THE METHOD OF SOLUTION OF OPTIMIZATION PROBLEMS

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Abstract - This article describes the method of mathematical and program solution of optimization problems. An approach of choosing an effective method of solution is shown.

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The problems of distribution of resources can be reduced to problems of conditional or unconditional optimization [1-2]. Further, it is possible to apply methods of optimization to these problems. The methods of optimization are described, in particular, in works [3-11] (see the references below).

Let's consider the following problem:

\[ F(x) \rightarrow \min \]

\[ x \in E^n, \]

\[ c_s \leq x_s \leq d_s, \quad s = 1, \ldots, n; \]

\[ q_v(x) \geq 0; \quad v = 1, \ldots, t_1; \]

\[ r_k(x) = 0; \quad k = 1, \ldots, t_2 \]

Where the functions

\[ F(x), q_v(x), r_k(x) \]

are convex.

When solving mathematical problems with an unexplored criterion function we face a problem of a choice of a concrete method of solution. The method chosen at random can converge very slowly to a point of a minimum of criterion function or not give a result at all. It is not always possible to receive characteristics of criterion function (convexity, gully, etc.). Therefore, when choosing a method, it is difficult to apply the heuristic reasons based on such characteristics.

Thus, a problem arises when trying to choose an effective method of solving a problem. This process can be automated.

The considered problem can be solved with a help of a computer system, which allows, depending on the information received during the search of the solution, to change not only methods of optimization, but also to replace values of parameters of a method on which the efficiency of methods also depends in many respects. Thus, it is possible to apply multiple methods to solve the problem without stopping the process of solution. The most effective algorithm is being chosen automatically at each stage of solving the problem.

The following approach of solving optimization problems lays in the basis of a system. We shall designate

\[ P_1, P_2, \ldots, P_n \]

- methods of the optimization, included in the system.

Various methods, based on characteristics, are included in the system, because the criterion function can have a complex structure.

The problem-oriented list of methods

\[ P_{k_1}, P_{k_2}, \ldots, P_{k_s} \]

is formed from a set of methods that are in the system, if any information is known about the criterion function (for example if it is known, that it is convex, gullied, square-law, etc.). The solution of a problem consists of steps. The most effective method from the list is being revealed on each step, and then the solution of a problem with the help of this method. The certain time intervals are allocated for revealing and solving the problem. The time, which was allocated for revealing the effective method, is used for promotion to a point of a minimum, because the current point is used during solving the problem. It allows to save time for solving the problem.

At a stage of finding of an effective method to all methods from the list
it is enabled to solve a problem during the allocated time interval $\Delta t$. After all methods had an opportunity to solve a problem during the allocated time, search of an effective method stops, gets out the most effective method of the list $P_{k_1}, P_{k_2}, \ldots, P_{k_s}$.

The size $\Delta t$ depends on time, spent by a computer on one calculation of criterion function and amount of calculation of criterion function on one iteration, and is calculated as follows:

$$\Delta t = \alpha \mu,$$

Where $\mu$ - time spent for one calculation of criterion function,

$$\alpha = \max_r \alpha_{k_r},$$

Where $\alpha_{k_r}$ - amount of calculations of criterion function on one iteration by a method $P_{k_r}$.

Thus, at this stage comes out the most effective method.

At the following stage the most effective method is used for solving the problem. The time allocated for solving a problem, is calculated as follows.

$$\tau_0 = \nu \Delta t, \quad \nu > 1,$$

$$\tau_s = q_s \tau_{s-1} + \tau_0.$$

The size $\nu$ is taken a priori, and depends on complexity and dimension of criterion function. The size

$$q_s = 1$$

if the same one method appeared as the most effective on two consecutive steps. If not, then we take

$$q_s = 0.$$

It means that the solution of a problem proceeds, if any method appeared effective on several consecutive steps.

Efficiency $E_i$ of a method $P_i$ is calculated based on the formula:

$$E_i = \frac{|F(x^k) - F(x^{k+1})|}{|F(x^k)| + \delta},$$

Where $x^k, x^{k+1}$ - is an initial and final points, when using a method $P_i$;

$F(x^k), F(x^{k+1})$ - values of criterion function in these points;

$\delta$ - is a small positive number. Efficiency of a method is equal to zero, if the value of a criterion function for allocated time has not decreased.

If for all methods $P_i$ from the list of methods $P_{k_1}, P_{k_2}, \ldots, P_{k_s}$ it happens that $E_i = 0$, then it means, that the considered list of methods cannot effectively solve the given problem. In this case the further search of an effective method by the given list of methods stops. To avoid such situation, it is necessary to include various methods into the list, so the list could be oriented to solve different types of problems.

If all the methods that are included in the system have finished their search of solution then we finish the work. The system gives out the saved up information about how the search of a solution to the problem was happening (carried out methods, values of criterion function, time of search, etc.).

Due to self-training, the system enables an automatic choice of an effective method of optimization from the available list $P_{k_1}, P_{k_2}, \ldots, P_{k_s}$ for solving specific problems.

Now let's consider the approach incorporated in the system to solve a problem of finding a global minimum of function

$F(x)$

Let functions
Usually, in practical problems there is no goal to find a global minimum with high accuracy. With methods of global optimization it is possible to find a good initial approximate solution to problem, and then to use effective local methods. Generally it is difficult to find out, whether the last found local minimum is the global solution. Except for some narrow classes of problems.

The search of a global minimum can be stopped, if in the current point we got a value of criterion function, which satisfies the requirements of a real life practice, or if we have used up the time that was dedicated to solve the problem.

The best received value of a function can be considered as a final solution of a problem.

When solving practical problems of global optimization the important value has a choice of good initial solution. In this case it is possible to find the minimum nearest to them.

In rare cases it is possible to conduct an analytical research of a criterion function and to receiving the information about the value of a global minimum, or its location.

But generally the considered problem is complex enough, though there is a big number of numerical methods and algorithms of global optimization. At the same time, there is no established classification, both methods of global optimization, and corresponding problems. We shall note, that the majority of authors adhere to classification of methods depending on the used information about the criterion function.

The process of search of a global minimum differs from the process of local optimization by that, that all methods of global optimization are not relaxational. Also, during the global optimization, each method should make a certain number of steps, only after that it is possible to make a decision on transition to another method. Thus the information received as a result of previous search, further is not always possible to use.

However, despite of these difficulties, we can control the process of finding a global minimum.

First, it is possible to change some parameters of this method, when we are looking for a global minimum.

Second, it is possible to do a transition to processes of a local search, from some current points of the process of a global search.

Third, it is possible to make the values of some variables fixed, during the search of a global minimum of the function that has the large number of variables, and to carry out global minimization on the rest of the variables.

These moments allow us to control the process of a global search.

References