Abstract - The objective of this study is to minimize the drag coefficient of aerodynamics bodies for a specified design Reynolds number regime. With the aerodynamics model the gradient of the objective function (drag coefficient) cannot be determined analytically. Furthermore, it is expected that the objective function is multi-modal, i.e. it shows more than one minimum. Therefore, the optimization algorithm must be efficiently applicable to such multi-dimensional, multi-modal and nonlinear objective functions. A powerful optimization procedure called Genetic Algorithms (GA) has been combined with the aerodynamics calculation to find the optimal shape with minimum drag coefficient. The aerodynamics calculation of flow field around the body of revolution is determined to get accurate drag coefficient. An effective numerical calculation called the Finite Volume Method is considered. For the laminar to turbulent transition locations, the linear stability analysis is applied to predict the natural transition location. The results were compared with those obtained from integral method and also experimental results and indicate a good agreement. It is concluded that for the purpose of the shape optimization of streamline bodies, the Genetic Algorithms and the Finite Volume Method with natural transition criterion is an essential approaches to indicate the optimal shape for various ranges of Reynolds number.

Keywords: Drag Reduction; GA; FVM; Natural Transition

1 Introduction

Recently there has been a growing interest in problem solving system based on principles of evolution and heredity: such systems maintain a population of potential solutions, and they have some selection process based on fitness of individuals and some recombination operators. One type of such system is a class of Evolution Strategies which imitate the principles of natural evolution for parameter optimization problems (Rechenberg, Schwefel). Fogel’s Evolutionary Programming is a technique for searching through a space of small finite state machines. Glover’s Scatter Search technique maintains a population of reference points and generates offspring by weighting linear combinations. Another type of evolution based systems are Holland’s Genetic Algorithms (GAs). For the large spaces, special artificial intelligence techniques must be employed. Genetic algorithms are a class of general search methods, which strike a remarkable balance between exploration and exploitation of the search space.

Drag reduction is important in considering the aerodynamic aircraft design. There have been many suggestions for reducing the skin friction drag on bodies of revolution, including extension of laminar flow regions and relaminarization of turbulent flow. For the aerodynamic design of three-dimensional fuselages with low skin friction drag, laminar bodies of revolution are often used as a design basis.

Laminar flow on a two-dimensional or an axisymmetric body can be achieved by designing the geometry so that there is an extensive region of favorable pressure gradient. This technique is frequently referred to as natural laminar flow control and may be implemented on a body of revolution by changing the location of the maximum body thickness as far aft as possible. A favorable pressure gradient and laminar flow may be maintained over a large percentage of body length, but this causes the flow separation at the remaining part of the body, with consequent increase in pressure drag.

The objective of our study is to find the optimal body shape with the minimum drag coefficient for specified Reynolds number regimes. For this purpose, an optimization process must be employed and linked to the aerodynamics calculation program. The combined computational program must be able to search through several body shapes and to choose those shapes with long laminar flow without separation. We also prove that the laminar and turbulent boundary layer calculations and the determination of the transition location must be extremely accurate for getting acceptable optimal body shape.
The aerodynamics calculation of streamline bodies is usually started from potential flow around the body to find inviscid velocity distribution. A numerical calculation called Finite Volume Method (FVM) is employed for both laminar and turbulent boundary layer calculation and those body shapes which show separation in the boundary layer domain must be rejected. Laminar to turbulent transition location is determined by using natural transition criterion based on linear stability theory. The total drag coefficient can be calculated directly.

Numerical shape optimizations were performed first by Parsons et al. [1]. In Pinebrook’s study [2], the body geometry is not optimized in a direct method. Instead, a source and sink singularity distribution on the axis is used to model the body contour and to calculate the corresponding inviscid flow field. A statistical technique derived from Rechenberg’s evolution strategy was also applied. In the shape optimizations presented by Lutz–Wagner [3], a semi-empirical \( \varepsilon^n \) criterion based on the linear stability theory was applied to determine the natural transition location. Nejati and Matsuchi [4] improved the Pinebrook’s work by employing a powerful optimization procedure called Genetic Algorithms to find optimal shape with the global minimum drag coefficient.

2 Boundary layer flow

For the purpose of numerical shape optimization, high computational efficiency is required, and a numerical procedure, FVM, which is consistent to the geometry and physics of the problem, is applied for solving laminar and turbulent boundary layer equations for airship bodies. The coordinate system \((x, y)\) is curvilinear, where \(x\) is parallel to the body surface and \(y\) is normal to it. This is illustrated in Fig. 1.

By assumption that flow is steady incompressible and axisymmetric, the governing equations for turbulent boundary layer are given as

\[
\frac{\partial}{\partial x} ru + \frac{\partial}{\partial y} rv = 0
\]

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{1}{r} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} - \rho u'v' \right)
\]

where \(\mu\) is viscosity, \(r(x, y) = r_0(X) + y \cos \alpha(X)\) and \(r_0(X)\) is the body radius. It is worth to note that both longitudinal and transverse curvatures effects are considered in the formulation. The transverse curvature effects of solution domain become quite important when the radius of the body is small compared with the boundary layer thickness.

In order to model the Reynolds stress term \(-\rho u'v'\), one of the most frequently used two equation models \(k - \varepsilon\) is employed. According to Launder and Spalding [5], at high Reynolds numbers, the final form of the transport equations for the turbulence kinetic energy \(k\) and its dissipation rate \(\varepsilon\) can be derived and by using Mangler transformation the transport equations adequate for steady and axisymmetric turbulent boundary layer flow can be written as

\[
\frac{\partial k}{\partial x} + \frac{\partial k}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} + \mu \left( \frac{\partial u}{\partial y} \right)^2 - \rho \varepsilon \tag{3}
\]

\[
\frac{\partial \varepsilon}{\partial x} + \frac{\partial \varepsilon}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial y}
+ \mu_t C_{\varepsilon_1} \frac{\varepsilon}{k} \left( \frac{\partial u}{\partial y} \right)^2 - \rho C_{\varepsilon_2} \varepsilon^2 \tag{4}
\]

![Figure 1. Curvilinear coordinate system for boundary layer on a body surface](image)

where \(\mu_t\) is turbulence viscosity given as

\[
\mu_t = C_{\mu} \rho \frac{k^2}{\varepsilon}
\]

In above equations \(C_{\mu}, C_{\varepsilon_1}, C_{\varepsilon_2}, \sigma_k\) and \(\sigma_\varepsilon\) are model constants.
Since the boundary layer width is not uniform, the curvilinear \((x, y)\) system is replaced by the mildly non-orthogonal \((\xi, \eta)\) system. The lateral extent of the mesh is boundary layer thickness \(\delta(x)\), which must be computed as a part of the solution procedure.

3 Transition predication

The reliable and consistent transition prediction is of essential importance for a successful shape optimization of laminar bodies. For this purpose a natural transition prediction such as linear stability theory should be employed. However transition prediction based on a complete analysis of stability theory requires too much computational effort and laborious calculations for the purpose of numerical shape optimization. An alternative is offered in the form of a database method.

The influence of the pressure gradient can be expressed by the use of a shape factor of the velocity profile and if this tedious calculation is carried out once for any shape factor, then a useful database reference can be generated. In this study, the exact velocity profile for axisymmetric body shape is calculated with the FVM and the dimensionless shape factor introduced by K. Pohlhausen can be calculated directly. For the determination of the instability point the method proposed by Schlichting and Ulrich (see Schlichting [6]) is used, and the transition point is determined based on the diagram found by Granville [7].

4 Genetic Algorithms

There are several optimization processes such as Rechenberg’s Evolution Strategy (RES) and Genetic Algorithms (GA). GA can reach minimum objective function faster through a better path compared to the other optimization procedures such as RES. Since GA represents a special artificial intelligence technique for large spaces, it’s one of the best optimization methods for such multi-dimensional, multi-model and nonlinear objective functions [8].

Genetic algorithms use a vocabulary borrowed from natural genetics. We would talk about individuals (or genotypes, structures) in a population; quite often these individuals are called also strings or chromosomes. This might be a little bit misleading: each cell of every organism of a given species carries a certain number of chromosomes (man, for example, has 46 of them); however, here we talk about one-chromosome individuals only. Chromosomes are made of units genes (also features, characters, or decoders) arranged in linear succession; every gene controls the inheritance of one or several characters. Genes of certain characters are located at certain places of the chromosome, which are called loci (string positions). Any character of individuals (such as hair color) can manifest itself differently.

Each genotype (here a single chromosome) would represent a potential solution to a problem; an evolution process run on a population of chromosomes corresponds to a search through a space of potential solutions. Such a search requires balancing two (apparently conflicting) objectives: exploiting the best solutions and exploring the search space. Hillclimbing is an example of a strategy, which exploits the best solution for possible improvement; on the other hand, it neglects exploration of the search space. Random search is a typical example of a strategy, which explores the search space ignoring the exploitations of the promising regions of the space. Genetic algorithms are a class of general-purpose (domain independent) search methods, which strike a remarkable balance between exploration and exploitation of the search space.

For the study we have selected two genetic algorithm implementations differing only by representation and applicable genetic operators, and equivalent otherwise: The binary implementation and floating-point implementation. Such an approach gave us a better basis for a more direct comparison. Both implementations used the same selective mechanism: stochastic universal sampling.

In particular, for parameter optimization problems with variables over continuous domains, we may experiment with real coded genes together with special genetic operators developed for them. The main objective behind such implementations is (in line with the principle of evolution programming) to move the genetic algorithm closer to the problem space. Such a move forces, but also allows, the operators to be more problems specific by utilizing some specific characteristics of real space.

In floating point representation each chromosome vector is coded as a vector of floating point numbers of the same length as the solution vector. Each element is initially selected as to be within the desired domain, and the operators are carefully designed to preserve this constraint (there is no such problem in the binary representation, but the design of the operators is rather simple; we do not see that as a disadvantage; on the other hand, it provides for other advantages mentioned below). The precision of such an approach depends on the underlying machine, but is generally much better than that of the binary representing. Of course, we can always extend the precision of the binary representation by introducing more bits, but this considerably slows down the algorithm.

The operators we use are quite different from the classical ones, as they work in a different space (real valued).
However, because of intuitive similarities, we will divide them into the standard classes, mutation and crossover. In addition, some operators are non-uniform, i.e., their action depends on the age of the population.

4.1 Crossover group

4.1.1 Simple crossover

Simple crossover, defined in the usual way, but with the only permissible split points between \( v \)'s, for a given chromosome \( x \).

4.1.2 Arithmetical crossover

Arithmetical crossover is defined as a linear combination of two vectors: if \( s'_i \) and \( s'_w \) are to be crossed, the resulting offspring are \( s_{v}^{i+1} = a.s'_i + (1-a).s'_w \) \& \( s_{w}^{i+1} = a.s'_w + (1-a).s'_i \). This operator can use a parameter \( a \), which is either a constant (uniform arithmetical crossover), or a variable whose value depends on the age of population (non-uniform arithmetical crossover).

Here we have some new mechanisms to apply these operators; e.g., the arithmetical crossover may be applied either to selected elements of two vectors or to the whole vectors.

4.2 Mutation group

4.2.1 Uniform mutation

Uniform mutation, defined similarly to that of the classical version: if \( x'_i = (v_1,\ldots,v_n) \) is a chromosome, then each element \( v_k \) has exactly equal chance of undergoing the mutative process. The result of a single application of this operator is a vector \( (v_1,\ldots,v'_k,\ldots,v_n) \), with \( 1 \leq k \leq n \), and \( v'_k \) a random value for the domain of the corresponding parameter domain.

4.2.2 Non uniform mutation

Non uniform mutation is one of the operators responsible for the fine tuning capabilities of the system. It is defined as follows: the non uniform mutation operator was defined as follows: if \( s'_i = (v_1,\ldots,v_m) \) is a chromosome and the element \( v_k \) was selected for this mutation (domain of \( v_k \) is \([I_k,u_k]\)), the result is a vector \( s_{v}^{i+1} = (v_1,\ldots,v_k,\ldots,v_m) \), with \( k \in \{1,\ldots,n\} \), and

\[
v_k = \begin{cases} v_k + \Delta(t,u_k-v_k) & \text{if random digit is } 0, \\ v_k - \Delta(t,v_k-l_k) & \text{if random digit is } 1, \\ \end{cases}
\]

where the function \( \Delta(t,y) \) returns a value in the range \([0,y]\) such that the probability of \( \Delta(t,y) \) being close to 0 increases as \( t \) increase. This property causes this operator to search the space uniformly initially (when \( t \) is small), and very locally at later stages. We have used the following function:

\[
\Delta(t,y) = y.\left(1-r^{\frac{1}{(1-t/T)}}\right),
\]

where \( r \) is a random number from \([0,1]\), \( T \) is the maximal generation number, and \( b \) is a system parameter determining the degree of non uniformity.

Moreover, in addition to the standard way of applying mutation we have some new mechanisms: e.g., non uniform mutation is also applied to a whole solution vector rather than a single element of it, causing the whole vector to be slightly slipped in the space.

5 Results and Discussion

Pinebrook [2] applied Rechenberg's Evolution Strategy for the optimization of the airship bodies. For one example with a certain initial set of singularity elements, the minimum drag after 9600 generations was reported as 0.0273. We also carried out this example by employing the available computational program, but it could run only up to 400 generations. The minimum drag was compared with our result obtained from genetic algorithms for the same number of generations (also with the same initial parameters and Reynolds number). Fig. 2 shows that the best drag coefficient for evolution strategy is 0.0283 whereas the minimum drag in genetic algorithms is 0.0234, which is better, even than the reported result. This is because Rechenberg's evolution strategy is a type of random search, which explores the search space ignoring the exploitation. To show that GA has a remarkable balance between exploration and exploitation a 3-dimensional demonstration of reduction of drag coefficient for an example is plotted in Fig. 2 and it indicates clearly how GA exploits the best solution.

In the aerodynamics model presented by Nejati–Matsuuchi [4], the integral method and empirical relations are used to calculate the laminar and turbulent boundary layers. For simplicity, we call this model IM-FT, which is abbreviation of Integral Method and Forced Transition criterion. In this
study a new model is proposed. To improve the boundary layer calculation, IM together with the empirical relations is substituted with a powerful numerical method FVM. Furthermore the transition prediction is also modified and the natural transition criterion is added to aerodynamics calculations to get better estimation of the transition location. This model is abbreviated as FVM-NT which is referred to Finite Volume Method and Natural Transition.

![Drag Curve](image1)

**Fig. 2 Comparison of drag curves**

The aerodynamics calculation is performed for a specified body shape with a volumetric Reynolds number \( \text{Re}_v = 9.39 \times 10^5 \). The boundary layer calculations using FVM are carried out for this typical body shape. The natural transition point in this calculation is found to occur at 45% of the body length. The free-stream velocity distribution and the mean velocity profiles for some stages on the body surface are plotted in Fig. 3. The velocity profiles indicate laminar form up to 45% of the body length and exactly before the transition point \( E \), flow approaches the laminar separation but transition occurs and flow becomes turbulent. It can be seen that around 50% of the body length, in point \( F \), the flow becomes laminar one which means relaminarization. Downstream of this point, flow stays turbulent and near the end of the body at point \( I \) flow is ready to separate, but flow separation is avoided.

![Free-Stream Velocity Distribution](image2)

**Figure 3. Free-stream velocity distribution and mean velocity profiles for specified body shape**

To show the natural transition location effects on the drag coefficient in the various ranges of the Reynolds number another example is considered for a specified body shape used by previous investigators. For this example the airship body R101 (see Lutz–Wagner [3]) shown in Fig. 4 is taken into account, and present FVM-NT model is used to determine the drag coefficient. By increasing the free-stream velocity the Reynolds number is increased. As a validation aspect, the drag curve calculated by other investigators for this airship body is also given in Fig. 4. The experimental investigations for this shape were conducted independently by Jones and Schirmer. Schirmer performed his measurements in the wind tunnel of the former Zeppelin Company. To find the drag coefficient for this example, Lutz and Wagner used the integral method with semi-empirical \( e^\alpha \) method. The calculated drag coefficient in this study shows satisfactory agreement with the experimental results, but in higher Reynolds number there occurs a turbulent separation almost near the end of the body and induce higher drag coefficient.

The shape optimization program is coupled with two different aerodynamics models, IM-FT and FVM-NT, in order to show the importance of aerodynamics calculation for achieving the optimal body shape. Lutz–Wagner [3] suggested five design regimes for the Reynolds number as shown in table 1. In the present research, we also carried out the optimization process for these five design regimes...
to compare our results with those obtained by Lutz–Wagner.

![Drag Coefficient for Airship Body R101](image)

**Table 1 Design regimes**

<table>
<thead>
<tr>
<th>Regime</th>
<th>Re (min)</th>
<th>Re (max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.00E+06</td>
<td>3.16E+06</td>
</tr>
<tr>
<td>II</td>
<td>3.16E+06</td>
<td>1.00E+07</td>
</tr>
<tr>
<td>III</td>
<td>1.00E+07</td>
<td>3.16E+07</td>
</tr>
<tr>
<td>IV</td>
<td>3.16E+07</td>
<td>1.00E+08</td>
</tr>
<tr>
<td>V</td>
<td>1.00E+08</td>
<td>3.16E+08</td>
</tr>
</tbody>
</table>

Several optimization processes with IM-FT and FVM-NT models are carried out to find the optimal body shape with the minimum drag coefficient in each design regime. The minimum drag coefficient curves related to these two models are depicted in Fig. 5 and the results of Lutz–Wagner using the integral method with the semi-empirical $e^n$ method are also drawn for comparison. Using FTC at 3% of the body length in IM-FT model gives bigger drag coefficient and has different drag curve.

**Fig. 5 Drag curve for range of $Re_y$**

The drag coefficient obtained by FVM-NT model has a curve much similar to that obtained by Lutz–Wagner. However in low Reynolds number we get bigger drag coefficient because the transition in the optimal body shapes occurs earlier. In Lutz–Wagner results for low Reynolds numbers, the maximum radius position is located very far from the body nose. This means the existence of long favorite pressure gradient and long laminar flow on the body surface. As a result, much lower drag coefficient can be achieved.

### 6 Conclusions

The Finite Volume Method for boundary layer calculation is employed in order to improve the drag reduction process and to obtain the reliable optimal body shapes. We summarize our study as follows:

1) Since the Integral Method gives no velocity profile in the boundary layer domain, the drag coefficient cannot be found directly and is calculated with somewhat poor accuracy. On the other hand, in the Finite Volume Method the drag coefficient is obtained directly from the boundary layer solution. In the Integral Method the separation of the turbulent flow is judged according to the value of the shape factor and therefore the results showing no separation may be unreliable. In contrary to the Integral Method, the Finite Volume Method can estimate the separation correctly because it evaluates the velocity profiles. Furthermore the transverse curvature effects are neglected in the Integral Method. The Finite Volume Method can take these effects into account completely.

2) The laminarization plays an important role in the shape optimization. It is concluded that the natural transition criterion should be used to estimate correctly the region of
natural laminar flow. As a result, the body geometry changes as the location of the maximum body thickness moves as far aft as possible.

3) Since Genetic Algorithms strike a remarkable balance between exploration and exploitation of search space, it can reach to minimum objective function faster through a better path, whereas Rechenberg’s evolution strategy is a random search type method ignoring the exploitation. Comparing the two methods, it is concluded that for large spaces, with multi-dimensional, multi-modal and nonlinear objective functions, GA is more accurate.

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8 References


