New Trends in Modeling and Identification of Loudspeaker with Nonlinear Distortion

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Abstract— *This paper provides recent developments in modeling and identification of nonlinear systems pertinent to loudspeaker with nonlinear distortion effect. It is known that when small loudspeakers are driven at high playback levels the nonlinear characteristics of these speakers become a major source of sound degradations. Consequently it is essential to find a good model that matches to the loudspeaker response for the purpose of predicting and preventing the nonlinear distortion. This becomes particularly important for the purpose of improving sound quality of mobile phones. This paper presents the loudspeaker operation, the issues of concern, and nonlinear modeling techniques that can reliably be used for its identification process. Frequency domain and state-space modelings are considered and emphasis is given towards polynomial nonlinear state-space models which can better tie to nonlinear identification of loudspeaker.*

Keywords: nonlinear systems, loudspeaker, modeling, identification, best linear approximation

1. Introduction

Acoustic transducers are part of our everyday life, and we use them intensively throughout the day using our cellphones, listening to the radio in our car, looking at the TV or playing games on computer at night. In all cases, sound distortion is present and has negative impact on the sound quality, diminishing listening pleasure and, worse, speech intelligibility. In some cases, texting is the only way to get your message across. In particular, cellphones, teleconference systems, PC systems use small loudspeakers driven at high-amplitude to get enough sound level greatly increasing nonlinear distortion. It is particularly critical when it comes to hands-free or speaker-phone situations. So, nonlinear distortion becomes increasingly prevalent, and yet there is still no satisfactory model for this phenomenon. The study of loudspeaker and its characterization based on sine response remained common approach for many years. Sine sweep, step by step or continuous, have been used to measure frequency response and distortion. For non-linear behavior characterization single tone is used to measure harmonic distortion and two tones are used for intermodulation and difference distortion. Many different and sophisticated variations of these basic measurements are used, but sine response doesn't predict reliably the music or speech quality. Multitone and random noise excitations, along with coherence analysis have been introduced [1] but have not gained in popularity. The different distortion measurements (harmonic distortion, intermodulation, multitone distortion, non-coherent power) are not related to each other by an underlying model, and remain purely symptomatic. It is natural to think that the loudspeaker industry could benefit from the modern techniques of nonlinear system identification to obtain a comprehensive and accurate model for diagnosis, quality control, simulation, prediction and ultimately, linearization. Following the advancements in nonlinear system theory, during the last 30 years, many attempts have been made in the identification and linearization of loudspeaker [2]-[3]. However due to the wide range of audio frequencies (20 Hz to 20kHz) and high human ear sensitivity, the loudspeaker identification and linearization remain an elusive goal. This paper reviews recent developments in the domain of loudspeaker identification and explores new possibilities to improve modeling that is better match to the loudspeaker response. First we present the loudspeaker operation principles and the major causes of distortion, then we explore the successive modeling approaches that have been investigated in the last 30 years. Finally we provide new directions of research in the frequency domain and propose two techniques based on state-space for modeling of loudspeaker which can effectively be used in identification process.

2. Preliminary Development

In this section, we give a brief overview of the loudspeaker operation and the modeling approaches that have been investigated in the last 30 years.

2.1 Loudspeaker Mechanism

The most common type of driver is electro-dynamic. The driving part, the motor, is a moving coil into a static magnetic field. The audio signal goes through the coil and creates a variable magnetic field that interact with fixed magnets and generate a mechanical force that is roughly proportional to the electrical current. The acoustic radiation is insured by a lightweight cone (diaphragm) attached to the coil. An elastic suspension maintains the coil and the attached cone in place into the frame ("basket"). The cone is also mechanically connected to the basket by an elastic

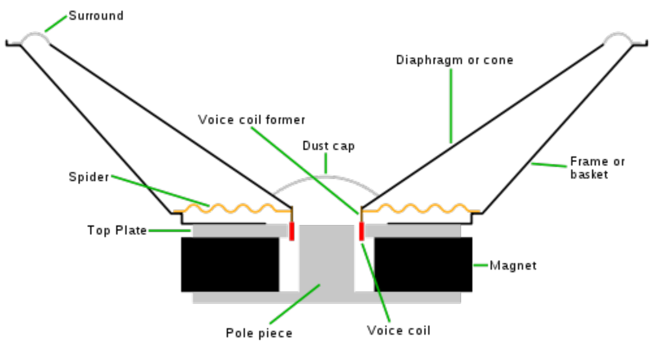

Fig. 1: Loudspeaker mechanism.

suspension called surround (see fig. 1). Designing a driver combines acoustic, mechanical, electrical and material science. A simplified linear model based on lumped parameters describes the loudspeaker mechanism at low frequencies. It is composed of 2 differential equations.

$$u(t) = Ri(t) + L\frac{di}{dt} + Bl\frac{dx}{dt} \tag{1}$$

$$Bli(t) = m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx(t) \tag{2}$$

Where u(t) is the input voltage, i(t) the current, x(t) the cone displacement and R, L, Bl, m, r, k are electromechanical parameters of the loudspeaker. It is important to note that the force factor Bl, the voice coil inductance L and the stiffness k are nonlinear function of the displacement x. Therefore nonlinearity is intrinsic to the driver's principle of operation. Beside the changing parameters just mentioned, there is a wide variety of non-linear behaviors [4]. For example, at high frequencies the cone and dome no longer behave as rigid bodies. They exhibit breakup modes and eventually the vibrations become nonlinear. Another distortion inherent to the fundamental principle of operation is the Doppler effect due to the fact that the sound is emitted from the diaphragm which is a moving source.

2.2 Approaches of System Identification

White Box (1980's): The first attempts of system identification applied to loudspeaker were based on the lumped model described by equations (1) and (2). A simple system identification method delivers a first prediction of the mechanical behavior of the loudspeaker for low frequencies, and small signals. It was applicable up to the cone breakup frequency where the cone still behaves as a rigid piston. The measurement method is based on sine excitation and proceeds in two successive parts, involving added mass or loudspeaker enclosure. In a seminal paper [2], the most prominent nonlinearities (force factor Bl(x), self-inductance L(x), stiffness k(x)) are approximated by polynomials, then expressed in term of Volterra series. Extensions of this work are reported in [5] (see also the references therein). The white box approach is limited to low frequencies and low order nonlinearities (typically 2 or 3).

Black-Box (1990's): Unlike white box approach, the black box scheme uses input/output model with no physical insight. One technique uses NARMAX in the time domain [6], described by:

$$y_t = f(y_{t-1}...y_{t-n}, u_t...u_{t-m}, e_{t-1}...e_{t-d}) + e_t \tag{3}$$

Where u and y represent input and output, e represents noise and f(.) is a nonlinear function (e.g. polynomial). Other attempts were made in the frequency domain, using a general Volterra model [7]. Volterra models are interesting because of their standard and general approach. They relate immediately to the frequency domain and provide generalized frequency responses, but their complexity is such that the order is limited practically to 3.

Block Model (2000's): Recent trends use a simplified Volterra model with diagonal kernels $h_n(t, ...t) \equiv h_n(t)$, incorporating Hammerstein scheme as shown in 2 with the output of the system represented by:

$$y(t) = \sum_{n=1}^{Q} u^n(t) * h_n(t) \tag{4}$$

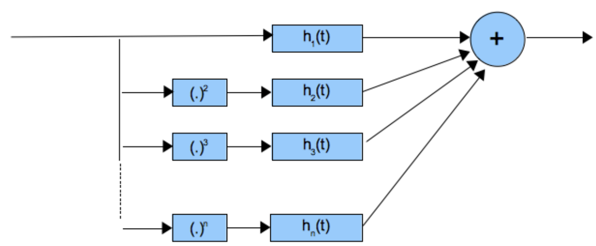

Fig. 2: parallel-hammerstein.

This model deviates from physical intuition, however attempts were made in [8] to obtain a proper identification of each path, more exactly of each transfer function h_n. Independently, a modified Wiener-Volterra model was proposed in [3] (see fig. 3), that has the property of having an exact inverse. This makes it suitable for loudspeaker linearization by derivation of a predistortion filter. It is interesting to note that the parallel-Hammerstein and the Wiener-Volterra models cover the full acoustic frequency range contrary to previous models.

2.3 Analysis

As we pointed out in the introduction, we are dealing with loudspeaker, not only for music and entertainment but also for communication. In the case of complex signals like speech or music, distortion sounds generally like a modulation noise that degrades the clarity of the signal. To demonstrate this effect, music has been played through a

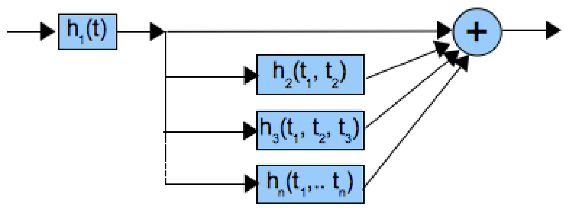

Fig. 3: Modified Wiener Volterra.

loudspeaker and the added non-linear distortion has been measured as the non-coherent power present in the acoustical signal [1]. Fig. 4 shows both the spectrum of the musical signal and the spectrum of the added distortion noise.

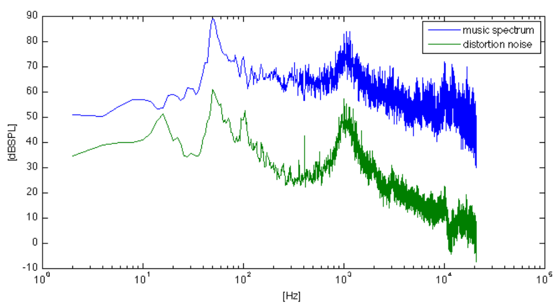

Fig. 4: Music spectrum and Distortion Noise from a Loudspeaker.

3. Proposed New Methodologies

In this section we present new models that can reliably be used in identification of loudspeaker.

3.1 Frequency Domain Block Model

In the last decade many papers have been published on the frequency domain approach ([9], [10] and references therein). The general approach is the following

1) Find the best linear approximation
2) Identify the added nonlinearity

which is practical and well-suited for weakly nonlinear systems.

This approach is justified by the following fact that a Volterra system subjected to Gaussian random input is equivalent to a linear system with an added noise source at the output (see fig. 5). The linear part Y_R contains all the contributions coherent with the excitation and the nonlinear part Y_S gathers all the contributions which are not coherent to the input. For each frequency $\omega, Y_S(\omega)$ is the sum of contributions like:

$$H_3(\omega_1, \omega_2, \omega - \omega_1 - \omega_2)U(\omega_1)U(\omega_2)U(\omega - \omega_1 - \omega_2) \quad (5)$$

Where H_3 is the Volterra generalized frequency response of order 3 and $\omega_1 + \omega_2 \neq 0$. For a random input U,

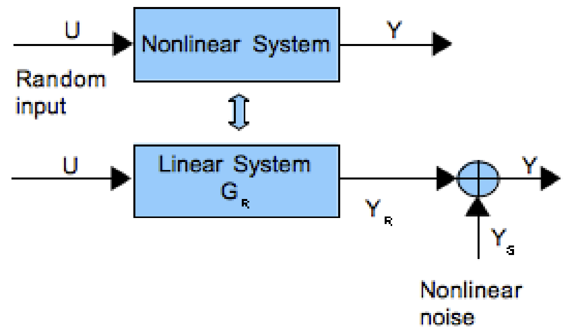

Fig. 5: Nonlinear system with random input and its equivalent linear system + nonlinear noise source.

each contribution is random, and Y_S cannot be distinguished from a noise. Note that odd degrees of nonlinearity can add coherent contribution to Y_R (e.g. insert $\omega_1 + \omega_2 \neq 0$ in (5)) that modify the best linear approximation G_R . Using this result a general purpose and flexible block model is proposed as shown in fig. 6. It is a parallel structure with

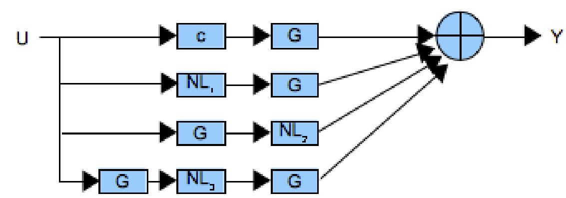

Fig. 6: Generic nonlinear model for frequency domain identification.

each branch representing a typical situation. The 1st branch is simply the linear case (c is a pure real gain). In the following branches NL_i are static polynomials systems. The 2nd branch is Hammerstein system. The 3rd branch is a Wiener system. The 4th branch is a cascade approximation of a nonlinear feedback. Note that linear block G is the same in all branches. That model is identified in successive steps. First the best linear approximation of the overall system is identified and inserted as G in all branches. Then the active branches are selected based on their power contributions. Finally the polynomial NL_i of the selected branches are identified. This approach seems well suited to loudspeaker identification. In particular, the nonlinear feedback that is part of electrodynamic loudspeaker mechanism can be identified. Our immediate goal is to apply this model to loudspeaker identification and compare it with previous methods.

3.2 Nonlinear State-Space Modeling

The most general representation of nonlinear system in state space notation can be expressed as:

$$\dot{x}(t) = f(x,u,t)$$
$$y(t) = h(x,u,t) \tag{6}$$

for continuous-time system, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^l$. The analysis and design of nonlinear system (6) is not a trivial task. Therefore, more attention has been given to special class of nonlinear systems. In particular, the class of nonlinear systems affine in the input attracted systems and control community for obvious reasons. The state equation of this class is given by:

$$\dot{x}(t) = f(x) + g(x)u \tag{7}$$

and the output equation is assumed linear in state i.e. $y(t) = Cx(t)$. It is not difficult to show that under the assumption of $f(x) = 0$ and continuous differentiability of f(x) and g(x), one can recast (7) in the pseudolinear form

$$\dot{x}(t) = A(x)x + B(x)u \tag{8}$$

which resemble linear system; however, the system matrices are state dependent. It should be pointed out that the choice of the matrix A(x) is not unique. Note also that the non-linearity is in multiplicative format. Due to this structure, control theoretical concepts can be developed to mimic the classical state-space design approaches. For example the state feedback design and its optimal control format based on Linear Quadratic Regulator (LQR) leads to state dependent Ricatti equation (SDRE), which can be solved to specify the feedback gain. Therefore it is also interesting to pursue research in identification schemes based on this model structure.

A. Polynomial Nonlinear State Space Model

Recently the following class of Lipschitz nonlinear systems has attracted a considerable attention [11]:

$$\dot{x}(t) = Ax + Bu + Ep(x,u)$$
$$y(t) = Cx + Du + Fq(x,u) \tag{9}$$

where $p(t) \equiv p(x,u)$ and $q(t) \equiv q(x,u)$ satisfy the Lipschitz condition. Note that in this case the nonlinearity is in additive format. It can be shown that applying functional expansion of the function **f** and **h** in (6) with various kinds of basis functions, one can arrive at (9). In this paper, a set of polynomial basis functions is chosen due to computational simplicity and its advantage in our application. The polynomial Nonlinear State Space (PNSS) model is defined by (9) consisting of the linear terms in x(t) and u(t) with constant coefficient matrices A, B, C, D, E, F and the vectors $p(t) \in \mathbb{R}^{n_p}$ and $q(t) \in \mathbb{R}^{n_q}$ containing nonlinear monomials in x(t) and u(t) of degree two up to a chosen degree r, where the coefficient matrices E and F contain the coefficients associated with those monomials. Note that the monomials of degree one are included in the linear part of the PNSS model structure. When a full polynomial expansion is carried out, all monomials up to degree r must be taken into account. First, a vector z is defined as the concentration of the state vector and the input vector as

$$z(t) = [x_1(t) \dots x_n(t) u_1(t) \dots u_m(t)]^T \tag{10}$$

As a consequence, the dimension of the vector z(t) is given by n_z= n+m. Then, using the conventional index notation for monomials we define:

$$p(t) = q(t) = z(t)_{\{r\}} \tag{11}$$

Note that the vector $z(t)_{\{r\}}$ as defined in (11) should contain all monomials with a degree between two and r. For instance, the vector $z_{\{3\}}$ with n_z= 2 denotes

$$z_{\{3\}} = [z_{(2)} z_{(3)}]^T = [z_1^2, z_1 z_2, z_2^2, z_1^3, z_1^2 z_2, z_1 z_2^2, z_2^3]^T \tag{12}$$

where we define $z_{(r)}$ as the vector of all the distinct monomials of degree r composed from the elements of vector z. The number of elements in vector $z_{(r)}$ is given by the following binomial coefficient

$$N_r = \binom{n_z + r - 1}{r} \tag{13}$$

Thus, the vector $z_{\{r\}}$ has the length

$$L_r = \binom{n_z + r}{r} - 1 - n_z \tag{14}$$

and corresponds to considering all the distinct nonlinear combinations of degree r, which is the default choice for the PNSS model structure. The total number of parameters required by the model in(9) , is given by

$$N = \left[\binom{n+m+r}{r} - 1 \right] (n+l) \tag{15}$$

B. Bilinear and State Affine Models

The class of bilinear state space models is described by

$$\dot{x}(t) = Ax + Bu + \sum_{k=1}^{m} N_k u_k x(t)$$
$$y(t) = Cx + Du \tag{16}$$

It is well known that bilinear state space models are universal approximates for continuous-time nonlinear systems within a bounded time interval. Unfortunately, this approximation does not hold in discrete time case. A more general class of state space models known as state affine models admit this approximation for discrete-time systems [12]. A state affine model of degree r for discrete-time systems is defined as

$$x(t+1) = \sum_{i=1}^{r-1} A_i u^i(t)x(t) + \sum_{i=1}^{r} B_i u^i(t)$$
$$y(t) = \sum_{i=1}^{r-1} C_i u^i(t)x(t) + \sum_{i=1}^{r} D_i u^i(t) \quad (17)$$

These models results in a natural way when describing sampled continuous-time bilinear state space systems [13]. The advantage of this model is that the states x(t) appear linearly in the state and output equations. As a consequence, subspace identification techniques can be used to estimate the model parameters. It is also interesting to see that state affine models form a subset of the PNSS model class.

4. Identification Procedure

It should be pointed out that a parallel treatment of previous section for discrete-time nonlinear system can be established. This is convenient for system identification process. In this case, without loss of generality, we see similar state space notation as follows:

$$x(t+1) = Ax + Bu + Ep(t)$$
$$y(t) = Cx + Du + Fq(t) \quad (18)$$

Due to the fact that nonlinearities are concentrated in the states, one can simplify PNSS representation by considering only $z(t) = x(t) = [x_1(t) \dots x_n(t)]^T$ and construct $x(t)_{\{r\}}$ which reduces the computational complexity. The identification procedure for PNSS model consists of three major steps. First, best linear approximation (BLA) of the system under test is determined non-parametrically in mean square sense. Then, a parametric linear model is estimated from BLA using frequency domain subspace identification method [14]. This is followed by solving a nonlinear optimization of the linear model. The last step consist of estimating the full nonlinear model by using again a nonlinear search algorithm that minimizes the model output error in regard to the measured output. The following steps summarizes the frequency domain subspace identification technique:

1) The BLA is obtained by a classical FRF measurement using periodic excitation (e.g. multitone) [9].
2) The FRF estimate is extended to the full unit circle and the Impulse Response coefficients $\hat{h}_i$ are obtained by IDFT of the FRF
3) The Hankel matrix is defined from the Impulse Response:

$$\hat{H} = \begin{pmatrix} \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_r \\ \hat{h}_2 & \hat{h}_3 & \cdots & \hat{h}_{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}_q & \hat{h}_{q+1} & \cdots & \hat{h}_{q+r-1} \end{pmatrix}$$

with number of rows $q > n$ and number of columns $r \geq n$, n being the order of the system.
4) The Singular Value Decomposition of the Hankel matrix is calculated and the system order is determined by selection of the n largest singular values.
5) The system matrices $\hat{A}, \hat{C}$ are estimated directly from the n left singular vectors.
6) The remaining system matrices $\hat{B}, \hat{D}$ are estimated by least-square optimization of the state-space model with respect to the measured FRF

5. Conclusion

This paper summarized the available techniques for modeling and identification of loudspeaker with its unavoidable nonlinear distortion phenomenon. Various frequency and state-space approaches have been analyzed. It is shown that when a general, black-box model of a nonlinear device is required, the PNSS model is a perfect tool to approximate the nonlinearity. This enables the identification procedure to be performed in a straightforward fashion using three simple steps; namely, best linear approximation, estimate a linear model, and finally solve a standard nonlinear optimization problem.

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