A pretopological multi-agent based model for an efficient and reliable Smart Grid simulation

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***Abstract*—Smart Grid is a typical complex system due to the heterogeneity of actors, economic issues and material aspects such as information technology and power generation. Moreover, the conflicting combination of human and artificial systems is an important complexity factor.**

In this paper, we provide a multi-agent based model for an efficient and reliable Smart Grid implementation. Our model combines adapted mathematical theories to obtain a more realistic modeling of Smart Grid. It takes into account the heterogeneity of components, links them with a generalized proximity concept, and ensures an optimal functioning of the whole system.

I. Introduction

A complex system is a system composed of many entities with simultaneous local interactions. The global behavior can not be deduced from the behavior of its components. An efficient modeling of Smart Grid needs an interdisciplinary approach owing to the heterogeneity of components, and also a holistic method due to the emergence phenomenon, in order to guaranty an optimal functioning of the global system.

In this paper, we are interested in Smart Grid modeling: computed and optimized electricity distribution networks to improve production and distribution of electricity. Smart grids implement heterogeneous actors with different interests: on one side, energy producers want to maximize their profits, on the other consumers try to cut costs, and finally, the state seeks to reduce carbon dioxide emissions, while optimizing the system to satisfy all customers. In our modeling, we need to aggregate the preferences of all stakeholders, to implement the concepts of sharing and redistribution of energy, and finally to ensure that the entire territory is covered in energy.

We will show that Smart Grid is a relevant example of complex system. Then we will present the Smart Grid concept and highlight the similarities with complex systems. Afterwards, we will propose a multi-agent based model founded on theoretical notions including specific algorithms to take into account specificities of Smart Grids properties. Eventually, we will present our model and discuss the results and future work.

II. Smart Grid

Currently, Smart Grid is a fuzzy concept: it is an objective to improve power generation and distribution that will integrate renewable and low carbon energies in the existent power grid. Smart Grid requires to satisfy essentials properties such as reliability, scalability, cost effectiveness but also requires real-time communications secure and sustainable [1]. Of course, this system must integrate all the heterogeneous actors.

In this section, we present the different ways of producing electricity and the relation with complex systems and networks.

A. Presentation of Smart Grid : from power generation to distribution

Smart Grids are composed of an enormous number of devices of various types, from smart meters and solar inverters to electrical substation equipment and sensors on power lines. Electricity can be produced by multiple processes: from the stable production of a nuclear plant, to the storage via electric vehicles, and integration of renewable energy which production may depend on environmental factors. A huge distribution and energy transport network has been created over the years but it is not mastered nor optimized. Our goal is to make powergrids more efficient by integrating renewable energies and taking advantages of information and communication technologies.

In the next paragraph, we present the notion of complex system to highlight that Smart Grid represents a perfect example.

B. Introduction to Complex Systems and Complex Networks

A system which consists of large populations of connected agents (or collections of interacting elements), is said to be complex if there exists an emergent global dynamics resulting from the actions of its parts rather than being imposed by a central controller. That is a self-organizing collective behavior difficult to anticipate from the knowledge of the agents' behavior [2].

The structure of complex systems is usually represented as a complex network which can be classified according to two criteria. The first concerns the categorization by scope. The second is a theoretical classification according to the mathematical properties of the network.

1) Network classification according to the domain: We can classify networks into four loose categories: social networks, information networks, technological networks and biological networks [3].

a) Social networks: A social network is a set of people or groups of people with some links or interactions between them [5], [6], [7]. It can represent friendships between individuals [8], [9], business relationships between companies [10], [11], etc... An epidemic disease affecting a population is also a good example of complex social network.

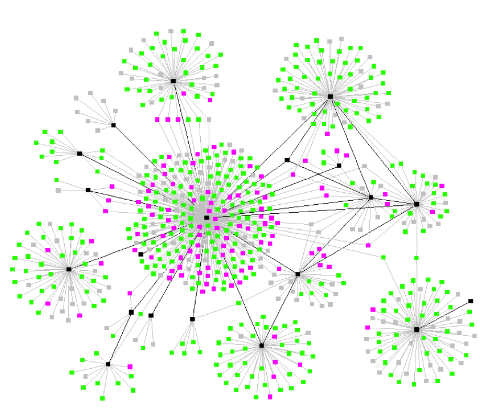

Fig. 1. Epidemic among colonies of individuals.

b) Information networks: The second network category is information networks (also sometimes called “knowledge networks”) [3]. The classic example is the network of citations between academic papers [12] in which the vertices are articles and a directed edge from article A to article B indicates that A cites B.

c) Technological networks: The third class of networks is technological networks, man-made networks designed typically for distribution of some commodity or resource, such as electricity or information [3]. The electric power grid and Smart Grids are good examples.

Lots of distribution networks have been studied over the time: airline routes [18], [13], roads [14], railways [15], [16], pedestrian traffic [17], but also power grids and recently Smart Grids [19], [20]. The logical structure of Internet is also a good example with all the communication lines (optic fiber, satellite, etc...)

d) Biological networks: The last class of complex network is biological networks [21]. Lots of biological systems can be usefully represented as networks, for example, a protein can be modeled as a network of amino acids, which may also be represented as networks of atoms such as carbon, nitrogen and oxygen.

We have seen some different types of real complex networks in the various fields, in the next paragraph we will see their mathematical properties.

2) Theoretical classification of networks: Theoretical research about network established that complex networks can be represented with particular random graphs. A random graph is a graph in which the edges are randomly distributed [2]. We can distinguish two types of random graph that fit with complex system modeling : scale-free networks and small world networks. In the case of Smart Grids, we have to deal

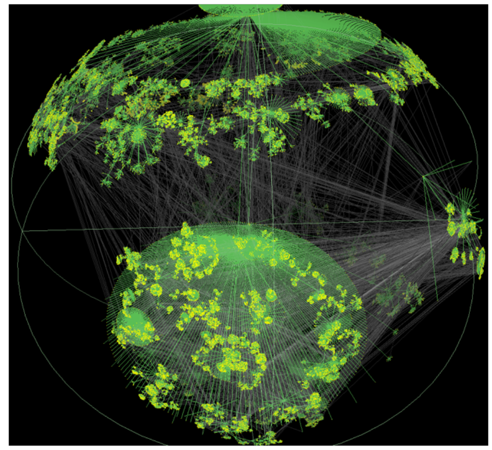

Fig. 2. Technological network : logical structure of Internet.

with heterogenous networks : a number of neighbors with large variations, compared to homogeneous networks which have a similar number of neighbors for each point. Our goal is to recreate the French national grid by generating a network with varying properties in term of network connectivity.

a) Scale-free: A scale-free network is a graph which degree distribution follows a power law.

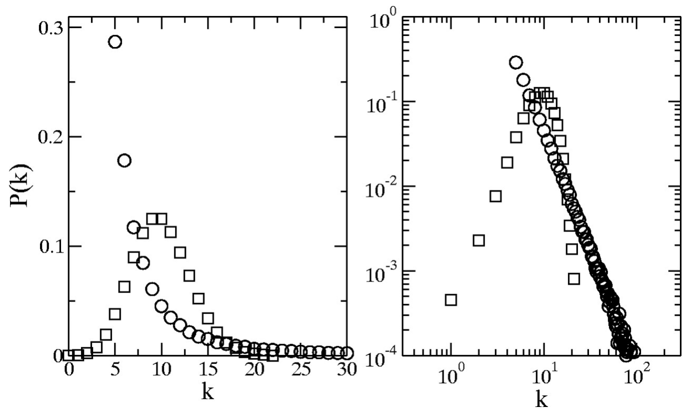

Fig. 3. Comparison between the degree distribution of scale-free networks and random graphs.

The bell-shaped degree distribution of random graphs peaks at the average degree and decreases fast for both smaller and larger degrees, indicating that these graphs are statistically homogeneous. In opposition, the degree distribution of the scale-free network follows a power law.

Scale-free networks occur in many areas of science and engineering, including the topology of web pages and the power grid of the western United States [4].

b) Small-world: A small-world network is a type of mathematical graph in which most nodes can be reached from every other by a small number of hops or steps.

Stanley Milgram, a social psychologist at Harvard, conducted a simple experiment in 1967 ([24]). He developed an easy methodology for studying how people are linked to others by giving a folder to a ”starting person” instructed to send it

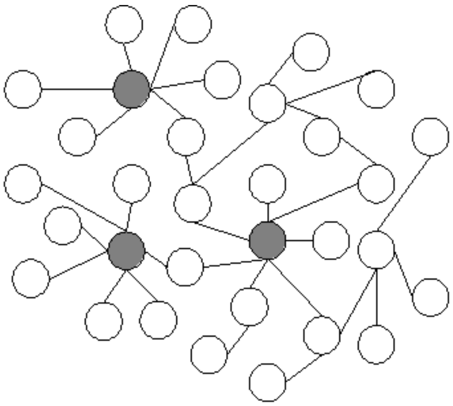

Fig. 4. Scale-free network.

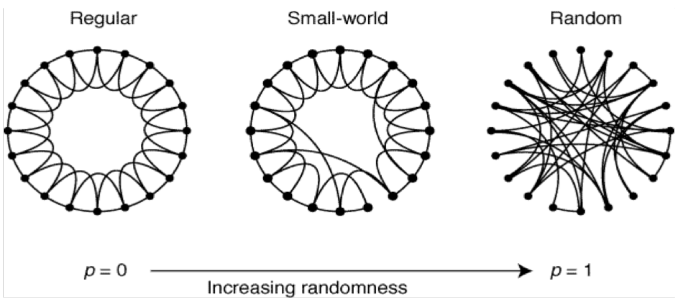

Fig. 5. Small World networks.

to a "target person". If the starter does not know the target, he send the folder to someone else among his friends, who will have to transmit the folder again until the target is reached. Thus Milgram could determine how closely linked any two people are in a population. In the original study, starters in Kansas tried to get folders to the wife of a divinity student in Cambridge, Massachusetts, and starters in Nebraska tried to get folders to a stockbroker in Sharon, Massachusetts. In the Nebraska study, Milgram found five intermediaries was the median number of links between the starter and target.

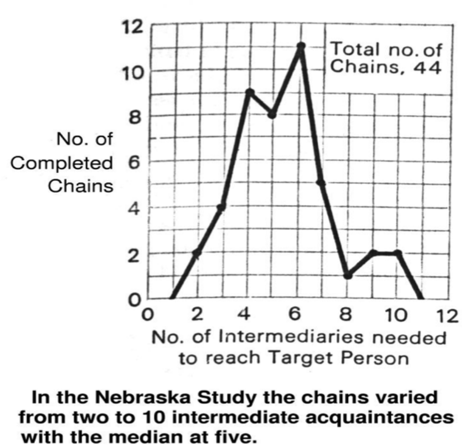

Fig. 6. Milgram Nebraska study

These properties are important issues to consider in our modeling because the scale-free is a property which clearly exists in the Smart grids as the distribution network follows this property.

Our goal is to try to find the relevant theories to solve problems of Smart Grid and to enable us to design a system with local rules leading to a global qualitative behavior. We will design the rules and a mathematics model to ensure the properties globally. In the next section, we present our theoretical model and theories used to build it.

III. Modeling

Primary, we have to distinguish models from simulation. A model is a simplified mathematical representation of a system at some particular point in time or space intended to promote understanding of the real system [2]. A simulation is the manipulation of a model in order to perceive interactions that would not otherwise be apparent because of their separation in time or space. A simulation should include as much detail as possible, whereas the model should include as little as possible to be easily generalized to other systems.[2]

Smart Grid represents a shared resource among multiple actors (or agents), with divergent interests. A complex system is characterized by its global behavior that is itself defined by local behavior. It is not obvious to do this in the form of equations, the use of multiple agents is therefore more suitable but we will be discussing that in more detail in the next section.

A. Agent-Based modeling

A multi-agent system (MAS) is a system composed of a set of multiple interacting intelligent entities (agents), an environment, a set of relations between entities and a set of operations that allows agents to perceive, produce, consume, transform and manipulate objects. Agents are specific objects representing the active entities of the system. Multi-agent systems can be used to solve problems that are difficult or impossible for an individual agent or a monolithic system to solve. Intelligence may include some methodic, functional, procedural or algorithmic search, find and processing approach.

The design of complex system is more realist with intelligent agents interacting together, because each agent can act as a separate mathematical model [30]. That is why agent-based modeling is one of the most evident approach for modeling complex system. MAS approach is used in lots of research topics such as pollution modeling [31], social structures...

An agent is a computational entity, like a computer program, a human or a robot, which can be viewed as perceiving and acting by its own on the environment. Agents in a multi-agent system have several important properties:[32]

- agents are partially autonomous: its behavior depends partly from its experience.
- agents don't have a full global view of the system but a local view
- there is no central controlling agent (it's not a monolithic system) [33]

Technically, a multi-agent system consists of a set of processes occurring at the same time. The key point of multi-agent systems lies in coordination between agents.

A MAS is said to be homogeneous if all agents are built on the same model (an ant colony), and heterogeneous if agents are built with different models from different hierarchical levels (eg the organization of a company). Concerning Smart Grid, we will use heterogeneous MAS to model all the different types of entities. Customers will even be modeled as autonomous and intelligent sub-systems. This will be further developed in this article.

One of the major benefits of MAS is to allow to study in silico the generic problem in risk-free space, while studying the global view of the system and setting up important parameters. Then, we can test the impact of various parameters in various scenari at very low-cost. Moreover, it corresponds to the reality because every agent act in real time simultaneously while having a possible individualized behavior.

However the only theory of multi-agent does not allow us to solve Smart Grids systems because there are theoretical problems. In the next section, we present in details the different theoretical aspects we use to enrich MAS to model efficiently Smart Grids. First, we will present Smart Grid as a complex network structure with behaviors tied with optimization and negotiations, so we will present game theory, a theory that models this behavior. Negotiations are based on notions of proximity between agents, but if we use a proximity metric that won't be appropriate and it will manage only one criterion, so we will use the pretopology. It will enable us to manage several relationships simultaneously using boolean functions, and that will better correspond to the reality. Moreover, in case of modification of one criterion we just have to change the corresponding adherence function. The pretopology is then one of the theories that brings lots of improvements to the existent model. Finally, we will present the percolation as a theory to validate our model. It will enable us to check if the network is always connected i.e. in terms of energy delivery, we can ensure that everyone is satisfied.

B. Smart Grid as a Complex Network

As seen in section 1, Smart Grid can be defined as a complex system. However, complex system structure is represented by a complex network, which is a combination of different graphs. Concerning Smart Grid, the complex network represent the transport and communication network. On one side, distribution network is the set of all power lines that allows the energy to pass through the grid and to be distributed. On the other side, the communication network allows agents communication, and collects the different statistics from smart meters.

In the case of Smart Grid, we represent the distribution network by an undirected graph: we consider the edges to be power lines, and the nodes to be junctions (i.e. substation, residentials, companies, ...) where the energy can go in or off a particular power line. Each edge has a maximum power supply that it is able to provide. Each node is an agent with some particular parameters (provide or consume energy, etc).

C. Game theory

In this part, we focus on game-theoretic ideas to solve the problem of negotiations concerning electricity distribution.

Smart Grids involves heterogeneous actors with different interests: the state, consumers, and companies are in perpetual conflict in the economic field. This is very close to game theory [34], [35]: it is indeed a strategic problem in which the actions of each player will influence the others.

Game theory is a method of applied mathematics used to study human and animal behaviors by modeling competing behaviors of interacting agents. Although it was initially developed in economics to understand a large collection of economic behaviors[34], the use of game theory in the social sciences has expanded, and has been applied to political, sociological, and psychological behaviors as well. Applications include auctions, bargaining, fair division, duopolies, oligopolies, social network formation, agent-based computational economics [36], mechanism design[38], voting systems[39], behavioral economics[40], and political economy [37].

Game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision-maker. It provides general techniques to analyze situations, such as in games, where an entity's success depends on the choices of the other entities [35]. From our point of view, we can see it as a model of conflicts and cooperations between agents and the *game* refers to the whole Smart Grid. Then, it will model electricity traffic patterns between the agents.

As seen above, the negotiations are based on notions of proximity between agents, but if we use metric distances that won't be appropriate and it will manage only one criterion. Conventional topology would be too cumbersome to be implemented so pretopology is one of the most relevant theories that brings a lot of improvement to the existent model. It will enable us to manage several relationships simultaneously using boolean functions, and that will better represent the reality. Moreover, in case of modification of one criterion we just have to change the corresponding function.

D. Pretopology

The pretopology is a mathematical theory, weaker than classical topology to express the structural transformation of sets composed of interacting elements such as the constitution of decisive coalition among a population, alliance phenomena, tolerance and acceptability processes and emergence of collective behavior [25], [26]. In our case, we use pretopology to model the concept of proximity.

Indeed, in the context of integrating renewable energy, it is necessary to express the notion of proximity in terms of algorithm. If we take the example of wind turbines, we do not speak about metric proximity, but functional proximity: the weather influences the electrical efficiency production, and the turbine closest from us is not necessarily the one that can provide all the energy required!

Definition We call *pseudoclosure* defined on a set E, any function a(.) from P(E) into P(E) such as:

- $a(\emptyset) = \emptyset$
- $\forall A \subset E, A \subset a(A)$

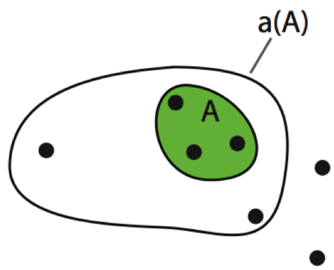

Fig. 7. Pseudo-closure of A.

Definition We call *interior* defined on a set E, any function i(.) from P(E) into P(E) such as:

- $i(E) = E$
- $\forall A \subset E, i(A) \subset A$

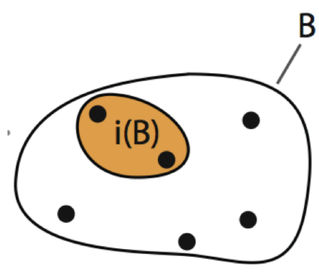

Fig. 8. Interior of A.

Definition Given, on a set E, a(.) and i(.), the couple $s = (a(.), i(.))$ is called pretopological structure on E and the 3-uple (E,a(.),i(.)) is called a pretopological space.

In pretopology, a complex network can be viewed as a family of pretopologies on a given set E [7].

Then, we can use different pretopological spaces on the same set to better model a phenomena. In our case, the integration of all the parameters of the Smart Grid can be done within a separate pretopological space for each one: weather influence, distances, maximum power supply, etc... By this way, an accurate setting can be done easily, quickly and efficiently.

With the pretopology, we can enrich our model using multivariate relationships between entities. However, the Smart Grid is dynamic so we need a method to verify that our model generates an overall system working properly: that is, it distributes electricity regularly to every customer. The percolation checks if a network is always connected so in terms of energy delivery, we can ensure that all customers are satisfied. In the next section, we present more in details this theory.

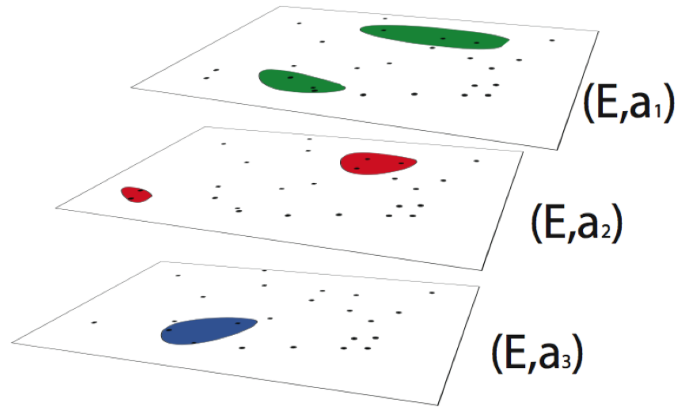

Fig. 9. Complex network in pretopology.

E. Percolation theory

Percolation is a mathematical theory used for determinist diffusion processes on a stochastic structure [27].

The theory of percolation can bring together many complex phenomena by analyzing the behavior of phase transition in these systems. It helps to highlight the underlying mechanisms. The word ”percolation” comes from Latin percolatio, meaning filtration. He referred in general to the concept of agglomeration and propagation in random media partially interconnected [29].

Physical problems such as Smart Grids are mathematically modeled as a network of points (n consumers) and the connections (or edges) between each two neighbors may be opened (allowing the electricity to pass through) with probability p (connected by bonds of p random efficiency), or closed with probability $(1 - p)$, and we assume they are independent. [28] So there exists a proportion $(1-p)$ of links that can be destroyed at random, which does not provoke any power failure visible for consumers, nor interfere with communication between a station and an another, as it is still possible to go through relay stations if p is greater than a critical value, called the percolation threshold p_c. Below the critical value p_c, the probability that two stations can communicate directly or indirectly is very low.

This mathematical model was originally developed for the study of physical phenomena, but it had been applied in various fields other than physics [41]. Indeed, the percolation theory has contributed, for example, in modeling complex systems in economics [42], marketing [43], sociology [44], computer science [45], ecology [46], or mathematics [47].

In our case, we will determine the acceptable threshold in order to have a proper power distribution. Eventually we will get a tool for decision support that will help to refine variables, change settings using modeling theories enriched by classical mathematical theories. We aim to achieve a very effective model to manage Smart Grid complexity in order to control it or enhance the performance.

IV. Simulation

In this section, we present an overview of our model developed with AnyLogic.

A. Anylogic

Anylogic, a simulation software provided by XJ Technologies, is the first and only tool that brings together System Dynamics, Process-centric (Discrete Event), and Agent Based methods within one modeling language and one model development environment.

The native Java environment provides multi-platform support, and models can be exported as standalone Java applications. It supports limitless extensibility including custom reusable Java code, external libraries, and external data sources. For example, we can integer the Java library PretopoLib [48] developped by Vincent Levorato and Marc Bui to manage pretopological functions and spaces.

B. Agents and parameters

We use several types of agent in our model. We focus on individual objects and describe their local behavior. We use a configurable scale-free structure in order to be able to modify some parameters and analyze the consequences of the structure on the system dynamic.

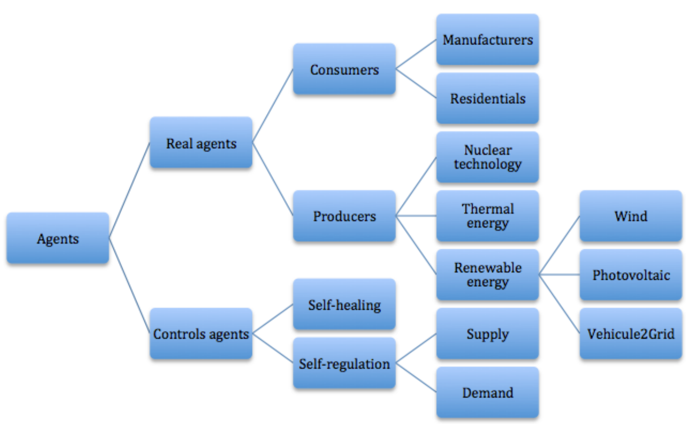

Fig. 10. Different types of agents.

We separated the work in different modules.

We stored all real data about energy consumption and production in an external database. Then, for each type of agent, we implemented mathematical models to reproduce their behavior as realistic as possible. Each type of agent has its own power management module to reproduce its consumption or production of electricity. Then, this module is connected to a decision making process module, based on game theory to compute the distribution of energy using a pretopological proximity measure. Agents can communicate with each others through a communication module, and are interconnected with a scale-free transport network. The "environment module" enables to integrate the management of external parameters having an influence on the system such as weather or pricing policy.

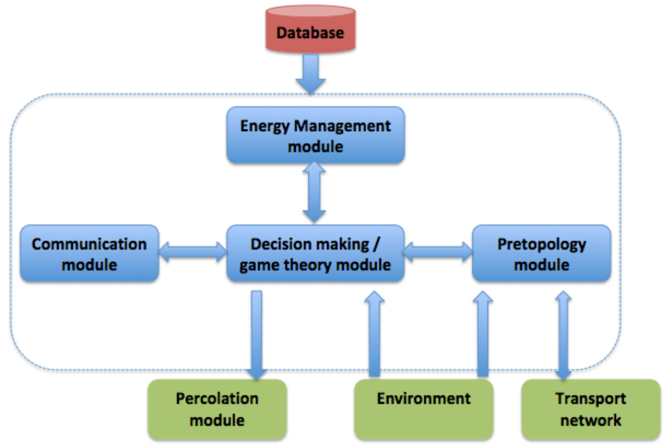

Fig. 11. Modules for each agent.

C. Futur work

Currently, our model developed on AnyLogic is a simplified model. We integrated the various agents presented in a scale-free network, and the distribution of electricity is made by calculation with pretopological distances. In a futur work, we plan to integrate game theory in our development and to test different scenario in order to optimize the system. Then, we will be able to change the value of some parameters and study their effects.

V. Conclusion

To conclude, we presented an efficient multi-agent based modeling to meet the problematics of Smart Grids.

By basing our model on theories adapted to fit each issue, our model allows to take into account the heterogeneity of the actors that makes the Smart Grid a complex system, and optimizes the existing network by integrating an overall independent supervision.

In a futur work, we will be able to present more results and will certainly adapt some points to calibrate our model at best compared to the reality of the existing network.

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