Traditional and new materials: A good cocktail for the Mathematics' learning in Engineering Schools.

Ana Belen Cabello[1], **Angel Martin del Rey**[2], **Gerardo Rodriguez**[3] **and Agustin de la Villa**[4]
[1], I.E.S. Joaquin Araujo, Madrid, Spain
[2]Applied Mathematics Department , E.P.S Avila, Salamanca University, Avila, Spain
[3] Applied Mathematics Department , E.P.S Zamora, Salamanca University, Zamora, Spain
[4] Applied Math. Department, Pontificia Comillas University and Polytechnic University, Madrid, Spain

Abstract - *We begin by offering some general considerations about University studies in Spain, guided by the Bologna process. Owing to the necessary changes, we make a proposal about the materials to be used for Mathematical courses in Engineering Schools and the main characteristics that each must meet. On the basis of such considerations, we analyze a teaching strategy, which should be a cocktail containing good "drinks" (materials) with an excellent combination of such "drinks" and excellent "barmen" (teachers and instructors). We conclude with an example of a subject -Linear Algebra- in which the considerations described above have been carried out.*

Keywords: CAS, b-learning, e-learning, Linear Algebra.

1 Introduction. The Bologna process in Spanish Universities.

The Bologna process, which has just been implanted in Spanish Universities in 2010-2011, has involved a profound change in University teaching practices. For more information see European Commission, Directorate-General for Education and Cultures (2009) "ECTS users' guide". Accessed via http://ec.europa.eu/dgs/education_culture/publ/

The milestones of the process are:

i) Teaching based on competences instead of an accumulation of concepts, which was typical of the earlier curriculum. (See Brown, R.B et al [3], Miguel, M. [12], Mulder, M. et al [13], Zabala et al [17])

To achieve this, it is necessary to establish the syllabus of the subject, providing students at the beginning of the course with information about a series of **generic** competences (common features for all mathematical topics in their curriculum), such as self learning, capacity for analysis and synthesis, planning and organization, communication and writing, communication and mathematical expression, science and technology, use of communications technologies, critical thinking, etc., and **specific** competences, such as knowledge, understanding and using the mathematical concepts in Linear Algebra, Calculus in one and several variables, Differential Equations, etc., the ability to apply knowledge, computation and technology to solve mathematical problems for Engineering, etc.

These competences should appear in **learning results**, which are those that should set the guideline for designing the subject.

ii) Organization of students' work, attempting to optimize with this the time assigned to the study plan (1 credit means approximately 30 hours of students' work).

This aspect is of utmost important for the success of the process. Until now, the only work demanded of students was to pass a **traditional** exam at the end of the academic year successfully, although it has been increasingly more frequent to offer periodic exams (along the semester) to assess the progress of their teaching and the students' learning. The exams comprised a series of problems and issues to be solved by the students in a short period of time.

iii) Monitoring of student's learning processes, through activities carried out along the period in which the mathematical topic is taught. For this, it is necessary to have a careful planning of such activities, attempting to make them of different types (traditional exams, laboratory sessions, group work, modeling, and the resolution of engineering problems, etc) so that they can cover and evaluate the competences and learning results.

In sum, the Bologna process demands a teaching style with clear aims and learning results, monitored by a series of varied activities along the whole of the academic period. One example of Calculus course according Bologna process can be shown in Garcia et al. [11].

2 The materials for designing the course.

Careful planning of the course: concepts to be addressed, estimated time allotted to each, the way of delivering them, etc, all lead to a choice of materials that will be useful for students. The choice of such materials is very important for ensuring success in their learning.

Essentially, the materials to be used are the following:

A **textbook**, whose choice should aim at being especially adequate for the students. Some examples of appropriate text books could be Diaz, A et al [5], Garcia, A. et al. [7].

Files for the application of CAS. These files should reinforce, and in some cases could substitute, the traditional teaching of theoretical concepts. The CAS used can be chosen according the needs and possibilities. The most popular are: DERIVE, MAXIMA, MATHEMATICA, MAPLE, MATLAB, etc. Examples of these files can be shown in Garcia, A. et al. [7].

A learning management system (MOODLE or equivalent), which in the case of face-to-face University teaching should aim at allowing students to access the materials easily and which can also be used as communications tools (chats, forums, E-mail) and for the management of self-assessment tests or as a revision tool, conforming a battery of tests or problems that are offered to the students on a "random" way.

Some examples of use can be shown in Polytechnic School of Zamora in the document "Teaching guides". http://campus.usal.es/~guias2009/centros/guiagrado.php?id=21, Rodriguez, G. [14] and Rodriguez, G. [15].

The choice of some **web sites**, selected by the instructor that could contain: examples of the application of the mathematical contents, visualization demos, mathematical computations, the possibility for experimenting with problems offered randomly to students, etc.

3 The teaching strategy.

It is undoubted that there has been a prominent evolution in the teaching of mathematics. Twenty years ago, the teaching of this subject was reduced to an ordered collection of definitions, examples, theorems, corollaries and problems. Currently, however, it could be said that we are almost at the opposite extreme. At the present, Engineering Schools –in general- are diminishing theoretical concepts, and the teaching is focused in problem-solving without the necessary background to understand the problems.

It is not possible to understand the teaching of maths without theoretical concepts, which students must understand and be able to handle. This is because the role of mathematics in Engineering Schools should be formative (mathematics allow one to "furnish" the heads of the students) and also informative, providing the mathematical concepts that will later be used in more technical subjects of their curriculum.

We must also be able to bring mathematics closer to our Engineering students, showing them -as far as they are able to understand- cases of Engineering that they will be able to handle with the mathematical knowledge they have acquired. The teaching strategy should be aimed at optimizing students' work, fully bearing in mind that their work load should be reasonable within their circumstances since it would be stipulated by the credits assigned to each mathematical theme. In the current University structure, most subjects are given on a semester basis with 6 credits (180 hours of work/student, of which 60 must be followed in face-to-face classes). Accordingly, the obligations of the instructor are to "generate" a work that will take students approximately 2 hours to do per face-to-face hour of class received.

In light of the above, we propose the following:

Textbooks. These books could be considered as **musical scores** that the instructor will interpret according to the needs. Their structure is that of an exposition of theoretical contents followed by a self-evaluation test, solved problems and unsolved problems, sequenced in the order of contents and difficulty. These books will act as guides for theoretical and practical (problem-solving) classes.

A balanced use of CAS. To accomplish this, the first task to be undertaken is to choose an appropriate CAS, which to a large extent will depend on economic and logistical possibilities, inherent to the organization of the course. Theoretically, it would be possible to consider two types of CAS: Simple ones, such as DERIVE and MAXIMA, and more powerful ones, such as MATHEMATICA, MAPLE and MATLAB. In the case of simple CAS, we propose that, after having elaborated simple tutorials regarding their handling, which that must be studied by the students previously, they should be included in traditional lectures in visualization tasks, heavy computations, experimentation, etc. In the case of the more powerful software, we must design sessions in a Mathematics laboratory. Different possibilities about the use of CAS can be shown in Garcia et al [8], [9] and [10].

Reinforcement tasks (mainly the most relevant issues appearing in the text book).

Solution of real problems, in keeping with the **problem-based learning** strategy.

Students must model the problem, apply the required mathematical concepts, and solve the problem with exact or approximate techniques. All this must be done under the supervision an instructor who will set and tutor the different steps in the process of solving the problem. See Cazes, C.; et al. [4].

All the information will be transmitted through a learning management system, in which students will be able to access chats, forums and other communications activities and assessment tasks.

4 An example.

We expose the main characteristics of a course adapted to the parameters quoted in the previous paragraphs. This example is devoted to the Linear Algebra topics.

A standard Algebra course in an Engineering School should have contents similar to those specified below:

i) Algebra´s basic tools: Matrices. Systems of linear equations. Determinants.

ii) Vector spaces: linear dependence and independence. Vector subspaces. Basis of a vector space. Dimension. Coordinates. Change of basis.

iii) Linear transformations: Definition and properties of linear transformations. Matrices and linear transformations. Eigenvalues and eigenvectors. Jordan's canonical form.

iv) Euclidean spaces: Inner products. Orthogonality. Projections. Least squares method. Orthogonal diagonalization. Orthogonal transformations.

The materials to be provided to the students are as follows:

Textbook: The course is based on the book Villa, A [16], recently published. The book contains: theoretical results, theoretical questions, solved and proposed problems. This book also contains a CD with files (produced with different software packages: DERIVE, MAXIMA and MATHEMATICA) showing the different possibilities for using CAS in a Linear Algebra course.

CAS: We integrate the CAS in the teaching, since they allow students to experiment with different situations because they do not have to make tedious calculations by hand and they can solve problems that are closer to real-life situations and not only canonical problems with the results prepared.

We promote the use of the following:

-By the instructor in class with different aims: demos involving graphical results, computations, problem-solving, etc.

-Laboratory sessions in which the students can do **computer exercises** about the knowledge already gained in theoretical lectures.

-Tutorials that the students have free access to and that contain explanations of the use of the CAS in a general way or the CAS commands that they will have to use.

Depending on the type of software, its use will be different since software with more features, such as MATHEMATICA demands a more rigid syntax and hence it will be necessary to employ more time to gain the use of the CAS in a fluid way.

Accordingly, the introduction of the CAS is performed through **tutorials** through the following files:

- A tour, which analyses the possibilities of using MATHEMATICA in a general way.

- General concepts that provide insight into the MATHEMATICA commands useful for working with vector spaces, matrices, linear applications, determinants and sets of equations.

- Autovalvect, which explains the commands of MATHEMATICA that are useful for studying the theory of eigenvalues and eigenvectors.

- Euclidean Spaces, which shows the main MATHEMATICA commands related to Euclidean space and orthogonal transformations.

The use of MATHEMATICA would imply that some sessions should be devoted to introducing it since after many years of experience using it in the field its syntax and its use in general advise this. Once students believe that they are able to handle the MATHEMATICA commands, they are recommended to solve the problems usually addressed in class by hand with the help of the CAS. As an example, all the problems proposed in the text book have been solved using MATHEMATICA.

If MAXIMA is used, which at the present time of budget restrictions is advisable since it is **free software**, see Abanades et al. [1] there is no need to devote any special sessions to explain its use because the syntax is very easy. The students merely receive tutorials: matrices, sets of equations, eigenvalues and eigenvectors and Euclidian space, explaining in the tutorials the commands of MAXIMA that allow problems related to the concepts analyzed in the tutorials to be solved. The instructor uses MAXIMA integrated in traditional classes and students must do certain exercises of those proposed in the work tasks using MAXIMA.

In the CD accompanying the book there are several files including the solution (using Maxima) of many of the problems proposed in the different chapters of the book.

Electronic material: Certain web-sites (not many) showing applications, the capacity to experiment, etc., are provided to the students. One of them, for example, would be: http://aix1.uottawa.ca/~jkhoury/app.htm

Projects: One part of the assessment is the development of a Project in groups of 3-4 students. A list, not very exhaustive, offered to the students in the 2010-2011 academic year is as follows: Kirchoff´s laws for electric circuits, discrete dynamic systems, matrix factorization methods, matrices and cryptography, magic squares, the distribution of temperature on a plate, the Fibonacci sequence and the golden number, Leontief models, applications of the spectral theory: Genetics, population growth.

In each model, the aims and bibliography (traditional and through the internet) are provided to the students and they must report their results to their peers.

With the materials, a deep organization of the work must be prepared. The students, at any time, must know what they should be doing, how to use the CAS, how to approach the project assigned, etc. And all this bearing in mind that the students' work-load must be as stipulated. And obviously also the assessment method according the course's design must be explained carefully. See Bokhove, C et al [2]. Let us show an example of this kind of projects (see [6]).

A surveillance device has access to the images from a security CCTV that focuses on four sides of a building. (See figure 1). The device is programmed in such a way that it only shows one of the sides of the building on the screen. After showing the same side for one minute, it may "choose" to maintain the image from the same camera, with probability *a* ($0 \le a \le 1$) or may access one of the two contiguous sides of the building, with an equal probability, which would be (1-a)/2. The security guard controlling the device introduces the value of *a*. See figure 1.

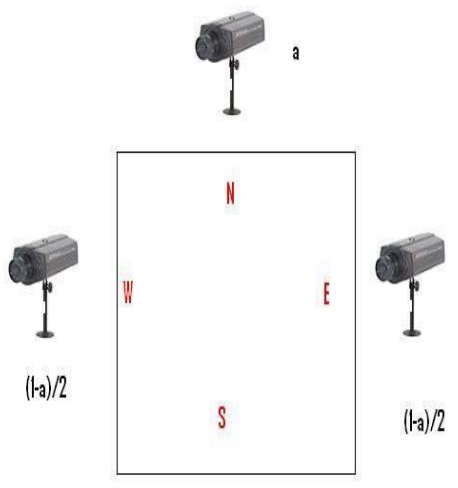

Figure 1: Surveillance device

i) Which value of parameter a should be introduced for displaying the same side of the building constantly?

ii) At 8:00 a.m. the device displays the Nord side. The guard introduces the value *a* = 1/2. Find the probability of showing each of the sides at 9:00 a.m. Analyze the same problem with different values of parameter *a*. Pay special attention to the cases *a* =0 and *a* =1.

iii) Study, for different values of a, the behavior of the device, after n minutes (with n very high).

iii) Study the same problem for different sides of the building n, distinguishing between n odd and even in the case *a* =1.

5 Conclusions

Bologna process has meant a profound change in University teaching, to which the teaching staff is now becoming adapted.

The characteristics of this process mean that the global work-load of the students must be suitably programmed.

To program the students' work-load the course materials must be selected with great care. We propose a mixture of traditional materials (textbooks and collections of problems) – with others that could be called e-materials, such as CAS, WEBs, etc., managed through a Learning Management System.

We also believe that students are moderately satisfied with the process, although these are early days and it would be imprudent to attempt to draw definitive conclusions, since we are still in the process of adaptation.

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