Situations, Deduction, Plausibility

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Abstract. We consider time-dependent systems and their possible states or situations. In order to do this, we define suitable algebraic structures, including time. Situation descriptions are represented with the corresponding formulas and we look at inferences from formulas, where the inference may be partially incorrect. We define the measure of correctness and show how it can be used to compare the plausibility of the described situations.

Keywords. Time-dependent systems. Developments and deduction. Measure of correctness of inference. Plausibility.

1. Introduction

In this work we consider time-dependent systems. These are systems, where the sets of elements and the properties of and relations between elements are dependent on time (see Lorents, Matsak 2011). At any moment of time t the system M is in a certain state M(t), or in other words, there is a certain situation in the system (see Jakobson 2011). At some moment of time t', which comes after the moment t, the corresponding situation can be the same, or shorter - M(t)=M(t'). However, it can also change. In the latter case, where M(t)≠M(t'), we speak of an event (see, for example, Jakobson 2007, 2011).

It is often reasonable to handle the time-dependence of systems in such a way that the relation between time moments and possible states is not necessarily one-to-one (see Lorents, Matsak 2011; Lorents 2006). In such a case we need instruments that would help to ***compare*** the possible situations at the next moment. This is primarily necessary in order to find the most plausible future situation before making a decision on how to act.

There are several approaches to assess plausibility. For example, we can rely on the corresponding probabilities (see, for example, Polya 1954). Another approach is to use a suitably defined notion of possibility and the corresponding distribution functions that are related to so-called fuzzy sets, which have the values of real numbers from the range of [0,1] (see, for example, Zadeh 1978; D. Dubois, H. Prade, R. Sabbadin 2001; Jakobson 2011). The plausibility can also be handled in the framework of a certain multi-valent logic (see Sigarreta, Ruesga, Rodriguez 2007).

One option for assessing and comparing plausibility is to use the *principle of correlation of developments and deduction*, according to which the more logical an inference, where the starting point is the description DesM(t) of the current situation M(t) of the system M and the where the result is the description DesM(t') of a possible following situation M(t'), the more probable it is that this situation develops in the system (see Lorents 1998, 2006; Lorents, Matsak 2011).

At this point we need to explain how to understand the phrase "more logical". For this we use the notion of the correctness of inference steps (see, for example, Takeuti 1975) and we define the measure of correctness. By using the measure of the correctness of inference steps (which is represented by the rational number that we get by applying the corresponding procedure), we can compare single inference steps and also the reasoning based on these inference steps. Based on this, the corresponding descriptions, and relying on the principle of correlation of developments and deduction, we can compare the following possible situations, in order to make a decision about which one of them is more plausible.

According to the chosen approach, the key question is how to define the measure of correctness of inference steps, so that it

(1) would be in accordance with the notion of correctness of inference steps used in mathematical logic

(2) would allow to operate with comparable values, preferably numbers, and would have the corresponding algorithm for getting these values

(3) would allow to assess the correctness of an inference consisting of single inference steps

(4) would allow (based on the principle of correlation of developments and deduction) to assess, which inference process produces a description of a situation that will more plausibly take place compared to some other situation.

2. Systems and time

When dealing with systems in this work, we use the notion of an algebraic system (see, for example, Maltsev 1970; Grätzer 2008) and we look at such ordered pairs, where the first is the collection of sets of elements of interest and where the second is the collection of the properties of or relations between the elements of interest.

Definition 1. A *system* is an ordered pair ⟨{X,Y,Z,…};{P,Q,R,…}⟩, where X, Y, Z, … are the *main sets of the system* (note that in a special case the system can have just one main set, for example X) and P, Q, R, ... are the *predicates of the system* (the properties or relations of the elements of the sets in question). The collection of predicates is called the *signature of the system*.

Example 1.1. Married people could conservatively be represented as the system ⟨{H};{♂,♀,Mar}⟩, where the only main set is the set H of humans and the signature consists of the properties "male", "female" and a binary marriage relation Mar. At the same time, we could look at the same people as the system ⟨{M, F};{Mar}⟩, which has two main sets: the set of men M and the set of women F; and which has only one binary relation, marriage relation Mar, in its signature.

According to the *Systematic principle*, things that can reasonably be represented as systems or belonging to systems, should be (see Lorents 1998, 2006). Therefore, we can also approach *time* from the systematic perspective:

Definition 2. *Time* is a system ⟨{T, D}; {Bef, Aft, Sim, Dur}⟩, where the main sets are the set of time moments T and the set of the lengths of time intervals D and where the signature consists of binary relations: the relation of being before Bef, the relation of being after Aft, the relation of being simultaneous Sim, and the ternary relation Dur, which relates any two observed time moments and the time interval, which includes all the time moments that occur between them.

Example 2.1. The budget year time

⟨{{01.01,02.01,…,30.12,31.12},{1,…,365}};{<,>,=,$-_{+1}$}⟩,

where $-_{+1}$ is a ternary relation, which relates two dates with the difference of the later date and the earlier date that is increased by one.

For example, if some work starts on the second of January and also finishes on the second of January (of the same year), then the number of days one should get paid for is $2-_{+1}2=2-2+1=1$ (NB! Not 2–2=0). If, however, the work begins on the second of January and ends on the twelfth of January, then one should get paid for $12-_{+1}2=12-2+1=11$ (NB! Not 12–2=10) days.

In order to define time-dependent systems (see Lorents, Matsak 2011) we must first identify the (concrete) system ∇=⟨{T,D};{Bef,Aft,Sim,Dur}⟩ that is our time and the classes CSets and CPred. The *elements* of class *CSets* are collections of sets or sets, where the elements are some other sets. The elements of class *CPred* are collections of predicates or sets, where the elements are predicates (let us recall that we have unary predicates as properties or subsets of some sets, and that we have k-order predicates (where k > 1) as relations between k elements or, in other words, the subset of the Cartesian product of some sets). Next we must identify the relations Set and Sig, which allow us to relate time moments with sets of elements from class CSets and signatures (or collections of the properties of elements and the relations between elements) from class CPred. Only now can we present the definition of a time-dependent system:

Definition 3 (Lorents, Matsak 2011)**.** An ordered triplet ⟨∇,Set,Sig⟩ is called a *time-dependent system*, if the following condition is fulfilled:

If, for some time moment t∈T, the collection of sets s∈CSets and the collection of predicates p∈CPred,

(1) Set(t,s) and (2) Sig(t,p),

then, without exceptions, all predicates from collection p must be the properties of or relations between the elements of the sets from collection s.

Let us agree that the *main sets of system* ⟨∇,Set,Sig⟩ *at time moment t can only be such sets* that are elements of collection s. Let us also agree that *only such predicates, which come from collection p, can be the predicates of system* ⟨∇, Set, Sig⟩ *at time moment t*. Finally, let us agree that *the possible state of the system* ⟨∇, Set, Sig⟩ *at time moment t* is the ordered pair ⟨s;p⟩, or a system where the collection of main sets is s (where Set(t,s)) and the signature (or collection of predicates) is p (where Sig(t,p)) and where, without exceptions, all predicates from collection p must be the properties of or relations between the elements of the sets in collection s). The collection of all possible states of the system is called the *development space of the system in time* ∇.

Example 3.1 Let us look at the system VF (Vooglaid family):

VF(2002)= ⟨**{**{Varro, Helena}**}**;{♂,♀,Mar}⟩ (Mar represents the relation "are married")

VF(2003)=⟨**{**{Varro,Helena},{Iida}**}**;{♂,♀,Mar,Par}⟩ (Par represents the relation "is a parent of"; the set {Varro,Helena} is the set of spouses in year 2003 and {Iida} is the set of children in year 2003)

VF(2004)=⟨**{**{Varro,Helena},{Iida}**}**;{♂,♀,Mar,Par}⟩

VF(2005)=⟨**{**{Varro,Helena},{Iida}**}**;{♂,♀,Mar,Par}⟩

VF(2006)=⟨**{**{Varro,Helena},{Iida,August}**}**;
{♂,♀,Mar,Par,BoS}⟩ (BoS represents the relation "is a Brother or Sister of")

… … …

VF(2008)=⟨{{Varro,Helena},{Iida,August}};
{♂,♀,Mar,Par,BoS}⟩

VF(2009)=⟨{{Varro,Helena},{Iida,August,Benita}};
{♂,♀,Mar,Par,BoS}⟩.

In here CSets={{{Varro,Helena}},{{Varro,Helena},{Iida}},
{{Varro,Helena},{Iida,August}},
{{Varro,Helena},{Iida,August,Benita}}} and

CPred={{Mar},{Mar,Par}, {Mar,Par,BoS}}.

Set ⊆ {02,03, …,09}×
×{{{Varro,Helena}},{{Varro,Helena},{Iida}},

{{Varro,Helena},{Iida,August}},{{Varro,Helena},
{Iida,August,Benita}}}

Set={⟨02,{{Va,He}}⟩,⟨03,{{Va,He},{Ii}}⟩,
⟨04,{{Va,He},{Ii}}⟩,⟨05,{{Va,He},{Ii}}⟩,
⟨06,{{Va,He},{Ii,Au}}⟩,⟨07,{{Va,He},{Ii,Au}}⟩,
⟨08,{{Va,He},{Ii,Au}}⟩,⟨09,{{Va,He}, {Ii,Au,Be}}⟩}

Sig ⊆ {02,03, …, 09}×{{♂,♀,Mar}, {♂,♀,Mar,Par},
{♂,♀,Mar,Par,BoS}}

Sig={⟨02,{♂,♀,Mar}⟩,⟨03,{♂,♀,Mar,Par}⟩,
⟨04,{♂,♀,Mar,Par}⟩,⟨05,{♂,♀,Mar,Par}⟩,
⟨06,{♂,♀,Mar,Par,BoS}⟩,⟨07,{♂,♀,Mar,Par,BoS}⟩,
⟨08,{♂,♀,Mar,Par,BoS}⟩, ⟨09,{♂,♀,Mar,Par,BoS}⟩}.

3. Describing situations

By studying the descriptions of time-dependent systems at one or another time moment, we can see that in a large part of the cases the descriptions consist of arguments in natural language text (for example, some elements have these properties but lack those properties; some elements are related somehow, but others are not; if some elements have such properties, then they are not related in that way, etc.). If we transform texts like this (for example, by using the DST dialogue system (see Matsak 2005; Matsak 2010; Lorents, Matsak 2011) we get certain predicate calculus formulas.

Example 3.1.1. We can describe the situation of family Vooglaid in 2007 with, for example, the following formulas: ♂(Va), ♂(Au), ♀(He), ♀(Ii), Mar(Va,He), Par(Va,Ii), Par(Va,Au), Par(He,Ii), Par(He,Au), BoS(Ii,Au).

For “aliens” who are not very familiar with Earthly affairs, we can add the following: ∀h(♂(h)∨♀(h)), ∀h(♂(h)⇔¬♀(h)), $\forall h_1h_2(\mathrm{Mar}(h_1,h_2)\supset(\neg(♂(h_1)\&♂(h_2))\vee\neg(♀(h_1)\&♀(h_2))$, $\forall h_1h_2(\mathrm{Mar}(h_1,h_2)\supset\neg\mathrm{BoS}(h_1,h_2))$, $\neg(\exists\forall h_1h_2)(\mathrm{BoS}(h_1,h_2)\&\mathrm{Mar}(h_1,h_2))$, etc.

By assigning to individual and predicate symbols suitable meanings drawn from the main sets and signature of the system, we get knowledge related to that specific system (see Lorents 2009). This knowledge is necessary both for handling the current state of the system, as well as for getting knowledge about the states of the system or situations at some later time moments.

One option for such forecasting is to rely on deduction, which people often do. However, they do not always use logically correct inference steps. On the other hand, we must admit that people often perform quite successfully even without “iron logic” (see, for example, Rescher 1976). In other words (NB!) *people can be relatively successful even if not all the inference steps in a deduction process are logically correct.* This brings us to the **main problems of this work:**

- how to assess the correctness of given inference steps or inferences that contain them? This problem is related to the observation that “good” decisions, which led to “good results” are often based on “better logic” (compared to others). Therefore, if it is not possible to build the decision process to full extent on logically correct inference steps, then what is our expectation that a situation matching the description that was generated in such a way (using somewhat broken logic) actually occurs?
- if and how can we ***compare*** the plausibility of the arrival of the described future situations, based on the measure of correctness of the inference steps that make up the deduction process?

The first step to solve these problems is to find a suitable definition for the measure of correctness of inference steps.

4. Measure of correctness of inference steps

In mathematical logic, we call an inference step *correct* if the following condition is met: for **any** interpretation, **if** for some interpretation φ **all** the direct premises of the observed inference steps are correct, **then** in the same interpretation φ the direct conclusion of the observed inference step must also be correct (see, for example, Takeuti 1975).

At this point we turn our attention to the phrase “for some interpretation φ **all** the direct premises of the observed inference steps are correct”. It is here where we exclude all interpretations, where all the direct premises are not correct at the same time (in the framework of the given interpretation), from our process of defining the measure of correctness. From the remaining interpretations we choose such interpretations that also have the correct direct conclusion. When we compare the collection of chosen interpretations to the previous collection, we find the corresponding proportion, which we take to be the measure of the correctness of the given inference step.

Definition 4. Let us observe an inference step, which has the formulas X, Y, …, Z as direct premises and the formula W as the direct conclusion. Let us look at interpretations, where the meanings of the individuals and predicates present in formulas come from a system with finite elements and signature. Let us look at the set of interpretations $Pres=\{\varphi| \varphi X=1\&\varphi Y=1\&\ldots\&\varphi Z=1\}$ and $Cons=\{\varphi| \varphi X=1\&\varphi Y=1\&\ldots\&\varphi Z=1\&\varphi W=1\}$. The *measure of correctness of the inference step S* is the number cor(S), where $cor(S)=E(Cons):E(Pres)$, if $E(Pres)>0$ and $cor(S)=0$, if $E(Pres)=0$, where E(H) represents the number of elements of a finite set H.

Example 4.1. From the reasoning of a two and a half year old child we find out that if it is cold outside then one cannot be naked outside, but if it is warm then one can (see http://childes.psy.cmu.edu/browser/index.php?url=Other/Estonian/Vija/20612.cha). This is an inference step, where the only direct premise is the formula $X\supset\neg Y$ and the direct conclusion is the formula $\neg X\supset Y$. It is not difficult to find that $E(Cons):E(Pres)=2:3$. Therefore, the measure of correctness of this inference step is 2:3.

Example 4.2. Why does our country have so many enemies, asks a youth from a known politician. Well, says the politician, on the one hand, of course, it is because if a country is wealthy and strong, many will want to be friends (or at least not enemies). On the other hand, wealth and strength is not liked by competitors and therefore all other countries are our enemies. In this case the wily politician seems to use the inference step, where the direct premises are the formulas $X\&Y\supset Z$ and $X\&Y\supset\neg Z$ and the direct conclusion is $\neg Z$. The measure of correctness of such an inference step is 1:2.

Note. The fact that an inference step is performed "according to rules" does not automatically guarantee that the inference step is correct!

Example 4.3. Let us consider a rule that belongs to sequential predicate calculus inference rules – *introducing disjunction into the antecedent* (see Gentzen 1936, Takeuti 1975). Let us now implement an inference step, where the direct premises are sequences $X\rightarrow X\&\neg X$ and $\neg X\rightarrow X\&\neg X$ and the direct conclusion is the sequence $X\vee\neg X\rightarrow X\&\neg X$. The measure of correctness of such an inference step is 0.

Let us now consider separately certain inference steps, which "introduce" quantifiers (that is, inference steps where the direct premise consists of formulas $F(b_1)$, $F(b_2)$, …, $F(b_m)$ and the direct conclusion is the formula $\forall xF(x)$ or the formula $\exists xF(x)$, where b_1, b_2, …, b_m are some elements of the finite system used for interpretation, $m\leq n$ and n is the number elements of the finite system used for interpretation). Based "directly" on the definition above (4), the measure of correctness in such inference steps is somewhat trivial: if there exists an element e in the system, so that $\varphi F(e)=0$, then the measure of correctness of the step that introduces the universal quantifier is 0 in every case. On the other hand, if the system contains an element d, so that $\varphi F(d)=1$, then the measure of correctness of the step that introduces the existential quantifier is 1 in every case.

However, people often use the above described inference steps in a somewhat different way: *the generalization is considered correct based on a finite number of suitable examples*. One possible explanation for this is people's weird (but ancient) belief that a collection of positive examples supporting some argument is enough to consider the argument proven for *all* cases. In such an approach, the measure of correctness of the step that introduces a universal quantifier is not at all important. In such cases some "measure of conviction" is much more important for people. There are undoubtedly several options for also defining this measure:

Let us observe an inference step, where the premises are the formulas $F(b_1)$, $F(b_2)$, …, $F(b_m)$ and the direct conclusion is the formula $\forall xF(x)$, where b_1, b_2, …, b_m are such elements of the finite system used for interpretation, that $\varphi F(b_1)=1$, $\varphi F(b_2)=1$, …, $\varphi F(b_m)=1$ where $m\leq n$ and n is the number of elements of the finite system used for interpretation. In such a case we can take as the *measure of conviction*, for example:

(Definition 4A). The number m:n (which shows, what proportion of the full collection is represented by the "positive examples" that are offered as proof).

(Definition 4B). The representation of the selection b_1, b_2, …, b_m in the n-element collection (see Gliner J. A., Morgan G. A., Leech N. L. 2009).

Note. It should be safe to agree with the argument: more correct is more convincing. The inverted argument: more convincing is more correct, is not very convincing.

Example 4.AB. (see Matsak 2010) A child (5 years and 9 months old) reasons why flowers do not talk. The child uses the premise that she has never heard the voice of a flower in order to reach this conclusion. By transforming the text we get to an inference step, where the premise consists of a finite number of formulas $\neg Speak(f_1)$, $\neg Speak(f_2)$, …, $\neg Speak(f_m)$ (where f_1, f_2, …, f_m correspond to the flowers that the child has experience with), and the conclusion is the formula $\forall f(\neg Speak(f))$. Convincing, right?

5. The measure of correctness of inference

The notion of the measure of correctness of inference is similar to the measure of correctness of inference steps.

In order to define it, we use the definition of a *tree shaped inference*:

Definition 5.

- Every inference step

$$\frac{X_1\ X_2\ \dots\ X_k}{Y}$$

is an inference, where the premises are the direct premises $X_1, X_2, \dots, X_k$ of this inference step and the conclusion is the direct conclusion Y of this inference step.

- Let us have inferences $D_1, D_2, \dots, D_n$, where the premises are correspondingly $A_{11}, A_{12}, \dots, A_{1m(1)}, A_{21}, A_{22}, \dots, A_{2m(2)}, \dots \dots, A_{n1}, A_{n2}, \dots, A_{nm(n)}$ and the conclusions are correspondingly $B_1, B_2, \dots, B_n$. In addition, let us have such an inference step, where the direct premises are $B_1, B_2, \dots, B_n$ and the direct conclusion is C. The inference of such a case is

$$\frac{D_1\ D_2\ \dots\ D_n}{C}$$

where the premises are $A_{11}, A_{12}, \dots, A_{1m(1)}, A_{21}, A_{22}, \dots, A_{2m(2)}, \dots \dots, A_{n1}, A_{n2}, \dots, A_{nm(n)}$ and where the conclusion is C.

Definition 6. Let us look at an inference D, which consists of inference steps $S_1, \dots, S_u$. Let the corresponding measures of correctness of the inference steps be $c_1, \dots, c_u$. The *internal measure of correctness* of the inference is the number $Incor(D)=\min\{c_1, \dots, c_u\}$.

Definition 7. Let us look at an inference D, where the premises are $A_{11}, A_{12}, \dots, A_{1m(1)}, A_{21}, A_{22}, \dots, A_{2m(2)}, \dots \dots, A_{n1}, A_{n2}, \dots, A_{nm(n)}$ and where the conclusion is C. Let us consider the interpretations, where the meanings of the individuals and predicates in the formulas come from a system with finite elements and signature. Let us look at the set of interpretations $Pres(D)=\{\varphi|\ \varphi A_{11}=1\&\varphi A_{12}=1\&\dots\dots\&\varphi A_{nm(n)}=1\}$, and the set $Cons(D)=\{\varphi|\ \varphi A_{11}=1\&\varphi A_{12}=1\&\dots\&\varphi A_{nm(n)}=1\&\varphi C=1\}$. The *measure of external correctness* of the inference D is the number Excor(D), where Excor(D)= =E(Cons(D)):E(Pres(D)), if E(Pres(D))>0 and Excor(D)=0, if E(Pres(D))=0, where E(H) represents the number of elements of the finite set H.

This raises the next **problem**: how are the external and internal measures of correctness of the inference related?

If we follow the ancient folk wisdom that the strength of the chain is determined by its weakest link (or in other words, the chain can be no stronger than the individual links), then the inequality Excor(D)≤Incor(D) should apply. If, however, we follow the idea that one can assemble quite reliable systems (for example, buildings, machinery, software etc.) from components which include some unreliable ones, then the inequality Excor(D)≥Incor(D) could apply.

Unfortunately, it turns out that neither case is true!

Theorem. (I) There exist inferences, where the external measure of correctness is strictly larger than the internal measure of correctness. (II) There exist inferences, where the external measure of correctness is strictly smaller than the internal measure of correctness.

Idea of the proof. To construct the necessary inferences, it is enough if in each so-called thread (see, for example, Takeuti 1975) the inference steps consist of such formulas, where the corresponding measures of correctness of the steps form a sequence of numbers, so that

- for "option (I)": no number is greater than E(Cons(D)):E(Pres(D)) and some number(s) are smaller than E(Cons(D)):E(Pres(D))
- for "option (I)": no number is smaller than E(Cons(D)):E(Pres(D)) and some number(s) are greater than E(Cons(D)):E(Pres(D)).

Example 7.1. Let us consider the tree shaped inference constructed in predicate calculus, which includes some incorrect inference steps:

$$\dfrac{\dfrac{X\to Y\vee X}{X\to Y\& X}\qquad \dfrac{Y\to Y\vee X}{Y\to Y\& X}}{X\& Y\to Y\& X}$$

The measure of correctness of the two inference steps in the top is 3:4. The measure of correctness of the bottom step is 1. The external measure of correctness of the entire inference is 1, but the internal measure of correctness is 3:4.

Example 7.2.

Let us consider the tree shaped inference constructed in predicate calculus, which includes some incorrect inference steps:

$$\dfrac{\dfrac{\to\neg X\vee Y}{\to Y}\qquad \dfrac{X\to Y}{\to Y}}{\to X\& Y}$$

The measure of correctness of the two inference steps in the top is 2:3. The measure of correctness of the bottom step is 1:2. The external measure of correctness of the entire inference is 1:3, but the internal measure of correctness is 1:2.

6. Inferences and comparing plausibility

We start with the approach of G. Jakobson, since it seems to best fit with the systematic approach that is based on the systematic principle (see section 2) and used by the authors. Therefore (see Jakobson 2011): *We define plausible future situations as situations that in some dynamic system with some degree of likelihood could happen at some time moment in future.*

The development of time-dependent systems, or in Jakobson's terminology - dynamic systems, in real macro world seems to happen according to the following principles (Lorents, Matsak 2011; Lorents 2006; Lorents 1998):

- *The principle of diversity of development opportunities* (A time-dependent system can have multiple different possible states for the next time moment)
- *The principle of being in one state* (Each time-dependent system is in each time moment t in exactly one state of all of its possible states)
- *The principle of no predestination* (The system arriving to a specific state in time moment t is based on chance – it is an event with some probability)
- *The principle of the correlation of developments and deduction* (A system is more probable to transit from the current situation to a new situation, when that situation's description is more correctly inferable from the current situation's description).

Presuming that greater probability corresponds with greater likelihood, we can use the above described to formulate the following principle:

- For the possible future situations of a time-dependent system, the degree of likelihood that is based on ***what is known*** is greater for a situation, where the description has a ***known*** greater measure of correctness in the inference that follows from the description of the current situation.

Note. The phrases "what is known" and "known" are very important in this part.

Indeed, in order to make decisions for a situation that may develop, we can only use our knowledge (of the system and situation in question) and what is provided by chance. Knowledge, however, is represented by certain atomic formulas (see Lorents 2009), which form the formulas (or arguments written down in a "very strict form") necessary to describe the system and the situations. When comparing the reasoning for the appearance of several possible future situations, we can still only use what we know and what we have. We just do not have anything else at our disposal. If we must decide which situation will occur in the situation, then we must very clearly identify the ***known*** descriptions of future situations and the inferences (or reasoning) that lead to those descriptions (since we are talking about the future!). Therefore, we can only compare ***known*** inferences, in order to make the decision on their measure of correctness. Thereat (NB!) it is possible that someone else who knows something else than we do at the moment, will provide a completely different assessment of the situation. There is no way around it.

Important question: Which measure of correctness did we just discuss?

Answer: Definitely only one of two – either internal or external measure of correctness, not two together or combined in some way!

Recommendation: It looks like it is "technically" easier to use the internal measure of correctness for plausibility. This is because the correctness of single steps is easier to assess.

Presuming that greater probability and likelihood corresponds to greater plausibility, we can formulate the most important principle of this work:

- For two possible future situations of a time-dependent system, the plausibility that is based on ***what is known*** is greater for such a situation, where the description follows the inference (that is based on the description of the current system) with the ***known*** higher measure of correctness, or in a more formal representation, if:

(1) at the time moment t', which follows the current time moment t, it is possible that the system M transits into situation $M_1(t')$ or into situation $M_2(t')$, and

(2) the reasoning used is based on the description of the current situation DesM(t), and

(3) it turns out that the reasoning $(DesM(t) \vdash_1 DesM_1(t'))$ behind the description $DesM_1(t')$ is "more correct" compared to the reasoning $(DesM(t) \vdash_2 DesM_1(t'))$ behind the description $DesM_2(t')$,

then ***for situations related with these specific descriptions and reasoning***, it is more plausible that the situation M(t) transits into situation $M_1(t')$ and less plausible that it transits into situation $M_2(t')$, or shorter -

$[Incor(DesM(t) \vdash_2 DesM_2(t'))<Incor(DesM(t) \vdash_1 DesM_1(t'))] \supset [M_2(t')<_{plaus}M_1(t'))]$.

Conclusion. The plausibility (based on what is known) of possible situations is greatest for the ones, where the description (consisting of corresponding formulas) has a logically impeccable (known!) inference.

Summary. In this work we considered how time-dependent systems transition to various possible situations and we explored which one of these transitions is more plausible. In order to do this we defined the necessary algebraic systems, including time as a system and the time-dependence of systems. It is possible to use formulas constructed from knowledge for describing the states of systems or situations. This way we can see if the description of one situation can be inferred from the description of another situation. We also considered a situation where not all inference steps are logically correct. By defining the measure of correctness we can compare the plausibility of possible future situations. We do this by comparing the measures of correctness of the inferences constructed to define and reason the corresponding descriptions.

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