Capacity of MIMO Fading Channels

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Abstract - Multiple antenna systems are projected to contribute a key role in potential multimedia wireless communication systems in the future. Such systems are predicted to provide high channel capacities in a limited bandwidth, which enables the efficient usage of spectrum. Essentially to the realization of these channel capacities are the characteristics it displays especially when analyzing the ergodic capacities and the outage capacities. To characterize the channel, certain methods have been employed. This paper reviews the basic background of MIMO systems and discusses the channel capacity using uniform power allocation in terms of ergodic and outage capacities.

Keywords: MIMO, Capacity, Fading channels, Ergodic capacities, outage capacities

1 Introduction

Various channel models have been presented over a period bits/sec, of the communication for each MIMO user can be improved. Therefore, this directly increases the network capacity and quality of service. This development enables MIMO systems to be viewed as an extension of the so-called radio propagation between two or more points. Radio propagation is an important aspect of any radio design or radio network planning. Therefore, a variety of analyses through process models have been identified. [1] One of the models is the MIMO configuration.

In the digital communication field, the recent introduction of MIMO - multiple input multiple output or can also known as volume-to-volume ‘wireless link’, has appeared as one of the most significant breakthrough in modern communications. [2] One of the main motivations for this advancement is the chance of resolving the bottleneck of traffic capacity in future Internet-intensive wireless networks.

To define MIMO systems simply, given an arbitrary wireless communication system, we consider a link for which the transmitting end as well as the receiving end is equipped with multiple antenna elements.[2] Saying this, it can clearly be seen that MIMO architectures are a form of receive and transmit diversity scheme. In addition to that, this form of configuration involving multiple antennas on the input and the output gives us the ability to dramatically improve capacity gains through increased spatial dimension. MIMO systems can provide significant increase in data throughput and link range without additional bandwidth or transmit power.

This basically means that the signals on the transmitting antennas on one end and the receiving antennas on another end are ‘combined’ in such a manner that the quality, with references to signal to noise ratio (SNR) and the data rates in bits/sec, of the communication for each MIMO user can be improved. Therefore this directly increases the network capacity and quality of service. This development enables MIMO systems to be viewed as an extension of the so-called ‘smart antennas’, where it uses antenna arrays along with signal processing algorithm to improve wireless transmissions. Multipath propagation, conventionally known to be a drawback of wireless communications, was made a turn around with the introduction of MIMO systems. This became a key feature of MIMO, where it uses multipath propagation properties to multiply transfer rates. A high rate signal is split into multiple lower rate streams. Then each of these stream is transmitted from a different transmit antenna in the same frequency channel. This is known as multipath, the signal taking different routes to reach the receiver. The different signals will then arrive at the receiver antenna with different multipath delay spread. Then the receiver can proceed to separate these different signals, creating parallel channels for free. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher Signal to Noise Ratio (SNR). [3] The panorama for magnitude improvement in wireless communication performance at no cost of extra bandwidth is largely responsible for the success of MIMO. This has encouraged tremendous progress in diverse areas such as channel modeling, information theory and coding, signal processing, antenna design and multi-antenna-aware cellular design, fixed or mobile. [3] This paper covers the capacity of MIMO going through a channel experiencing Rayleigh fading. The first portion of this paper will be introducing the system model of MIMO systems. Follow up on that would be the MIMO signal model where certain analysis of the model with respect to the system model will be explained. Next section would see the analysis of the simulation results about the ergodic capacity and 99% outage capacities of MIMO fading channels.
2 System Modal and Signal Model

2.1 System Model

The way it has always been traditionally for wireless communications is the usage of a single antenna for transmission and a single antenna for reception. Such systems are known as single input single output (SISO) systems. As the times began to change, significant progress has been made in developing systems that employ multiple antennas at the transmitter and the receiver in order to achieve better performances. [4]

Fig. 1 Block diagram representing multiple transmitting antennas and multiple receiving antennas

Fig.1 depicts the system model of the MIMO systems. As seen, on the transmitter end we have M transmit antennas, represented by XM. On the receiver end, we have N receive antennas. Transmission from the transmitter's M antennas can go in any path, therefore forming multipath propagation before it is being received at the various receiving antennas. There are several benefits of using multiple antennas. Firstly, the link budget and spatial diversity improvements. Spatial diversity refers to the fact that the probability of having all antennas at bad locations is significantly lower as the number of antennas increases. Link budget improvement on the other hand, refers to the fact that the signals from the various antennas can be combined to form a signal stronger than any of the individual signals. For receive spatial diversity, signals received on multiple antennas are weighted and combined. Example of this is Maximum Ratio Combining (MRC), where the receiver co-phases the signals and sums them together, weighting each branch with a gain proportional to the amplitude on the received signal on that branch. Other diversity methods used at the receiver includes Selection Combining where we simply select the branch with the strongest SNR to be the output signal and Equal Gain Combining where the receiver co-phases the signals and sums them up. There are two types of transmit spatial diversity, open- loop and closed-loop. Open-loop transmit diversity involves transmitting signals from multiple antennas in some deterministic pattern, that does not depend on the channel. Open-loop techniques include cyclic delay diversity (CDD) and space-time block codes (STBC). Closed loop transmit diversity techniques, in contrast, require channel information to guide transmissions. An example is the Transmit Beam Forming (TxBF), where proper magnitude and phase weights computed from the channel estimation are applied again across antennas to aim the signal in a given desired direction. MIMO systems with spatial diversity achieve better performance. This basically means that it has a longer range for a given data rate, as compared to SISO systems at a given same location. The other advantage to exploit rich spatial dimensionality is via spatial multiplexing. This means transmitting and receiving multiple data streams from multiple antennas at the same time, and in the same frequency spectrum. This is possible because the signals received at different antennas are unique combinations of the transmitted data streams. Advanced digital signal processing algorithms can be used to recover the original data. Spatial multiplexing can be implemented in either open-loop or closed-loop. In open-loop spatial multiplexing, different streams are simply transmitted from different antennas. In closed-loop spatial multiplexing, every stream is transmitted from all of the antennas using weights computed from the channel estimation. MIMO systems with spatial multiplexing achieve higher peak data rates and increases spectrum efficiency.

2.2 Signal Model

In order to design efficient communication algorithms for MIMO systems and to understand the performance limits, it is important to understand the nature of MIMO channels. For a system with M transmit antennas and N receive antennas, assuming frequency-flat fading over the bandwidth of interest [5], we obtain the following:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_N \\
\end{bmatrix} = \begin{bmatrix}
h_{1,1} & \cdots & h_{1,M} \\
h_{2,1} & \cdots & h_{2,M} \\
\vdots & \ddots & \vdots \\
h_{N,1} & \cdots & h_{N,M} \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_M \\
\end{bmatrix} + \begin{bmatrix}
n_1 \\
n_2 \\
\vdots \\
n_N \\
\end{bmatrix}
\]

Or can also be denoted as:

\[
y = Hx + w \\
\]

We see that y denotes the N x 1 received signal vector and x is represented by the M x 1 transmitted signal vector. n is the N x 1 zero-mean complex Gaussian noise vector with independent, equal variance real and imaginary parts, thus giving us N (0, 2 \sigma^2) [6]. The matrix H is the N x M normalized channel matrix. Each element h(N, M) represents the complex gains between the Mth transmit and Nth receive antenna.

For a memory less 1x1 (SISO) system the capacity is given by:

\[
C = \log_2 \left( 1 + \rho |h|^2 \right) \text{bits / sec} / \text{Hz}
\]
Where h is the normalized complex gain of a fixed wireless channel or that of a particular realization of a random channel. In (2) and subsequently, $\rho$ is the SNR at any receiver antenna. As we deploy more receiver antennas the statistics of capacity improve and with N receiver antennas we have a single-input multiple-output (SIMO) system with capacity given by:

$$C = \log_2 \left( 1 + \rho N \right) \text{bits/Hz/sec}$$

(3)

Where $h_i$ is the gain for these receiver antennas, i. Note the fundamental element of (3) is that increasing the value of N only results in a logarithmic increase in average capacity. Correspondingly, if we decide to opt for transmit diversity, in the common case where the transmitter does not have channel knowledge or state information, we have a MISO system with M transmit antennas and the capacity is given by:

$$C = \log_2 \left( 1 + \frac{\rho}{M} N \right) \text{bits/Hz/sec}$$

(4)

Where the normalization by M ensures a fixed total transmitter power and shows the absence of array gain in that case, as compared to the case in (3) where the channel energy can be combined coherently. Again, note that capacity has a logarithmic relationship with M. Now we consider the use of diversity at both transmitter and receiver giving rise to MIMO systems. For M transmitting antennas and N receiving antennas we have the now famous capacity equation:

$$C = \log_2 \left( \frac{\rho}{M} N \right) \text{bits/Hz/sec}$$

(5)

Where $\text{tr} \left( \mathbf{R} \right)$ denote the power constraint. Note that for equal power uncorrelated sources, $\mathbf{R} = \frac{\rho}{M} \mathbf{I}_N$ and (6) becomes (5). This is optimal when H is unknown at the transmitter and the input distribution maximizing the mutual information is the Gaussian distribution. It is also demonstrated that the capacity in (5) grows linearly with $\min(M, N)$ rather than logarithmically. This result can be intuited as follows: the determinant operator yields a product of $\min(M, N)$ non-zero eigenvalues of its (channel-dependent) matrix argument, each eigenvalue characterizing the SNR over a so-called channel eigen-mode. An eigenmode corresponds to the transmission using a pair of right and left singular vectors of the channel matrix as transmit antenna and receive antenna weights respectively. Thanks to the properties of the log, the overall capacity is the sum of capacities of each of these modes, hence the effect of capacity multiplication. Clearly, this growth is dependent on properties of the eigenvalues. If they decayed away rapidly then linear growth would not occur. However (for simple channels) the eigenvalues have a known limiting distribution and tend to be spaced out along the range of this distribution. Hence it is unlikely that most eigenvalues are very small and the linear growth is indeed achieved. With the capacity defined by (5) as a random variable, the issue arises as to how best to characterize it. Two simple summaries are commonly used: the mean or also known as the ergodic capacity given as

$$C = E \left[ \log \left( \frac{\rho}{M} N \right) \right] \text{bits/Hz/sec}$$

(7)

and also, the outage capacity, $C_o$. Outage capacity measures are often denoted by capacity values supported (a), for example, 90% or 99% of the time, and indicating the system reliability. Therefore we have:

$$\text{Prob}(C(H) > C_o) = \alpha$$

(8)

A full description of the capacity would require the probability density function or equivalent. Some vigilance is required when interpreting the above equations. The important thing to consider is that our discussions concentrate on single user MIMO systems. Although so, many results can also apply to multi-user systems with receive diversity. [7]

Finally the linear capacity growth is only valid under certain channel conditions. It was originally derived for the independent and identically distributed flat Rayleigh fading channel and does not hold true for all cases. For instance, if large numbers of antennas are packed into small volumes then the gains in H may become highly correlated and the linear relationship will reach a state of little or no growth at all due to the effects of antenna correlation. On the other hand, other propagation effects not captured in (5) may provide a means to highlight the capacity gains of MIMO such as multipath delay spread. This can be shown in the case when the transmit channel is known but also in the case when it is unknown. In general, the consequences of the channel modeling are critical. Environments can easily be chosen which give channels where the MIMO capacities do not increase linearly with the numbers of antennas. [1] However, most measurements and models available to date do give rise to channel capacities which are of the same order of magnitude as the promised theory. [7] Also the linear growth is usually a reasonable model for moderate numbers of antennas which are not extremely close-packed. Here the capacity is given by CEP in (5). This was derived and showed $\mathbf{R} = \frac{\rho}{M} \mathbf{I}_N$, that is, optimal for identical and independent Rayleigh fading channels. Also, this was derived starting from an equal power assumption.
3 Simulation Results and Analysis

In this section, we examine the ergodic capacities and 99% outage capacities of a MIMO Rayleigh fading channel with M transmit antennas and N receive antennas for SNR ranging from 0 dB to 40 dB. We consider $M=N=1$; $M=1, N=2$; $M=2, N=1$ and $M=N=2$ respectively.

3.1 The ergodic Capacity comparison between the different number of M and N.

At first, we assume the channel information is known at transmitter (CSIT). So, it used the waterfilling solution. Then we can see the capacity is increased while the number of antennas is increased. (Fig. 3.) In the SISO case, where both M and N are equal to 1, the capacity at SNR equals to 5 dB is 1.8 bit/s/Hz while 3.6 bit/s/Hz in $2 \times 2$ MIMO case which is approximately twice as the SISO case. This feature keeps from low SNR to high SNR situation. Secondly, the capacity of asymmetrical antennas at transmitter and receiver is nearly the same at $2 \times 1$ antennas case and $1 \times 2$ antennas case.

Then, we examined when the channel information is unknown at transmitter, so we used the equal power allocation. (Fig. 4.) In this case, the performance of symmetrical number of antennas is as same as the previous case, double the capacity when the antennas increased from $1 \times 1$ to $2 \times 2$. However, the capacity of $2 \times 1$ model (MISO) is different from the $1 \times 2$ (SIMO) one. The more receiver antennas achieve higher capacity compared to the more transmitter antennas’ case. The performance is worse at receiver when the channel state information is unknown at transmitter, especially at low SNR. That is because at low SNR with Full CSI, waterfilling can provide a significant power gain.

3.2 The 99% outage capacities.

The result is shown in Fig. 5.

3.3 Ergodic capacity increases linearly in $\min(M,N)$ at high SNR.

Result is shown in Fig. 6.
The ergodic capacity between different $M$ and $N$ is unknown at the transmitter. The ergodic capacity:

$$C = \varepsilon_H \left[ \log \det \left( I_N + \frac{1}{\sigma^2} M I_M H H^H \right) \right]$$

$$= \varepsilon_H \left[ \sum_{i=1}^{m} \log (1 + \frac{p}{m} \lambda_i) \right], \quad (9)$$

where $m = \min (M, N)$ and $\lambda_1, \ldots, \lambda_m$ are the eigenvalues of the Wishart matrix $W = HH^H$.

For large SNR, $C_{EP} = \min(M, N) \log P + N$, the capacity grows linearly with $\min(M, N)$.

### 4. Conclusion

As a conclusion, this paper has covered the major reviews of MIMO systems and its potential use for further enhancement in future wireless networks. Information theory reveals that great capacity gains can be realized from employing MIMO systems. Whether we achieve this fully or at least partially in practice, it can be seen that this depends on a sensible design of transmit and receive signal processing algorithms. It is obvious that the success of MIMO algorithm has integrated into various commercial standards such as 3G, WLAN and beyond and this clearly will rely on a fine compromise between the maximization of data rates and diversity, example space time coding, solutions. This also includes the ability to adapt to the time changing nature of the wireless channel using some form of feedback. To this end more progress in modeling, not only the MIMO channel but its specific dynamics, will be required. As new and more specific channel models are being proposed it will useful to see how those can affect the performance trade-offs between existing transmissions algorithms and whether new algorithms, tailored to specific models, can be developed. Finally, upcoming trials and performance measurements in specific deployment conditions will be a key to evaluate precisely the overall benefits of MIMO systems in real-world wireless scenarios.

### 5. References


