From Glider to Chaos: A Transitive Subsystem Derived From Glider $\bar{\mathrm{B}}$ of CA Rule 110

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Abstract— *Rule 110, a member of Wolfram's class IV and Chua's hyper Bernoulli-shift rules, has been proved to be capable of supporting varieties of mobile self-localizations referred as gliders. This paper is devoted to a careful study for glider $\bar{B}$ of rule 110 from the viewpoint of symbolic dynamics. A transitive subsystem is revealed based on existing glider $\bar{B}$, and its complex dynamics such as having positive topological entropy and density of periodic points are proved. These results therefore suggest that rule 110 is chaotic in the sense of both Li-Yorke and Devaney.*

Keywords: cellular automata; glider; directed graph; symbolic dynamics; topologically transitive; chaos

1. Introduction

Cellular automata (CA) are a class of spatially and temporally discrete, deterministic mathematical systems characterized by local interactions and an inherently parallel form of evolution, and able to produce complex dynamical phenomena by means of designing simple local rules [1]. The study of topological dynamics of CA began with Hedlund in 1969, who viewed one-dimensional CA (1-D CA) in the context of symbolic dynamics as endomorphisms of the shift dynamical system [2], where the main results are the characterizations of surjective and open CA. In 1970, Conway proposed his now-famous "*Game of Life*" [3], which received widespread interests among researchers in different fields. In the early 1980's, Wolfram introduced space-time representations of 1-D CA and informally classified them into four classes by using dynamical concepts like periodicity, stability and chaos [4, 5]. In 2002, he introduced his monumental work *A New Kind of Science* [6]. Based on this work, Chua *et al*. provided a nonlinear dynamics perspective to Wolfram's empirical observations via the concepts like characteristic function, forward time-τ map, basin tree diagram and Isle-of-Eden digraph [7-10].

Rule 110 in Stephen Wolfram's system of identification [6] has been an object of special attention due to the structures or gliders which have been observed in evolution space from random initial conditions. A list of gliders is presented in [12]. Wolfram established the conjecture that rule 110 could perform universal computation [13]. Lindgren and Nordahl around 1992 studied the transitional role of rule 110 and its relation with class IV rules sitting between Wolfram's classes II and III [14]. In 1999, Cook gave a brief introduction about the complex activity of gliders [12], and made a comparison between rule 110 and "*Game of Life*", finding some similarities and suggesting to call it as left life. Further, he demonstrated that rule 110 is universal via simulating a novel cyclic tag system (CTS) [12, 15] with well-defined blocks of gliders by means of collisions.

Gratefully, the research of CA has drawn more and more scientists' attention in the last 20 years. Many concepts of topological dynamics have been used to describe and classify them [11, 16-19]. The dynamical properties of some robust Bernoulli-shift rules with distinct parameters have been studied in the bi-infinite symbolic sequence space [20-22]. Based on Chua's classes, Bernoulli-shift pattern for rule 110 changes with length, and Bernoulli-shift dynamics of some gliders parameters are relatively large, so it's difficult to investigate their invariant subsystems according to references before. And some gliders with small parameters may not have chaotic subsystems. Thus, this work extends the investigation of Bernoulli-shift dynamics of glider $\bar{B}$ of rule 110, and reveals its complex dynamics under the framework of bi-infinite symbolic sequence space.

The rest of the paper is organized as follows: Section 2 presents the basic concepts of 1-D CA, symbolic dynamics and glider. Section 3 identifies the subsystem of glider $\bar{B}$, and shows the chaotic dynamics of the subsystem. It is indeed remarkably that rule 110 is topologically transitive, possesses positive topological entropy and has a dense periodic set. Therefore, it is chaotic in the sense of both Li-Yorke and Devaney. Finally, Section 4 highlights the main results of this work.

2. Symbolic Dynamics and Glider

2.1 Symbolic sequence space and CA

Let $S = \{0, 1\}$, and $\Sigma_2 = \{x = (\cdots, x_{-1}, \overset{*}{x}_0, x_1, \cdots) | x_i \in S,\ i \in Z\}$ with distance "*d*": $d(x, y) = \sup\left\{\frac{\rho(x_i,y_i)}{2^{|i|}} \mid i \in Z\right\}$, where

$$\rho(x_i, y_i) = \begin{cases} 1 & \text{if } x_i \neq y_i \\ 0 & x_i = y_i \end{cases} \qquad (1)$$

It is known that Σ_2 is a compact, perfect, and totally disconnected metric space.

If $x \in \Sigma_2$ and $I = [i, j]$ is an interval of integers, put $x_{[i,j]} = (x_i, x_{i+1}, \cdots, x_j)$ $(i < j)$, $x_{[i,j)} = (x_i, \cdots, x_{j-1})$.

Table 1: Truth table of local function of rule 110

(x_{i-1}, x_i, x_{i+1})	$\hat{f}_{110}(x_{i-1}, x_i, x_{i+1})$
(0, 0, 0)	0
(0, 0, 1)	1
(0, 1, 0)	1
(0, 1, 1)	1
(1, 0, 0)	0
(1, 0, 1)	1
(1, 1, 0)	1
(1, 1, 1)	0

Let $x_{(-\infty,i]} = (\cdots, x_{i-1}, x_i)$ and $x_{[j,+\infty)} = (x_j, x_{j+1}, \cdots)$ denote the left and right half infinite sequence, respectively. For a finite sequence $a = (a_0, \cdots, a_{n-1})$, if there exists an $m \in Z$ such that $x_{m+k} = a_k$ $(k = 0, 1, \cdots, n-1)$, then a is said to be a subsequence of x, denoted by $a \prec x$. The left-shift map σ_L and right-shift map σ_R are defined by $\forall x \in \Sigma_2$, $[\sigma_L(x)]_i = x_{i+1}$ and $[\sigma_R(x)]_i = x_{i-1}$, respectively, where $[\sigma_L(x)]_i$ $([\sigma_R(x)]_i)$ stands for the i-th element of $\sigma_L(x)$ $(\sigma_R(x))$.

By a theorem of Hedlund [2], a map $f : \Sigma_2 \to \Sigma_2$ is a CA iff it is continuous and commutes with σ, where σ is left-shift map σ_L or right-shift map σ_R. Furthermore, if f is a CA, then there exists a radius $r \geq 1$ and a local function $\hat{f} : S^{2r+1} \to S$ such that $[f(x)]_i = \hat{f}(x_{[i-r,i+r]})$. If $r = 1$, then f is an elementary CA (ECA). Any CA f defines a dynamical system (Σ_2, f). A subset $X \subseteq \Sigma_2$ is f-invariant if $f(X) \subseteq X$, and strongly f-invariant if $f(X) = X$. If X is a closed and f-invariant, then (X, f) or simply X is called a subsystem of (Σ_2, f).

Each ECA rule can be expressed by a local function, the logical truth table of rule 110's local function $\hat{f}_{110}$ is shown in Table 1. Obviously, the output binary sequence of the rule is 01101110, its decimal number is $N = 110$.

2.2 Glider $\bar{B}$

A glider is a compact group of non-quiescent states traveling along CA lattice, and is a periodic structure moving in time [15, 25-27]. From the viewpoint of symbolic dynamics, a glider can be defined as the evolutionary orbit starting from a special initial configuration in the bi-infinite sequence space Σ_2 [28].

It was known that the speed of glider $\bar{B}$ of rule 110 is $-6/12$, the lineal volume is 22, and the even number of periodic margin on the left (or right) border in the ether pattern are 3 [26]. In symbolic sequence space, the ether pattern and glider can be defined as the evolutionary orbit starting from a special initial configuration [28]. The ether factor of the ether patterns e_r (or e_l) is $a = (1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0)$, and one of glider factors of $\bar{B}$ is b=(1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0), i.e., an ether pattern of rule 110 is the evolutionary orbit $Orb_{f_{110}}(a^*) = \{a^*, f_{110}(a^*), f_{110}^2(a^*), \cdots\}$ and glider $\bar{B}$ is the evolutionary orbit $Orb_{f_{110}}(\bar{x}) = \{\bar{x}, f_{110}(\bar{x}), f_{110}^2(\bar{x}), \cdots\}$ in the CA lattice space, where $a^* = (\cdots, a, a, a, \cdots)$ is a cyclic configuration and $\bar{x} = (\cdots, a, a, a, b, a, a, a, \cdots)$. That the speed of $\bar{B}$ is $-6/12$ implies this glider shifts to left by 6 bits in every 12 iterations under rule 110, i.e., $f_{110}^{12}(\bar{x}) = \sigma_L^6(\bar{x})$. The ether pattern and glider $\bar{B}$ are shown in Figure 1.

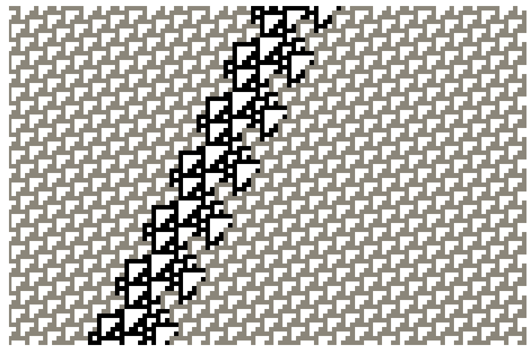

Fig. 1: Ether pattern and glider $\bar{B}$ of rule 110.

3. Subsystem Derived From Glider $\bar{B}$

3.1 Shift of finite type

Based on glider $\bar{B}$, one can obtain a 25-sequence set $\mathcal{B} = \{q | q = \bar{x}_{[i,i+24]}, \forall i \in Z\}$ from $\bar{x} = (\cdots, a, a, a, b, a, a, a, \cdots)$, where $a = (1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0)$ is the ether factor and $b = (1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0)$ is a glider factor of $\bar{B}$. More specifically, $\mathcal{B} =$
{0000111100100110111110001, 0001001101111100010011011,
0001001101111110000111100, 0001111001001101111100010,
0010011011111000100110111, 0010011011111100001111001,
0011011111000100110111110, 0011011111000100110111111,
0011011111100001111001001, 0011110010011011111000100,
0100110111110001001101111, 0100110111111000011110010,
0110111110001001101111100, 0110111110001001101111110,
0110111111000011110010011, 0111100100110111110001001,
0111110001001101111100010, 0111110001001101111110000,
0111111000011110010011011, 1000011110010011011111000,
1000100110111110001001101, 1000100110111111000011110,
1001001101111100010011011, 1001101111100010011011111,
1001101111110000111100100, 1011111000100110111110001,
1011111000100110111111000, 1011111100001111001001101,
1100001111001001101111100, 1100010011011111000100110,
1100010011011111100001111, 1100100110111110001001101,
1101111100010011011111000, 1101111100010011011111100,
1101111110000111100100110, 1110000111100100110111110,
1110001001101111100010011, 1110001001101111110000111,
1110010011011111000100110, 1111000011110010011011111,
1111000100110111110001001, 1111000100110111111000011,
1111001001101111100010011, 1111100001111001001101111,
1111100010011011111000100, 1111100010011011111100001,
1111110000111100100110111}.

The decimal code set $D(\mathcal{B})$ of $\mathcal{B}$ is $D(\mathcal{B}) =$ {1986033, 2554011, 2554940, 3972066, 5108023, 5109881, 7309758, 7309759, 7324617, 7944132, 10216047, 10219762, 14619516, 14619518, 14649235, 15888265, 16292834, 16292848, 16530587, 17770232, 18054221, 18054686, 19331227, 20432095, 20439524, 24923633, 24923640, 25042509, 25662332, 25804326, 25804559, 26442829, 29239032, 29239036, 29298470, 29608382, 29679379, 29679495, 29998630, 31581407, 31616905, 31616963, 31776531, 32567919, 32585668, 32585697, 33061175}.

Let $\Lambda_0 = \Lambda_{\mathcal{B}} = \{x \in \Sigma_2 | \ x_{[i,i+24]} \in \mathcal{B}, i \in Z\}$. Since the 12 times iteration of the local function $\hat{f}_{110}$ is a map $\hat{f}^{12}_{110} : S^{25} \to S$, and obviously, $\hat{f}^{12}_{110}(q) = q_{i+6}$ for any $q = (q_{i-12}, \cdots, q_i, \cdots, q_{i+12}) \in \mathcal{B}$. Thus, it follows that $f^{12}_{110}(x) = \sigma^6_L(x)$ for $x \in \Lambda_0$. Furthermore, let

$$\Lambda = \bigcup_{i=0}^{11} f^i_{110}(\Lambda_0) \tag{2}$$

The following propositions can be easily verified.

Proposition 1: Λ is closed f_{110}-invariant set, and $f^{12}_{110}(x) = \sigma^6_L(x)$ for $x \in \Lambda$.

Proposition 2: Λ is a subshift of finite type of σ_L (SFT).

Let $\mathcal{A}$ is a determinative block system of Λ, then, $\Lambda = \Lambda_{\mathcal{A}}$, where $\mathcal{A}$ is a 25-sequence set consisting of 435 elements. Due to space limitations and for simplicity, the decimal code set $D(\mathcal{A})$ of $\mathcal{A}$ is placed in Appendix.

3.2 de Bruijn diagram

The invariant set $\Lambda = \Lambda_{\mathcal{A}}$ in Propositions 1 and 2 can be described by a finite directed graph, $G_{\mathcal{A}} = \{\mathcal{A}, E\}$, where each vertex is labeled by a sequence in $\mathcal{A}$, and E is the set of edges connecting the vertices in $\mathcal{A}$. The finite directed graph $G_{\mathcal{A}}$ is called the de Bruijn diagram of $\mathcal{A}$ (or $\Lambda_{\mathcal{A}}$). Two vertices $a = (a_0, \cdots, a_{24})$ and $b = (b_0, \cdots, b_{24})$ are connected by an edge $(a_0, \cdots, a_{24}) \to (b_0, \cdots, b_{24})$ if and only if $(a_1, \cdots, a_{24}) = (b_0, \cdots, b_{23})$. One can think of each element of $\Lambda_{\mathcal{A}}$ as a bi-infinite path on the diagram $G_{\mathcal{A}}$. Whereas a de Bruijn diagram corresponds to a square transition matrix $A = (A_{ij})_{435\times435}$ with $A_{ij} = 1$ if and only if there is an edge from vertex $b^{(i)}$ to vertex $b^{(j)}$, where $|\mathcal{A}| = 435$ is the number of elements in $\mathcal{A}$, and i (or j) is the code of the corresponding vertex in $\mathcal{A}$, $i, j = 1, 2, \cdots, 435$. Thus, $\Lambda_{\mathcal{A}}$ is precisely defined by the transition matrix A. The de Bruijn diagram $G_{\mathcal{A}}$ associated with $\mathcal{A}$ is shown in Fig. 2, where the code of vertex represents the location of corresponding element in $\mathcal{A}$.

Remarkably, a $0-1$ square matrix A is irreducible if for any i, j, there exists an positive integer n such that $A^n_{ij} > 0$; aperiodic if there exists an $n > 0$, such that $A^n_{ij} > 0$ for all i, j, where A^n_{ij} is the (i, j) entry of the power matrix A^n. If $\Lambda_{\mathcal{A}}$ is a SFT of (Σ_2, σ), then σ is topologically transitive on $\Lambda_{\mathcal{A}}$ if and only if A is irreducible; σ is topologically mixing if and only if A is aperiodic. Equivalently, A is irreducible if and only if for every ordered pair of vertices $b^{(i)}$ and $b^{(j)}$

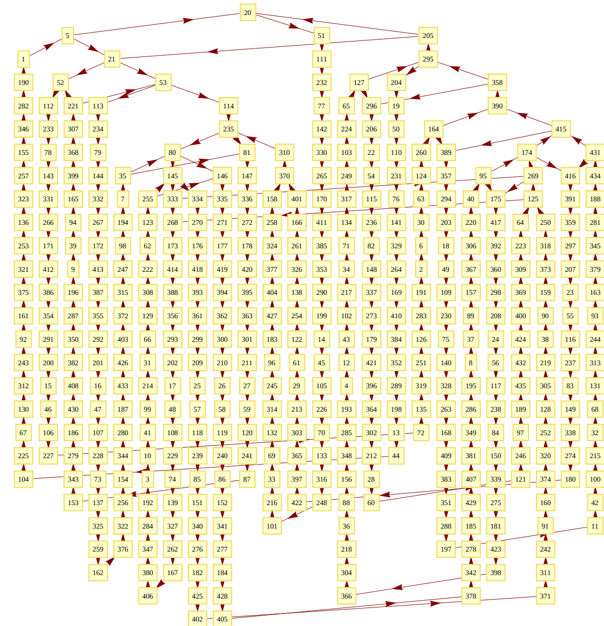

Fig. 2: The de Bruijn diagram $G_{\mathcal{A}}$ associated with glider $\bar{B}$.

there is a path in $G_{\mathcal{A}}$ starting at $b^{(i)}$ and ending at $b^{(j)}$ [23, 24].

3.3 Chaoticity

In the subsection, the chaoticity of rule 110 on $\Lambda_{\mathcal{A}}$ will be revealed.

Proposition 3:
(1) σ_L is topologically transitive on Λ;
(2) f_{110} is topologically transitive on Λ.

Proof: (1) In fact, it can be verified that for every ordered pair of vertices $b^{(i)}$ and $b^{(j)}$ in $\mathcal{A}$ there is a path in the de Bruijn diagram $G_{\mathcal{A}} = \{\mathcal{A}, E\}$ starting at $b^{(i)}$ and ending at $b^{(j)}$, thus, the transition matrix $A = (A_{ij})_{435\times435}$ corresponding to $G_{\mathcal{A}}$ is irreducible, so σ_L is topologically transitive on Λ [23, 24].
(2) Similar to [29], the topologically transitive of f_{110} on Λ can be proved. ■

Proposition 4: The set of periodic points of f_{110}, $P(f) = \{y \in \Lambda | \ \exists \ n > 0, f^n(y) = y\}$, is dense in Λ.

Proof: For any $x \in \Lambda$ and $\epsilon > 0$, there exists an positive integer M (> 12) such that $\sum_{i=M+1}^{\infty} \frac{1}{2^i} < \epsilon/2$, and for $(a_0, \cdots, a_{2M}) = x_{[-M,M]} \prec x \in \Lambda$, it follows that $(a_{2M-24}, \cdots, a_{2M})$, $(a_0, \cdots, a_{24}) \in \mathcal{A}$. Since σ is topologically transitive on Λ, there exists a path from $(a_{2M-24}, \cdots, a_{2M})$ to $(a_0, \cdots, a_{24})$ in $G_{\mathcal{A}} = \{\mathcal{A}, E\}$. Let $\tilde{b} = (a_{2M-24}, \cdots, a_{2M}, b_0, \cdots, b_{k_0}, a_0, \cdots, a_{24})$ be the sequence corresponding to this path. Then, its any 25-subsequence belongs to $\mathcal{A}$.

Now, construct a cyclic configuration $y = c^* = (\cdots, c, c, c, \cdots)$, where $c = (a_0, \cdots, a_{2M}, b_0, \cdots, b_{k_0})$. Obviously, $y \in \Lambda$ and $\sigma^m(y) = y$, where $m = |c|$ is the length of c. Thus, $f^{12m}(y) = \sigma^{6m}(y) = y$ and $y_{[-M,M]} = x_{[-M,M]}$, i. e., y is a periodic point of f_{110} and $d(x, y) < \epsilon$. Therefore, the set of periodic points $P(f)$ is dense in Λ. ∎

Proposition 5: The topological entropy of f_{110} is positive.

Proof: The topological entropy of $f_{110}|_\Lambda$ satisfies $ent(f_{110}) \geq ent(f_{110}|_\Lambda) = \frac{6}{12} ent(\sigma_L|_\Lambda) = \frac{1}{2} log((A)) = \frac{1}{2} log(1.07466) \approx 0.0360 > 0$, where (A) is the spectral radius of the transition matrix A corresponding to $\mathcal{A}$. ∎

It is well known that positive topological entropy implies chaos in the sense of Li-Yorke [24], and topological transitivity and density of periodic points imply chaos in the sense of Devaney [30, 31]. Thus, one has the interesting result.

Theorem 1: f_{110} is chaotic in the sense of both Li-Yorke and Devaney on Λ.

4. Conclusion

In this work, we uncover some dynamic properties of glider $\bar{B}$ of rule 110. That is, glider $\bar{B}$ defines one subsystem, on which it is topologically transitive and possesses positive topological entropy and the density of periodic points. Hence, the rule is chaotic in the sense of both Li-Yorke and Devaney on the subsystem. One important problem is how to find the biggest subsystem associating with an existing glider and how to find the relation between universal computation and dynamics of CA, which are important topics for further research in the near future.

Acknowledgments

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Appendix

The determinative block system $\mathcal{A}$ of Λ is a 25-sequence set consisting of 435 elements. For a sequence

$a = (a_0, a_1, \cdots, a_{24}) \in \mathcal{A}$, its decimal code is defined as

$$D(a) = \sum_{i=0}^{24} a_i \cdot 2^{24-i}. \tag{3}$$

For simplicity, $\mathcal{A}$ is replaced by its decimal code set $D(\mathcal{A})$. $D(\mathcal{A}) =$
{638502, 771295, 1152760, 1156646, 1277005, 1542590, 1827439, 1866829, 1986033, 2305521, 2309112, 2313293, 2520800, 2526459, 2539896, 2551290, 2553926, 2553996, 2554009, 2554010, 2554011, 2554015, 2554039, 2554081, 2554328, 2554864, 2554940, 2618104, 3008248, 3085180, 3333907, 3391369, 3403375, 3458271, 3654879, 3688088, 3733659, 3864731, 3972066, 4073208, 4611042, 4618225, 4626587, 5041601, 5052919, 5079793, 5102580, 5107852, 5107993, 5108019, 5108020, 5108022, 5108023, 5108031, 5108079, 5108162, 5108657, 5109729, 5109881, 5236209 6016497, 6021743, 6170360, 6230908, 6451081, 6667814, 6778377, 6782739, 6806751, 6868921, 6916542, 7083913, 7266209, 7308389, 7309518, 7309725, 7309731, 7309744, 7309755, 7309758, 7309759, 7309820, 7310201, 7310865, 7314830, 7323406, 7324617, 7376177, 7467319, 7729463, 7748344, 7790747, 7860535, 7944132, 8146417, 8244721, 8335241, 8845467, 9222084, 9236450, 9239451, 9253175, 10001378, 10083202, 10105838, 10159586, 10205160, 10215705, 10215987, 10216039, 10216040, 10216044, 10216046, 10216047, 10216063, 10216158, 10216324, 10217315, 10219459, 10219762, 10472418, 12032994, 12043487, 12340721, 12461816, 12679323, 12902163, 13066003, 13335629, 13556755, 13565478, 13613502, 13737843, 13833084, 14167827, 14422908, 14532419, 14577545, 14616779, 14619037, 14619451, 14619462, 14619488, 14619510, 14619516, 14619517, 14619518, 14619641, 14620403, 14621731, 14629661, 14646812, 14649235, 14655934, 14719970, 14752354, 14934639, 15136923, 15458927, 15496689, 15581495, 15596059, 15721071, 15808452, 15888265, 15957069, 16270939, 16289007, 16292315, 16292403, 16292613, 16292791, 16292833, 16292834, 16292835, 16292839, 16292847, 16292848, 16293835, 16299931, 16310554, 16373995, 16489442, 16511203, 16529656, 16530587, 16584177, 16656120, 16670483, 17096467, 17162863, 17353596, 17355539, 17690935, 17710630, 17770232, 17931772, 18037616, 18040445, 18047164, 18052861, 18054179, 18054214, 18054220, 18054221, 18054223, 18054235, 18054256, 18054380, 18054648, 18054686, 18086268, 18281340, 18444169, 18472900, 18478903, 18506351, 18621260, 18709581, 18813820, 19331227, 19788087, 19892670, 20002756, 20166404, 20211676, 20319172, 20410320, 20431410, 20431975, 20432078, 20432081, 20432088, 20432093, 20432095, 20432126, 20432316, 20432648, 20434631, 20438919, 20439524, 20651388, 20672589, 20707483, 20899576, 20944836, 21199949, 21396941, 21777905, 23008124, 23116877, 23310217, 23988670, 24065988, 24086974, 24105183, 24137201, 24345677, 24575245, 24681442, 24755750, 24912685, 24921719, 24923373, 24923417, 24923522, 24923611, 24923632, 24923633, 24923635, 24923639, 24923640, 24924133, 24927181, 24932493, 24964213, 25032817, 25042044, 25042509, 25069304, 25105276, 25325449, 25358647, 25454014, 25454985, 25632531, 25662332, 25743102, 25796024, 25797438, 25800798, 25803646, 25804305, 25804323, 25804326, 25804327, 25804333, 25804344, 25804406, 25804540, 25804559, 25820350, 25917886, 26087846, 26132006, 26184126, 26442829, 26671259, 26723551, 26993263, 27102910, 27113510, 27130957, 27227004, 27377190, 27475686, 27666168, 28281278, 28335654, 28432324, 28771551, 28829807, 28845816, 28950054, 29064838, 29155091, 29233558, 29238075, 29238902, 29238924, 29238977, 29239021, 29239032, 29239033, 29239035, 29239036, 29239282, 29240806, 29243462, 29259322, 29293624, 29298238, 29298470, 29311868, 29329854, 29439940, 29504223, 29504708, 29593481, 29608382, 29648767, 29675228, 29675935, 29677615, 29679039, 29679368, 29679377, 29679379, 29679382, 29679388, 29679419, 29679486, 29679495, 29687391, 29736159, 29821139, 29869279, 29998630, 30138991, 30273847, 30328671, 30465811, 30917855, 30993378, 31162991, 31192119, 31252243, 31426335, 31442143, 31529327, 31573956, 31581407, 31601599, 31614830, 31615183, 31616023, 31616735, 31616900, 31616904, 31616905, 31616907, 31616910, 31616925, 31616959, 31616963, 31620911, 31645295, 31687785, 31776531, 31846711, 31914139, 31941551, 32010121, 32403337, 32490383, 32541879, 32564194, 32567919, 32578015, 32584631, 32584807, 32585227, 32585583, 32585666, 32585668, 32585669, 32585671, 32585678, 32585695, 32585697, 32587671, 32599863, 32621108, 32700571, 32747991, 32782276, 32978884, 33022407, 33059313, 33061175, 33070050, 33127501, 33168354, 33312241, 33340966}.