Optimization of Unscented Kalman Filter Algorithm for 3-D Point Based Rigid Registration

A. F. Zamani[1], S. Asadallahi[2], S. Cheng[3], P.Verma[4], and A.A. Beheshti Shirazi[5]
[1234]Electrical Engineering Department, University of Oklahoma, Tulsa, OK., USA
[5]Electrical Engineering Department, University of Science and Technology, Tehran, Iran

Abstract - *This paper proposes an optimized algorithm for 3-D Point Based Rigid Registration. This algorithm uses an Unscented Kalman filter (UKF) for estimating the state vector of transformation, which can be interpreted as a nonlinear function of translation and rotation. In the previous work, we showed that the drawback of the UKF algorithm in estimating high range rotations is due to its sensitivity to initial state vector. To address this drawback, we proposed using a pre-registration step to find an appropriate initial state vector [9]. In this paper we optimize our proposed algorithm to trade off running time with accuracy by selecting the initial state vectors out of a uniformly sampling and using a large error threshold for stopping the pre-registration stage. It is shown that by applying these strategies we can have an enhanced UKF algorithm, which can robustly estimate any rigid transformation with high accuracy and acceptable time consumption.*

Keywords: rigid body registration, absolute orientation problem, unscented kalman filter.

1 Introduction

As imaging industries grow, the need for registering various images grows incredibly. Image registration is the process of searching for the best alignment that transforms the points in the subject image to the corresponding points in the reference image.

One of the important processes in the image registration procedure is finding the best transformation, which aligns an input image to the reference image [1]. In the rigid body model, the object has no shape change, so the distance between two points in the first image is preserved after mapping into the second image. In the case of point-based rigid body image registration, it is essential to estimate the true transformation, which is containing translation and rotation, which is called the absolute orientation problem, and by this transformation we can register two 3-D point based data sets belonging to rigid objects. This type of registration has many applications in medical and computer vision images. It plays an essential role in procedures such as surgical planning, image-guided surgery and other systems that are used for both diagnosis and therapy. Also in the field of computer vision, 3D registration has many applications in areas such as object reconstruction and reverse engineering. This involves taking multiple scans of an object, which are then registered and combined to form the complete surface model [1, 2]. From 1966 up to now, many attempts have been done to find the best solution for absolute orientation problem. In 1997, Pennec and Thirion [3] proposed an iterative estimator by using the Extended Kalman Filter (EKF) for estimating the parameters of transformation. The Unscented particle filter (UPF) algorithm in 2004 was proposed by Ellis and Ma [4] for registering small data sets in presence of Gaussian noise but this algorithm converges very slowly due to the large number of particles and thus is not suitable for large data sets. Because of these limitations, in 2007 Hejadzi and Aboulmasoumi [5, 6] proposed the Unscented Kalman Filter (UKF) algorithm for rigid body registration in comparison to the EKF algorithm. In the previous work we pointed out an UKF algorithm drawback in estimating transformation parameters when the translations are in high ranges. Moreover, we proposed to use pre-estimation before applying the UKF [7]. In this paper we will address the high computational complexity of our previous algorithm. This new optimized algorithm can estimate any transformation in any range quickly and accurately with a significantly shorter running time.

The rest of the paper is organized as follows: In the next section, we will give a brief review of the Kalman filter. Then we will review the UKF and UKF registration algorithms in Sections 3 and 4. We will describe our proposed algorithm in Section 5. Experimental results will be given in Section 6 and a brief conclusion will follow.

2 Kalman Filter

Kalman filtering was proposed in 1960 by E. Kalman for finding the recursive and incremental solution for discrete data linear problems. Kalman filtering has many applications such as navigation and target tracking. In general, a Kalman filter estimates the state vector of a general discrete time control process model as follows:

$$X_i = Ax_{i-1} + n_{i-1}^x \tag{1}$$

In addition, the observation model is a linear function as below:

$$y_i = Cx_i + n_i^y \tag{2}$$

where A and C are defined by the system dynamics, y_i is the observation vector at time i , n_i^x and n_i^y represent the process and the observation noise at time i, respectively, and are independent Gaussian random variables with distributions $N(0,\sum_i^x)$ and $N(0,\sum_i^y)$. The main idea is to estimate sequentially the state vector x by minimizing its mean squared error. The algorithm assumes the knowledge of initial value for the state vector and its covariance matrix. We estimate the state vector x from the process model as follows (time update):

$$\bar{\hat{x}}_i = E[x_i] = AE[x_{i-1}] = E[n_{i-1}^x] = A\hat{x}_{i-1}$$
$$\bar{p}_{\hat{x}_i} = E[(x_i - \bar{\hat{x}}_i)(x_i - \bar{\hat{x}}_i)^T] =$$
$$AP_{\hat{X}_{i-1}} A^T + \sum_{i-1}^x \tag{3}$$

Secondly, the filter uses the observation vector and the observation model to update its estimated state vector in the previous step as follows (measurement update):

$$\hat{x}_i = \bar{\hat{x}}_i + K(y_i - \bar{\hat{y}}_i)$$
$$P_{\hat{x}_i} = E[(x_i - \hat{x}_i)(x_i - \hat{x}_i)^T] = (I - K_i C)\bar{P}_{\hat{X}_I} \tag{4}$$

where $\bar{\hat{y}}_i = C\bar{\hat{x}}_i$ and K_i is called Kalman gain that is calculated as follows:

$$K_i = \frac{P_{x_i y_i}}{P_{y_i}} = \frac{E[(x_i - \bar{\hat{x}}_i)(y_i - \bar{\hat{y}}_i)^T]}{E[(y_i - \bar{\hat{y}}_i)(y_i - \bar{\hat{y}}_i)^T]}$$
$$= \bar{P}_{\hat{X}_I} C^T (C\bar{P}_{\hat{X}_i} C^T + \sum_i^y)^{-1} \tag{5}$$

If the process and the observation models are defined by linear equations, and if the process and observation noises are independent Gaussian random variables, then the estimation is optimal. However, in a general case, the process and the observation models can be governed by nonlinear equations:

$$x_i = f(x_{i-1}, n_{i-1}^x),$$
$$y_i = h(x_i, n_i^y), \tag{6}$$

f and h are nonlinear functions. In this situation, the Kalman filter estimation of the state vector is not optimized since the $P_{\bar{x}_i}, P_{x_i y_i} = E[(x_i - \bar{\hat{x}}_i)(y_i - \bar{\hat{y}}_i)^T]$ and $P_{y_i} = E[(y_i - \bar{\hat{y}}_i)(y_i - \bar{\hat{y}}_i)^T]$ cannot be calculated in closed form manner. In rigid registration, we encounter the nonlinear function of transmission which we should estimate sequentially so we cannot use the Kalman filter in the basic manner. For solving this problem, there are two solutions, which deal with nonlinearities in the process of the observation model. The first option is using the Extended Kalman Filter (EKF) and the second one is the Unscented Kalman Filter (UKF). The UKF [8, 9] is more accurate and less computationally intensive than the EKF for rigid body registration. It is concluded that because of these benefits using UKF is the best solution for this problem [5] [6].

3 Unscented Kalman Filter

The UKF algorithm, which uses the Unscented transform basis, computes the mean and covariance matrix of the state vector x_i governed by the nonlinear process model and nonlinear observation model as follows (similar notations to [5,6] have been used):

a) Define the state random variable X_i^a as the concatenation of the original state and noise variables as:

$$X_i^a = [x_i^T, n_i^{x^T}, n_i^{y^T}]_{1\times n_a}^T \equiv [x_i^{xT}, x_i^{n_x T}, X_{iI}^{n_y T}]^T \tag{7}$$

We initialize the parameters as follows:

$$\hat{x}_0 = E[x_0]$$
$$\hat{x}_0^a = E[x_0^T, n_0^{x^T}, n_0^{y^T}]^T = [\hat{x}_0^T, 0, 0]^T$$
$$P_{\hat{x}_0} = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \tag{8}$$

$$P_{\hat{x}_{i0}^a} = E[(x_0^a - \hat{x}_0^a)(x_0^a - \hat{x}_0^a)^T] = diag(P_{\hat{x}0}, \sum_0^x, \sum_0^y)$$

b) The sigma points will be calculated as follows:

$$\tilde{x}_{0,i} = \hat{x}_i^a \tag{9}$$

$$\tilde{x}_{k,i} = \hat{x}_i^a + (\sqrt{(n_a + \lambda)P_{\hat{X}_I^A}})_K \qquad k = 1,\ldots, n_a$$

$$\tilde{x}_{k,i} = \hat{x}_i^a + (\sqrt{(n_a + \lambda)P_{\hat{X}_I^A}})_K \qquad k = n_a + 1, \ldots 2n_a$$

c) In the time update stage, we propagate sigma points through the process and observation models to estimate the means and covariance matrices:

d) We update the measurements as follows:

$$P_{y_i} = \sum_{k=0}^{2n_a} w_k^c [\tilde{y}_{k,i} - \bar{\hat{y}}_i][\tilde{y}_{k,i} - \bar{\hat{y}}_i]^T$$
$$P_{x_i y_i} = \sum_{k=0}^{2n_a} w_k^c [\bar{\tilde{x}}_{k,i}^x - \bar{\hat{x}}_i][\tilde{y}_{k,i} - \bar{\hat{y}}_i]^T$$

$$\tilde{x}^{\bar{x}}_{k,i} = f(\tilde{x}^{x}_{k,i}, x^{n_x}_{k,i}),$$

$$\bar{\hat{x}}_i = \sum_{k=0}^{2n_a} w^m_k x^{\bar{x}}_{k,i}$$

$$\bar{P}_{\hat{x}_i} = \sum_{k=0}^{2n_a} w^c_k [\tilde{x}^{\bar{x}}_{k,i} - \bar{\hat{x}}_i][\tilde{x}^{\bar{x}}_{k,i} - \bar{\hat{x}}_i]^T$$

$$\tilde{y}_{k,i} = h(\tilde{x}^{\bar{x}}_{k,i}, \tilde{x}^{n_y}_{k,i}),$$

$$\bar{\hat{y}}_i = \sum_{k=0}^{2n_a} w^m_k \tilde{y}_{k,i}$$

$$K_i = P_{x_i y_i} P^{-1}_{y_i}$$
$$\hat{x}_i = \bar{\hat{x}}_i - K_i P_{y_i} K_i^T \qquad (10)$$
$$p_{\hat{x}_i} = p_{\bar{\hat{x}}_i} - K_i P_{y_i} K_i^T$$

4 UKF Registration Algorithm

As mentioned before, the UKF algorithm is based on the unscented transform theory. The unscented transform is the optimal solution for estimating the nonlinear state vector which is the combination of translation and rotation. The UKF algorithm, which is proposed for rigid registration, has the accuracy equal to the second order of the Taylor series. In this algorithm the point of moving data sets append to the estimation process incrementally, so this utilizes the algorithm to use real time in applications. In the previous work [7] we reviewed the UKF algorithm in the case of isotropic noise and also with the assumption that the corresponding points between two data sets are unknown. In that case, we had two data sets. The first one is the fixed data set, which is used as the reference image, and the other one is the moving data set which is the transformed points with rigid transformation containing translation and rotation. We assumed the correspondence points in the two data sets are unknown so we should find the corresponding point in the two data sets at the first stage. For this reason in the UKF algorithm, we utilized the Closest Point function, which searches for each transformed point its correspondence point in the fixed data set. Since searching the correspondence point is a computational procedure, we use the KD- tree search, which can accelerate the searching procedure. [10, 6]

In the UKF registration algorithm, the state vector is $x=[t_x,t_y,t_z,\theta_x,\theta_y,\theta_z]^T=[x_t^T,x_\theta^T]^T$ and the state model is defined as, $x_i = x_{i-1} + N(0,\sum_Q)$ with the initial value and covariance matrix x_0 and P_x^0 .Here $N(0,\sum_Q)$ is a zero-mean Gaussian random vector with a covariance matrix $\sum_Q$. In this case, because the observation model is a nonlinear function of state vector X, it is necessary to use the UKF algorithm for estimating this nonlinear transform. Although the EKF can also be used, according to comparison between UKF and EKF, the UKF is more effective and has higher accuracy [5]. The procedure of the UKF algorithm is shown in Fig. 1[6].

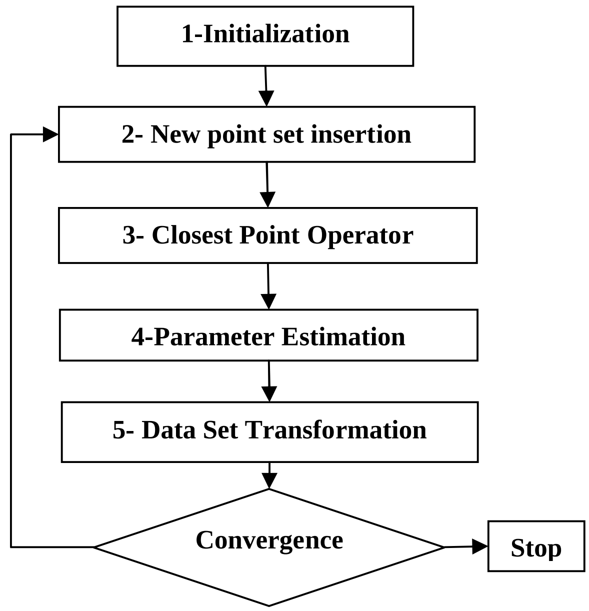

Fig.1. Flowchart of the UKF Algorithm [3]

1) Initializing the state vector x and its covariance matrix with zero and the identity matrix, respectively. Also set the covariance matrix of the observation model, $\sum_i = \sum_i^{y'} + \sum_i^{\text{int}}$ .Then, set the covariance matrix of the process model for every point in the data sets, $\sum_i^x$,to the initial uncertainty for each transformation parameter.

2) Append the i random point from the moving data set U to the previously selected points from that data set.

3) Find the corresponding points between the selected points from the moving data set U and the fixed data set Y by using $y_i = f_{CIP}(Y, Ru_i + t)$ and the KD-tree algorithm [10],[6], and determine the mean square distance error, E[d2], among the estimated point matches.

4) Use the estimated corresponding points to compute the rotational and translational parameters.

5) Use the estimated transformation parameters to update the moving data set U:

$$u_i = R_{(\hat{x}_\theta)} u_i + \hat{x}_t \qquad i = 1,\dots,N \qquad (11)$$

6) If $E[d^2]$ is less than a certain threshold, stop the algorithm; otherwise, update the covariance matrix of the observation model with $\sum_i = \sum_i^{y'} + E[d^2]$

Also, anneal $\sum_i^x$ with the factor of 0.95 and go to Step 2.

5 Extended UKF Algorithm

As it was mentioned in the previous work [7] the sensitivity of the UKF algorithm depends on selecting the initial state vector appropriately. We compared different strategies and it is shown that the sequence of points has no significant influence on the UKF performance. In this regards we proposed an enhanced UKF algorithm, which uses a pre estimation stage in the registration procedure. It uses 200 random initial state vectors in the range of between 0 and 90 degree and millimeters which indicate transformation parameters and for each of these initial state vectors we used UKF registration for registering the limited points of data sets. At the end of this pre-registration algorithm, we have the correspondence error, which belongs to each initial state vector. Among the 200 correspondence errors, which are generated after registering limited points, we select the initial state vector which generates the minimum average error. This state vector is optimum among others and we can use it for registration according to the UKF flowchart. The flowchart of this algorithm is shown in Fig. 2. By this pre-estimation algorithm, we can compensate the UKF limitation for estimating state vectors with large rotation. For examining the performance of this enhanced algorithm, in the pre-estimation stage, with the 200 initial random points the UKF registration algorithm is run according to the flowchart shown in Fig. 1. For each initial state vector the registration procedure continued up to appending the 50th point from the moving data set. At the end of each trial, after registering 50 points from the 200 points data set, we save the correspondence errors. The correspondence point at each trial is generated by applying the nearest closed point function. The minimum average correspondence error indicates the best selection for the initial state vector.

By using this pre-estimation to find an appropriate initial state vector in the enhanced UKF algorithm, it can be seen that the performance of registration grows incredibly and there is not any limitation for registering the high range transformations as shown in Fig. 3. We compare the performance of the UKF with the proposed enhanced UKF algorithm in Fig. 3. It is obvious that the time of registration will be increased by the enhanced UKF registration method. In this paper, we are developing this algorithm by applying some changes in our algorithm for improving the time consuming and also the accuracy of the algorithm.

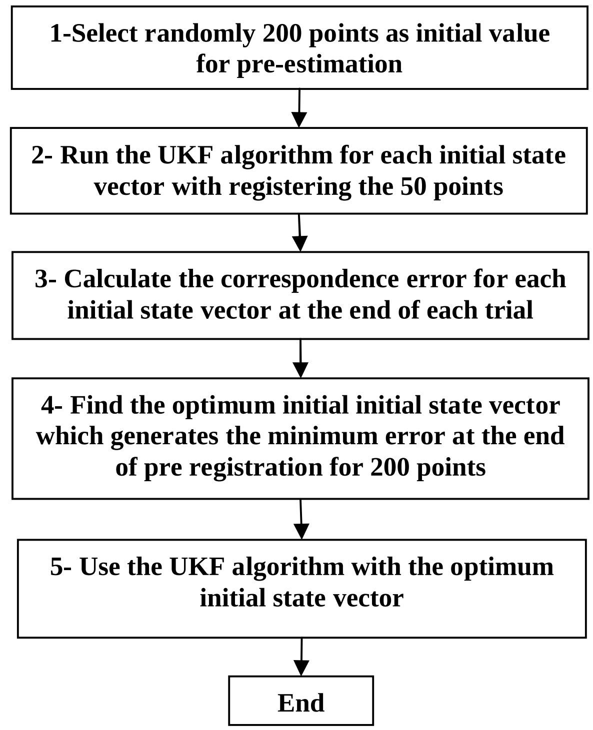

Fig .2.Flowchart of Extended UKF algorithm

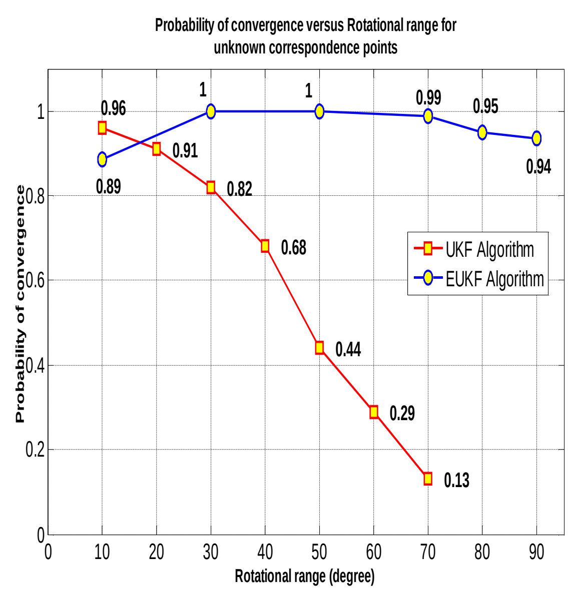

Fig. 3.Comparing UKF and Extended UKF algorithms' accuracy

6 Experimental Results

To increase the accuracy of the enhanced UKF algorithm, we sample the initial state vector uniformly for the pre-estimation stage. For examining this strategy in each interval of 10 degrees and millimeters, we selected 10 initial state vectors that are totally 198 initial state vectors. The comparison results between randomly selected initial state vector and the uniformly selected initial state vector are shown in Fig 4. We can see that by these changes made in the algorithm, we need fewer points for convergence of the enhanced UKF algorithm for reaching the solution.

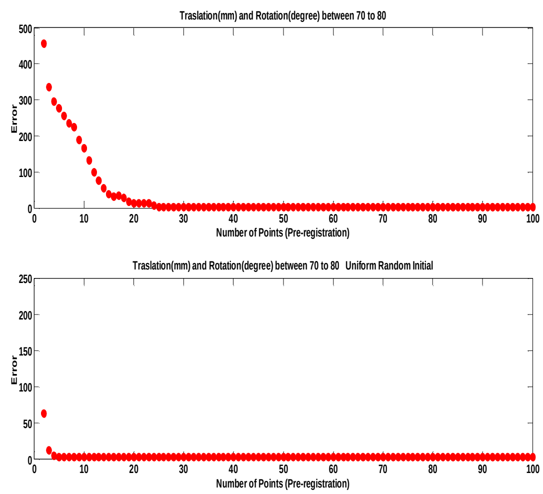

Fig. 4.Comparing the Extended UKF performance by using a) random sampling b) uniform sampling of state vectors in the pre-estimation stage

For example, in the range of 70 to 80 degrees for rotation parameters, by uniformly selecting the random state vector we need approximately 10 points for the pre-estimation stage. Otherwise according the previous method, we need approximately 25 points. The decrease in the number of points for registration has direct influence on running time for the registration procedure. We can see the influence of using uniformly the random state vector on time consumption in Fig. 5.

The second strategy for improving the enhanced UKF algorithm is to consider the certain threshold for registration error in the pre-estimation stage. In other words, instead of using a constant number of points for pre-estimation in the enhanced UKF algorithm, we can find the best threshold to minimize registration error. For this reason, we repeated the experiment in pre-estimation until the registration error is below the threshold and among these errors we select the state vector which produces the minimum error for using in the UKF registration algorithm. By this method, we can reduce the time significantly. For example, for rotation between 70 to 80 degrees we compared the performance of the algorithm for different strategies. First of all, we examined three different cases: 1) Without any enhancement; 2) Uniformly selecting the state vector; 3) Uniformly selecting the state vector and applying the error threshold 750-3000 for stopping the pre-estimation algorithm. In the last strategy, we used 5 thresholds of error equal to 750, 1500, 2000, 2500 and 3000 which indicate the square of correspondence error which is determined by the nearest closed point function. This error is the distance between two correspondence data sets. In Fig 5, we can easily compare these various strategies' performance and for each method we can see the accuracy of the algorithm for estimating the 6 parameters of transformation and also the time which is needed for registration. According to this comparison with a good tradeoff between time and accuracy, the combination method, which uses uniform distribution for initial state vectors in the pre-estimation stage with applying the pre-estimation threshold for a corresponding error equal to 2000, is the best choice for rigid registration in these simulations. To do the simulations, MATLAB Version 7.1.0.246 software from Math Works, running on a desktop PC with 1 GB RAM and Intel (R) Core (TM) 2 CPU 2.00GHz, has been used.

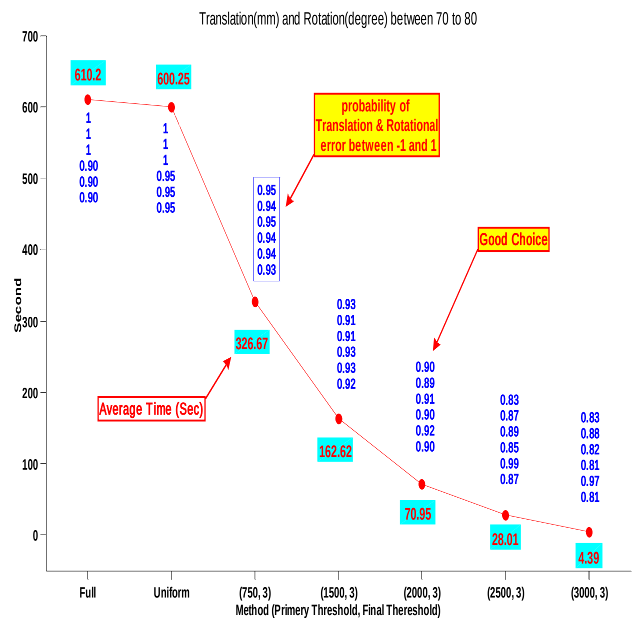

Fig.5. Comparison of the different strategies for applying in extended UKF algorithm a) randomly without any enhancement b) Uniformly selecting the state vector c) Uniformly selecting the state vector and applying the error threshold 750-3000 for stopping the pre-estimation algorithm

7 Conclusions

We conclude that for the rigid registration of 3-D points in the case of unknown correspondence data sets with high transformation, we can use the proposed enhanced UKF registration algorithm that is an effective method for estimating transformation between two data sets instead of UKF. We compared different strategies and found that the sequence of points has no significant influence on the UKF performance, but selecting the appropriate initial state vector has a strong influence on UKF performance in high range rotations. We compared the performance of enhanced UKF to UKF for convergence and accuracy and we found that the enhanced UKF algorithm was sufficiently accurate in estimating high rotations despite the UKF, which cannot estimate high range rotations. In this paper, we optimized the enhanced UKF registration algorithm to decrease the running time of the algorithm. We proposed to uniformly sample initial state vectors and using various thresholds for stopping the pre-estimation stage in the enhanced UKF algorithm and we compared running time and accuracy of each method. We showed that the combination of uniform sampling and primary threshold = 2000 and secondary threshold = 3 identity gives the most favorable time accuracy trade-off. This makes the enhanced UKF algorithms very strong and efficient in estimating any range of rigid transformation and makes it an efficient tool that can handle much larger rotating angle for rigid image registration. .

8 References

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