B(t) \} \) defined as the variance-to-mean ratio of the number of busy servers in the fictitious infinite server group,

\[
    z_{X}[B] = \frac{\text{Var}[S(t)]}{E[S(t)]} \quad (1)
\]

Where, \( S(t) \) is the number of busy servers in the fictitious infinite server group at time \( t \). Ref. [13] and [15] propose the peakedness in discrete time and the modified peakedness which encompasses point process and fluid models under a common framework and verify them respectively. The peakedness function in discrete time of the arrival stream with respect to geometric holding time distribution is given by [13],

\[
    z_{\text{geo}}[\mu] = 1 + K^* \left( \frac{1 - \mu}{2 - \mu} \right)^{-1} \quad (2)
\]

Where, \( K^* \left[ 1 - \mu \right] \) is the z-transform of the autocorrelation function of the arrival process.

Based on peakedness in discrete time, ref. [13] presents the peakedness result of Markov modulated batch Bernoulli process (MMBBP). In MMBBP, we have a discrete time Markov process as a modulating process. In each state of the modulating Markov-process, batch arrivals are generated according to a general distribution corresponding to the state. Let \( P \) and \( D \) denote the transition probability matrix and the steady-state distribution vector of the modulating Markov process respectively,

\[
    P = \begin{bmatrix}
        p_{1,1} & p_{1,2} & \cdots & p_{1,n-1} & p_{1,n} \\
        p_{2,1} & p_{2,2} & \cdots & p_{2,n-1} & p_{2,n} \\
        \vdots & \vdots & \ddots & \vdots & \vdots \\
        p_{n-1,1} & p_{n-1,2} & \cdots & p_{n-1,n-1} & p_{n-1,n} \\
        p_{n,1} & p_{n,2} & \cdots & p_{n,n-1} & p_{n,n}
    \end{bmatrix} \quad (3)
\]

\[
    D = [\pi_1, \pi_2, \cdots, \pi_{n-1}, \pi_n] \quad (4)
\]

Owing to the Markov property of the process, we have the following formulas,

\[
    \sum_{i=1}^{n} p_{i,j} = 1 \quad (5)
\]

\[
    \sum_{j=1}^{n} \pi_j = 1 \quad (6)
\]

\[
    DP = D \quad (7)
\]

Let \( M \) and \( M^* \) be diagonal matrices corresponding to the first and second moments of the number of arrivals in the corresponding states, let \( I \) be the identity matrix and \( e \) be a vector of all ones.

\[
    M = \begin{bmatrix}
        m_{1,1} & 0 & \cdots & 0 & 0 \\
        0 & m_{2,1} & 0 & \cdots & 0 \\
        \vdots & \vdots & \ddots & \vdots & \vdots \\
        0 & 0 & \cdots & m_{n-1,1} & 0 \\
        0 & 0 & \cdots & 0 & m_{n,1}
    \end{bmatrix} \quad (8)
\]

\[
    M^* = \begin{bmatrix}
        * & * & \cdots & * & * \\
        * & * & \cdots & * & * \\
        \vdots & \vdots & \ddots & \vdots & \vdots \\
        * & * & \cdots & * & * \\
        * & * & \cdots & * & *
    \end{bmatrix} \quad (9)
\]
$M_2 = \begin{bmatrix} m_{1,2} & 0 & \cdots & 0 & 0 \\ 0 & m_{2,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & m_{n-1,2} & 0 \\ 0 & 0 & \cdots & 0 & m_{n,2} \end{bmatrix}$  \hspace{1cm} (9)

$I = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$  \hspace{1cm} (10)

$e = [1,1,\cdots,1,1]$  \hspace{1cm} (11)

where, $m_{i,1}$ and $m_{i,2}$ are respectively the first and second moments of the number of arrivals corresponding to state $i$.

We can express the mean number of arrivals as $m_1 = M_1 e D'$ and the second moment as $m_2 = M_2 e D'$. The auto-covariance function of the arrival process is given by $k[i] = DM_1 P^i M_1 e - m_1^2$. Then, according to (2), the peakedness function of MMBBP $\tilde{z}_{geo} [\mu]$ with respect to geometric holding time distribution defined as (12).

$$\tilde{z}_{geo} [\mu] = \frac{1}{2-\mu} \left[ 2(1-\mu)DM_1 P^{(1-\mu)P^{-1}M_1 e + m_1} \right] \frac{m_1}{\mu}$$  \hspace{1cm} (12)

3 The Wireless Channel Approximate Modeling

In this section, we propose a simple wireless channel approximate modeling method based on equivalent process burstiness, by which we can get the simple wireless channel approximate model (SWCAM), whose peakedness function value equal to that of the channel process, by measuring first-order moment $m_1$ and peakedness function value $\tilde{z}_{geo} [\mu]$. First, we get the auto-covariance function of the arrival process by measuring $m_1$ and $m_2$ respectively, and which similarly satisfy formula (5), (6) and (7).

3.1 The First-Order and Second-Order Moment of SWCAM

Let service rate $\mu = 1$, then formula (12) equally transformed into,

$$\tilde{z}_{geo} (1) = \frac{m_2}{m_1} - m_1$$  \hspace{1cm} (13)

Then, we can get the second-order moment of channel service process by measuring $m_1$ and $\tilde{z}_{geo} (1)$,

$$m_2 = (\tilde{z}_{geo} (1) + m_1) m_1$$  \hspace{1cm} (14)

According to the principle of equivalence, the first-order and second-order moment of the SWCAM should equal to $m_1$ and $m_2$ respectively,

$$\begin{cases} Em_1 = m_1 \\ Em_2 = m_2 \end{cases}$$  \hspace{1cm} (15)

3.2 The Variable $Y_i$

Let $\omega_i = 1 - \mu_i$ formula (2) may be equivalently transformed into,

$$K^* (\omega_i) = \left( \tilde{z}_{geo} (\mu_i) - 1 \right) (\omega_i + 1) + 1$$  \hspace{1cm} (16)

Using eq. (12) and (14), we can compute,

$$K^* (\omega_i) = \frac{2\omega_i DM_1 P (1-\omega_i P) M_1 e + \tilde{z}_{geo} (1) - 2\omega_i}{1-\omega_i} m_1$$  \hspace{1cm} (17)

Then, we define variable $Y_i$ as,

$$Y_i = Y(\omega_i) = \frac{m_1}{2\omega_i} \left( K^* (\omega_i) + \frac{2\omega_i}{1-\omega_i} m_1 - \tilde{z}_{geo} (1) \right)$$  \hspace{1cm} (18)

which can be computed by measuring $m_1$ and $\tilde{z}_{geo} [\mu]$ of the channel service process.

3.3 The Simple Wireless Channel Approximate Modeling

Using eq. (17), eq. (18) equivalently transformed into,
\[ Y_i = Y(\omega_i) = DM_i P(I - \omega_i P)M_i e \]  \hspace{1cm} (19)

Defining vector \( M_0 \) as
\[ M_0 = M_i e = [m_{1,1}, m_{2,1}, \ldots, m_{n-1,1}, m_{n,1}] \]  \hspace{1cm} (20)

defining row matrix \( X(\omega_i) \) as,
\[ X(\omega_i) = [x_{1,1}, x_{1,2}, \ldots, x_{i,a-1}, x_{i,a}] = DM_i P(I - \omega_i P)^{-1} \]  \hspace{1cm} (21)

Using eq. (20) and (21) eq. (19) equally rewritten as,
\[ X(\omega_i) M_0 = Y(\omega_i) \]  \hspace{1cm} (22)

Eq. (21) can be equally transformed into
\[ X(\omega_i)(I - \omega_i P) = DM_i P^{-1} \]  \hspace{1cm} (23)

Then, using eq. (23), we can get the system of equations (24) (given in APPENDIX). Using eq. (5) and (6), adding Left side and Right side of system of equations (24) respectively, and then we can get,
\[ x_{i1} + x_{i2} + \cdots + x_{ia} = \frac{m_i}{\omega_i} \]  \hspace{1cm} (25)

Using system of equation (24) and eq. (25), we can get the expressions of variables \( x_{i1}, x_{i2}, \ldots, x_{ia} \) for different \( \omega_i \), which only includes the state transition probability and constants, and using these expressions and eq. (22), we can get formula (26) (given in APPENDIX) with state transition probability for different \( \omega_i \). For the Simple Wireless Channel Approximate Modeling based on equivalent peakedness, the state transition probability and steady-state distribution of SWCAM should similarly satisfy eq. (26), i.e. satisfy eq. (27) (given in APPENDIX). And \( ED \), \( EM1 \) similarly satisfy the following formula,
\[ ED \times EM1 = EM1 \]  \hspace{1cm} (28)

Then accord to the number of variables which determined by the states of SWCAM, we can have the system of equations (29) (given in APPENDIX).

When we get the SWCAM by system of equations (29), we should determine the times of \( z_{geo}[\mu] \) measuring according to the number of states of the SWCAM. Using system of equations (29), we can get the SWCAM of original channel with equivalent peakedness by numerical method, and we will verify system of equations (29) effective by numerical examples in section 4.

4 Numerical Results

Our objective in this section is mainly to validate the simple wireless channel approximate modeling method introduced in Section 3. For giving wireless channel service process model in multi-states Markov model, we can get the correspondingly SWCAM by the methods introduced in Section 3.

Table 1 (given in APPENDIX) lists the SWCAM in simple two-state Markov model correspondingly to 10-state, 5-state and 3-state original Markov model of wireless channel model correspondingly, which shows the feasibility of the methods introduced in Section 3.

All the figures in this section compare the peakedness of multi-state original wireless channel Markov model (Multi-states OWCMM) and that of correspondingly two-state simple wireless channel approximate model (Two-state SWCAM). Figs. 1-3 reveal the peakedness equivalent of two-states SWCAM and the correspondingly wireless channel model in 10-state, 5-state and 3-state Markov models respectively listed in Table 1, and validate the simple wireless channel approximate modeling method introduced in Section III. Comparing curves in Figures. 1-3, it can be deduced that the orders of SWCAM and original channel model more close, the peakedness of SWCAM and original channel model more consistent, and correspondingly SWCAM more accurate. In Fig.2 and Fig.3, the order difference between SWCAM and original channel model are respectively 1 and 3, which are all small, and the peakedness of SWCAM and original channel are all well consistent; but the order difference between SWCAM and original channel model in Fig.3 is 8, which are bigger than that in figures1-2, then the peakedness consistent between SWCAM and original channel is inferior to that in Figs.1-2.
the peakedness of SWCAM and original channel model more consistent, and correspondingly SWCAM more accurate. In Fig.2 and Fig.3, the order difference between SWCAM and original channel model are respectively 1 and 3, which are all small, and the peakedness of SWCAM and original channel are all well consistent; but the order difference between SWCAM and original channel model in Fig.3 is 8, which are bigger than that in figures 1-2, then the peakedness consistent between SWCAM and original channel is inferior to that in Figs.1-2.

5 Conclusions

Modern wireless communication system complexity and wireless channel variability make accurate wireless channel modeling be highly complex, which makes the computation complexity of queuing analysis in wireless channel QoS performance exponentially increase with the number of states of the arrival process, the number of states of the PHY Markov model, or the maximum queue length of the system. Meanwhile, the burstiness behavior of wireless channels service process is a critical factor which affects the wireless link QoS performance. With the proposed burstiness approximate modeling method, one can get a simpler model of the wireless channel, whose burstiness behavior is equivalent to that of the corresponding wireless channel service process. To validate this method, we analyze the peakedness equivalent of three-order, five-order and ten-order Markov wireless channel models with the corresponding two-order equivalent burstiness low complex wireless channel Markov model respectively. The application of this method to wireless channel QoS analysis will be one of our future works.

6 Acknowledgment

This work is supported in part by National Natural Science Foundation of China (No.60972038), Ministry of Education (China) Ph.D. Programs Foundation (No.20103223110001), and the graduate student scientific research innovation project of Jiangsu Province, China (No.CXZZ11_0392).

7 References


8 Appendix

\[
\begin{align*}
& x_n \left(1 - \alpha p_{i,1} \right) - x_1, 2 \alpha p_{1,2} - \cdots - x_n \alpha p_{n,1} = \pi_{1,1} \pi_{1,2} \cdots \pi_{n,1} + \pi_{2,1} \pi_{1,2} \cdots \pi_{n,1} + \cdots \pi_{n,1} \pi_{1,1} \pi_{2,2} \cdots \pi_{n,1} \\
& - x_1, 2 \alpha p_{1,2} \cdots - x_n \alpha p_{n,1} = \pi_{1,1} \pi_{1,2} \cdots \pi_{n,1} + \pi_{2,1} \pi_{1,2} \cdots \pi_{n,1} + \cdots \pi_{n,1} \pi_{1,1} \pi_{2,2} \cdots \pi_{n,1} \\
& - x_1, 2 \alpha p_{1,2} \cdots - x_n \alpha p_{n,1} = \pi_{1,1} \pi_{1,2} \cdots \pi_{n,1} + \pi_{2,1} \pi_{1,2} \cdots \pi_{n,1} + \cdots \pi_{n,1} \pi_{1,1} \pi_{2,2} \cdots \pi_{n,1} \\
& f_{20, \gamma} \left( p_1, n \cdots, p_{n-1}, 1 \cdots, p_{n,1}, \cdots, p_{n,1} \right) = \gamma \left( \alpha_i \right) \\
f_{20, \gamma} \left( p_1, n \cdots, p_{n-1}, 1 \cdots, p_{n,1}, \cdots, p_{n,1} \right) = \gamma \left( \alpha_i \right)
\end{align*}
\]
\[
f_{20,Y(\alpha)}(E_{p_{1,n}} \cdots E_{p_{1,n-1}} \cdots E_{p_{n,1}} \cdots E_{p_{n,n-1}} E_{\pi_{1}} \cdots E_{\pi_{n}}) = Y(\alpha)
\]

\[
f_{20,Y(\alpha)}(E_{p_{1,n}} \cdots E_{p_{1,n-1}} \cdots E_{p_{n,1}} \cdots E_{p_{n,n-1}} E_{\pi_{1}} \cdots E_{\pi_{n}}) = Y(\alpha)
\]

\[ED \times EM = ml\]

\[\sum \pi_j = 1E_{\pi_j} = 1\]

\[ED \times EP = ED\]

**TABLE I. SERVICE PROCESS AND SWCAM OF WIRELESS CHANNEL**

<table>
<thead>
<tr>
<th>The number of states</th>
<th>State Transition Matrix P</th>
<th>M_i</th>
<th>EP</th>
<th>EM_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.4000 0.2000 0.4000 0.6000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.4000 0.2000 0.1000 0.2000 0.1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.4000 0.2000 0.4500 0.2000 0.1000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
0.4000 & 0.2000 & 0.1000 & 0.2000 & 0.1000 \\
0.1000 & 0.3000 & 0.2500 & 0.2000 & 0.1500 \\
0.1100 & 0.2900 & 0.3000 & 0.2000 & 0.1000 \\
0.0500 & 0.1500 & 0.1000 & 0.3000 & 0.4000 \\
0.1000 & 0.1000 & 0.1000 & 0.3000 & 0.4000 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
2.0769 & 0 & 0 & 0 & 0 \\
0 & 3.1491 & 0 & 0 & 0 \\
0 & 0 & 0.4783 & 0 & 0 \\
0 & 0 & 0 & 0.2803 & 0 \\
0 & 0 & 0 & 0 & 1.3866 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
32.4247 & 0 & 0 & 0 & 0 \\
0 & 32.0724 & 0 & 0 & 0 \\
0 & 0 & 0 & 185.1371 & 0 \\
0 & 0 & 0 & 0.7000 & 0 \\
0 & 0 & 0 & 0.4000 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
83.6332 & 0 & 0 & 0 & 0 \\
0 & 33.4739 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 144.1527 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.4000 & 0.6000 \\
0.8000 & 0.2000 \\
0.2000 & 0.8000 \\
0.6000 & 0.4000 \\
0.3000 & 0.7000 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.4000 & 0.2000 & 0.1000 & 0.2000 & 0.1000 \\
0.1000 & 0.3000 & 0.2500 & 0.2000 & 0.1500 \\
0.1100 & 0.2900 & 0.3000 & 0.2000 & 0.1000 \\
0.0500 & 0.1500 & 0.1000 & 0.3000 & 0.4000 \\
0.1000 & 0.1000 & 0.1000 & 0.3000 & 0.4000 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
2.0769 & 0 & 0 & 0 & 0 \\
0 & 3.1491 & 0 & 0 & 0 \\
0 & 0 & 0.4783 & 0 & 0 \\
0 & 0 & 0 & 0.2803 & 0 \\
0 & 0 & 0 & 0 & 1.3866 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
32.4247 & 0 & 0 & 0 & 0 \\
0 & 32.0724 & 0 & 0 & 0 \\
0 & 0 & 0 & 185.1371 & 0 \\
0 & 0 & 0 & 0.7000 & 0 \\
0 & 0 & 0 & 0.4000 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
83.6332 & 0 & 0 & 0 & 0 \\
0 & 33.4739 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 144.1527 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]