# High Performance Grid Computation of the Scattered Field Formulation for N<sup>th</sup> Order Debye Modeling of the General Dispersive Media

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Abstract - With the advent of the current wireless communication revolution and the increase in its applications, the electromagnetic researchers in conjunction with the medical physicians take the initiative of studying the price of this beneficial technical revolution. It is the health hazards due to the electromagnetic radiation on the human body. The most cost efficient mean of studying these effects is the numerical simulation using the popular FDTD numerical technique. The FDTD simulates the existence of the human body tissues using many fitting models. One of the most famous one is the Debye model. In this paper, the Nth-order Debye model for modelling the human tissues is represented using the scattered field formulation for deducing the FDTD update equation. The scattered field formulation is an accurate method of implementing the different excitation mechanisms for the waves that impinging on the human dispersive tissues media. Although the FDTD is an efficient method, it is a heavy computational one. It may take time of several hours or even days to simulate a single run of a specific problem. In this paper, a parallel scheme is utilized to speed up the running time of the 3D FDTD that includes The Nth-order Debye model. The parallel computations are running on VO (Virtual organization) of the EUMED grid platform. The speed up and behaviour over different number of processors is monitored.

**Keywords:** FDTD, Dispersive media, Grid Computation, parallel processing.

# **1** Introduction

The FDTD method has been applied successfully to a wide variety of problems including complex interaction of electromagnetic fields within materials. These materials include biological tissues, optical materials and ferrite [1] all of which exhibits dispersive behaviour. Three approaches based on the Auxiliary Differential Equation method, (ADE) are developed [2- 5]. The first one is the direct time integration method in which, generalized synchronized scattered field formulation for both Debye and Lorentz models are presented [2- 5]. The second approach is the Nth order ADE which is based on the state equations [4] and is applicable for both Debye and Lorentz media. The third

approach is called the polarization current algorithm for ADE [5] which is generalized to update the scattered electric field [6].

In this paper, the scattered field formulation for the Nth order Debye model [8] in conjunction with other five FDTD update equations are paralyzed using the Massive parallel interface (MPI) library.

Our parallel code is executed on the EUMED grid. ERI has been participated as a partner in the EUMED (European Mediterranean) Grid project entitled ("Empowering E-science across the Mediterranean") that is a project co-funded by the European Union. It was built using the GLite middleware. Network Time Protocol (NTP) with a time server is used for node synchronization.

#### **2 Problem formulation**

Based on the infinite impulse response (IIR) filter design, Sullivan [7] uses the Z-transform technique to calculate the electric field from the electric flux density. In this paper, the analysis is extended to include the N-order Debye model, with the optimum usage of the memory requirement. Then, the scattered field formulation is obtained [8]. The dispersive media update equation in conjunction with the free space; the Uni-axial Perfect Matched Layer UPML, perfect conductor, lossy media, and the thin wire approximation for wire antenna are paralleled as a single patch. The analysis of the dispersive media has the focus in this paper because it is the more general update equation in the FDTD algorithm.

Let's start with the displacement vector which is defined as;

$$\overline{D}(\omega) = \varepsilon_0 \varepsilon(\omega) \overline{E}(\omega) \tag{1}$$

Where,  $\varepsilon(\omega)$  is the relative permittivity function of the media.

Let's start with the N<sup>th</sup> order Debye model with the permittivity of the medium described as follows

$$\varepsilon(\omega) = \varepsilon_{\infty} + \sum_{p=1}^{N} \frac{\Delta \varepsilon_{p}}{1 + j\omega \tau_{p}}$$
(2)

 $\mathcal{E}_{\infty}$  is the relative permittivity at infinite frequency,  $\Delta \mathcal{E}_{p}$  is the change in relative permittivity due to the Debye pole, N is the number of poles, and  $\tau_{p}$  is the pole relaxation time.

Using the Infinite Impulse Response (IIR) filter design [7], the permittivity function can be represented in the Z-domain as follows

$$\varepsilon(z) = \frac{\varepsilon_{\infty}}{\Delta t} + \sum_{p=1}^{N} \frac{\Delta \varepsilon_p / \tau_p}{1 - e^{-(\Delta t / \tau_p)} Z^{-1}}$$
(3)

Transforming equation (2) into time domain results in

$$\overline{D}(t) = \varepsilon_0 \int_{-\infty}^{\infty} \varepsilon(\tau) \overline{E}(t-\tau) d\tau$$
(4)

The convolution integral of equation (4) is converted to a multiplication in the Z-domain, and a factor of  $\Delta t$ , the time increment, is included as follows

$$\overline{D}(z) = \varepsilon_0 \varepsilon(z) \overline{E}(z) \Delta t \tag{5}$$

Assuming N auxiliary variables  $\bar{I}_p(z)$  each one corresponds to one Debye pole of the permittivity function  $\varepsilon(z)$  such that equation (5) can be rewritten as

$$\overline{D}(z) = \varepsilon_0 \varepsilon_\infty \overline{E}(z) + \sum_{p=1}^N \overline{I}_p(z)$$
(6)

Where

$$\bar{I}_{p}(z) = \frac{\varepsilon_{0} \Delta \varepsilon_{p} \Delta t / \tau_{p}}{1 - e^{-(\Delta t / \tau_{p})} Z^{-1}} \overline{\mathrm{E}}(z)$$
<sup>(7)</sup>

Rewriting equation (7) in a more convenient form results

$$\left[1 - e^{-(\Delta t/\tau_p)} z^{-1}\right] \bar{I}_p(z) = \varepsilon_0 \Delta \varepsilon_p \left(\Delta t/\tau_p\right) \overline{\mathrm{E}}(z)$$
<sup>(8)</sup>

Now, equation (8) can be transformed into discrete time domain simply by shifting each field component multiplied by  $z^{-1}$  in the Z-domain, one time step later in the discrete time domain giving

$$\overline{I}_{p}^{n+1} = e^{-(\Delta t/\tau_{p})} \overline{I}_{p}^{n} + \varepsilon_{0} \Delta \varepsilon_{p} \left( \Delta t/\tau_{p} \right) \overline{\mathbb{E}}^{n+1}$$
(9)

Thus the auxiliary variable  $\overline{I}_{p}^{n+1}$  can be calculated from its previous value and the present electric field sample. Now, proceeding to get the electric field updating equation by transforming equation (6) into discrete time domain giving

$$\overline{D}^{n+1} = \mathcal{E}_0 \mathcal{E}_\infty \overline{\mathbf{E}}^{n+1} + \sum_{p=1}^N \overline{I}_p^{n+1}$$
(10)

Equation (10) reveals that the electric  $\overline{E}^{n+1}$  cannot be updated from  $\overline{I}_p^{n+1}$ , since it is calculated from the electric field at the same time step, so it is useful to substitute for  $\overline{I}_p^{n+1}$  by its value given in equation (9) and proceeding again

to get the electric fields in terms of the previous value of  $\bar{I}_p^n$  giving

$$\overline{E}^{n+1} = \left(\frac{1}{C_e}\right)\overline{D}^{n+1} - \sum_{p=1}^{N} \left(\frac{C_{xp}}{C_e}\right)\overline{I}_p^n \tag{11}$$

Where  $\overline{D}$  is the electric flux density

$$C_{e} = \varepsilon_{0} \left[ \varepsilon_{\infty} + \sum_{p=1}^{N} \Delta \varepsilon_{p} \left( \Delta t / \tau_{p} \right) \right]$$
(12-a)

$$C_{xp} = e^{-(\Delta t/\tau_p)}$$
(12-b)

Now we can summarize the programming sequence and summary of the updating equations within the time increment loop for the scattered field formulation by assigning the summation of the incident and the scattered fields instead of the total fields.

$$\begin{split} \frac{\partial \overline{D}(t)}{\partial t} &= \nabla X \overline{H}(t) + \varepsilon_0 \frac{\partial \overline{E}_i(t)}{\partial t} \\ \overline{E}_s^{n+1} &= (1/C_e) \overline{D}^{n+1} - \sum_{p=1}^N (C_{xp}/C_e) \overline{I}_p^n - \overline{E}_i^{n+1} \\ \overline{I}_p^{n+1} &= e^{-(\Delta t/\tau_p)} \overline{I}_p^n - \varepsilon_0 \Delta \varepsilon_p \left( \Delta t/\tau_p \right) \left( \overline{E}_s^{n+1} + \overline{E}_i^{n+1} \right) \\ \frac{\partial \overline{H}_s(t)}{\partial t} &= -\frac{1}{\mu} \nabla X \overline{E}_s(t) \end{split}$$

Where  $\overline{H}_{s}(t)$  is the magnetic field intensity.

The previous sequence is the general sequence for both the scattered field formulation and the total field formulation. The total field formulation can be easily set by considering the incident fields equal zero as the case of studying the effect of the scattering from mobile wireless devices on the human (user) tissues, so that the total field will be equal to the scattered field. Finally, one can say that the scattered field formulation may be considered as the general case.

## **3** Analysis of the parallel formulation

In this paper, the FDTD algorithm treats six types of media; the perfect conductor [9], the thin wire approximation [10] with infinitesimal gap [11], free space, general lossy media, Uni-axial perfect matched layer [9], and then the Nth order Debye model for modeling general dispersive media [6]. In our serial code, six functions are assigned to compute the three electric field components and the three magnetic field components at each cell location. Each function has a selection between the update equations for the six media. From the above discussion, it is worth mentioning that, in each function there is only a selection of one group of update equations for only one medium at a specific location. Figure 1 shows the locations of the field components in each cell. This ensures no Amstrong complexity in the analysis. Amstrong complexity occurs when the data structures are subjected to a sequence of instructions rather than one set of instruction. In this sequence, one instruction may perform certain modifications that have an impact on other instructions in the sequence at the run time. By analyzing the time axis, the electric and magnetic fields at each time step are evaluated from the neighborhood fields in the previous time step as shown in Figure 1. So, this axis cannot be distributed between processors. By analyzing the spatial axes x, y, or the z axis, the electric field or its corresponding displacement vector D or the auxiliary variables  $\bar{I}_p^n$  are calculated from the neighborhood magnetic fields. From this fact, one can divide one axis from the three spatial axes between processors to



calculate the fields as illustrated in Figure 2.

Fig.1 A unit cell from the discretized domain with fields' components' positions [9].



Fig.2 Data Dependencies [13]

### 4 **Results and Discusions**

Let us calculate the reflection coefficient at the interface between the air and the muscle tissue. The permittivity of the 2/3 muscles is assumed because the average permittivity of human body tissues is close to that for 2/3 muscles. The

associated parameters are:  $\varepsilon_{\infty} = 19$ ,  $\varepsilon_{s1} = 10019$ ,  $\varepsilon_{s2} = 42$ ,  $\tau_1 = 0.71 \times 10^{-6} / 2\pi$ , and

 $\tau_2 = 0.75 \times 10^{-10} / 2\pi$ . The one dimensional problem assumes problem space of 1000 cells, 500 of which are used to represent the air, and the remaining 500 cells are used to represent the 2/3 muscle equivalent material. The cell size is taken 37.5  $\mu m$  and time step  $\Delta t = \Delta x / 2c$ . The assumed incident pulse takes the form  $E(t) = 1000e^{-(t-t_0)^2/T^2}$  where  $t_0 = 400\Delta t$ , and  $T = 152\Delta t$ . Figure 3 illustrates the reflection coefficient of the numerical FDTD solution compared to the analytical one that is given by

$$\left| R(\omega) \right| = \frac{\left| \sqrt{\varepsilon_0} - \sqrt{\varepsilon(\omega)} \right|}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon(\omega)}} \right|. \tag{13}$$



Fig.3 The reflection coefficient at the interface between the air and the 2/3 muscle tissue material.

The parallel 3D FDTD is applied to calculate the effect of rectangular microstrip antenna, that is most used now in mobile phones, [12]. The antenna is 5 cm away from the head side. The space domain enclosing the human head and the antenna is equal to  $60 \times 60 \times 90$  cells. The microstrip antenna has a substrate material of relative permittivity 2.2, a substrate

thickness of 6.73 mm and rectangular patch size of 134.6 mm in the x direction and 111 mm in the z direction. The feed is performed via a microstrip line of 67.3 mm length in the x direction and 37 mm width in the z direction. A noticeable time reduction is observed up to 25 processor over the network "ce0.m3pec.u-bordeaux1.fr:2119/jobmanager-pbseumed" in the EUMED grid using the MPI liberary. The serial code over the same network takes time of 9520 second. Figure 4 shows the time reduction by applying the same problem on EURO-EMD Grid parallel copmutation enviromrnt. From Figure 4, one can notice a linear reduction of the time up to 14 processor while still exist a reduction even for 25 processors. The calculation time for the 3D FDTD is reduced by 38 times (2.6% only of the serial execution time) over 25 processors which illustrates the merites of applying parallel computation to the algorithm. If the number of processors increase than 25, the reduction in the execution time approximately stopped due to the effect of commuication time which at this point becomes comparable or even greater to the execution time.



Fig.4 The run time of the parallel 3D FDTD over the EUMED grid

# 5 Conclusions

In this paper, a parallel processing algorithm is developed for the FDTD code including six update equations. The focus of this study is in illustrating the parallel strategy of including the dispersive properties of the human tissues in a parallel code. The derivation of the scattered field formulation for including the dispersive properties of the human media is by using Nth order debye model. Good accuracy is observed when calculating the reflection coefficient from a two pole debye tissue. The parallel 3D FDTD code is applied on the scattering from the human head due to the excitation by microstrip antenna. The reduction of the execution time is observed up to 25 processor by about 96%. The parallel code is executed on the virtual organization of EUMED grid.

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