Application of quarter Iteration of FRFT in BRATUMASS for Weak Signal Extraction

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Abstract *- In this paper, we present a quarter iterative of FRFT algorithm to solve the problem, which is the extraction of weak signal from the back wave in BRATUMASS. [1] The energy of the same frequency in back wave add together and to discard the signal phase. The energy of the modulus as leading evolution function, the weak back signal in BRATUMASS can be separated out. Experiments showed that the quarter iterative of FRFT algorithm plays a bigger role in the extraction of the weak back signal.*

Keywords: BRATUMASS, quarter iterative of FRFT, extraction of the weak back signal

1 Introduction

As an active microwave imaging system, BRATUMASS can image the breast tissues for their dielectric constants which have the obviously different between malignant tissue and normal tissue. The transceiver antenna is placed on the breast surface and transmits microwave to breast internal. Backscatter will be happened and the back wave signals will be produce when microwave signals meet different breast tissues. The properties of the detecting target can be obtained by analyzing the back wave. Thus, the main problem, for gaining the target characteristic and reconstructing of detecting space, lies in the extracting of the back wave. The antenna will receive two main kinds of signals: one is the back wave signal radiated from the antenna main lobe to the target; the other is radiated directly from the antenna side lobe to the receiver. In addition, the energy amplitude of the latter is larger than the former. That will make the BRATUMASS, a frequency related system, create abundant problems of fuzzy object and interference in receiver. Usually, clutter can be eliminated by filter; however, the filters inevitably reduce the useful information component, and also made a few small targets back wave loss. We propose a quarter iterative of FRFT to solve this problem. Experiments show that the method can effectively extract the small target from the weak back signal in BRATUMASS.

2 The signals of BRATUMASS

Detecting points are located in the surface of breast. The system uses transceiver antenna, which are shown in Figure 1D. Side lobe signal of the transceiver antenna is $f_p(t)$. The system noise is $N(t)$. The position of the detecting points and the structure of the antenna are shown in Figure 1[2].

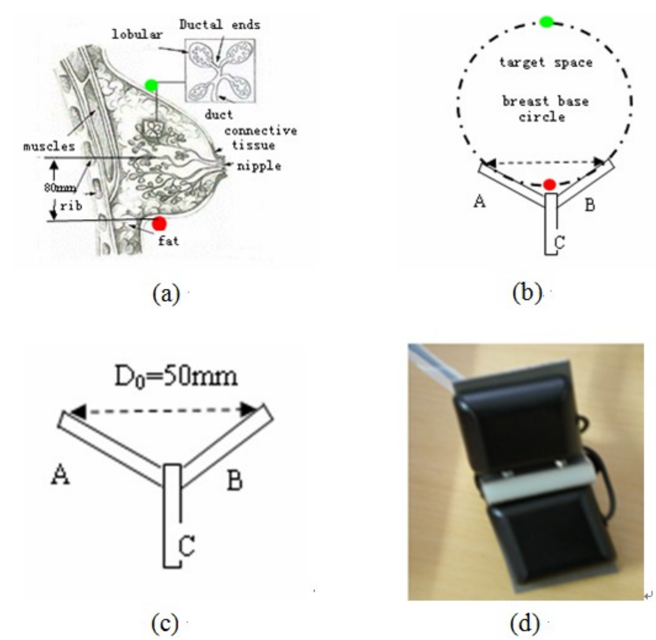


Figure1. The position of the detecting points and the structure of the antenna, (a)The structure of breast. (b)The schematic of *BRATUMASS* detecting position, Red point is the position of antenna and green point is the position of metal slice in (a) and (b).(c) The schematic of transceiver antenna. A is Transmitting Antenna, B is Receiving Antenna and C is Center Clapboard. (d) The photo of transceiver antenna.

The transmitting signal of BRATUMASS is:

$$S\ (t) = rect\,(\frac{t}{T})\exp\left\{ j2\pi[f_0 t + \frac{1}{2}kt^2] \right\} \qquad (1)$$

Where, $rect(t)$ is rectangular envelope; f_0 is initial frequency; T is time width; k is frequency modulation slope.

Side lobe signal of the transceiver antenna $f_p(t)$ is:

$$f_{\mathrm{p}}(t) = rect(\frac{t-t_{\mathrm{p}}}{T})\exp\left\{ j2\pi[f_0(t-t_{\mathrm{p}}) + \frac{1}{2}k(t-t_{\mathrm{p}})^2] \right\} \quad (2)$$

Where, t_p is the transmission delay of side lobe signal.

The signal obtained by receiver antenna chiefly includes two parts: $N(t)$ and $f_p(t)$. Figure 2 illustrates the structure of frequency mixing. $S_f(t)$ is the output of mixer, $S_f(n)$ is obtained from A/D sampler.

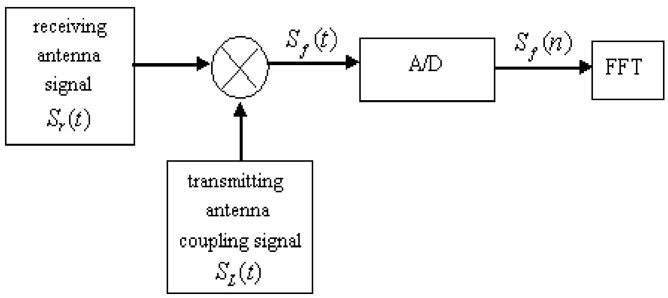


Figure2. The structure of frequency mixing of *BRATUMASS*

The output of the mixer can be expressed as:

$$S_f(t) = S_r(t) \times S_L(t) \quad (3)$$

Where, $S_r(t)$ is receiving signal from transceiver antenna.

$S_L(t)$ is coupling signal from transmitting antenna which satisfy the requirements of frequency mixing of zero IF. Suppose γ is a coupling coefficient, τ_0 is coupling delay. The $S_L(t)$ is:

$$S_L(t) = S(t-\tau_0) \times \gamma \quad (4)$$

The state, which has no target in the detecting space, is S_0, so the receiving signals include side lobe $f_p(t)$ and noise $N(t)$. The state, which only has one target in detecting space, is S_1, so the receiving signals obtain side lobe $f_p(t)$, noise $N(t)$ and the back wave of the single target. By analogy, S_n represents the state which has N target in the detecting space.

In the state S_0, equation (4) is substituted into equation (3):

$$S_f(t)\Big|_{s_0} = \gamma \times S(t-\tau_0) \times \left(f_p(t) + N(t)\right) \quad (5)$$

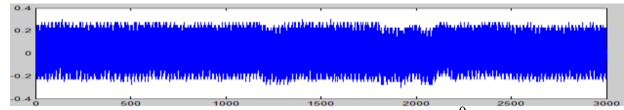
$S_f(t)\Big|_{s_0}$ is abbreviated to $S_f^0(t)$.

After Fourier transform, Equation (5) is changed to:

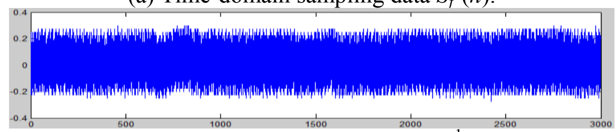
$$S_f^0(\omega) = A \times S(\omega) \otimes (F_p(\omega) + N(\omega)) \quad (6)$$

Where, $A = \frac{\gamma}{2\pi} \times \exp(j\omega\tau_0)$, $S(\omega)$, $F_p(\omega)$ and $N(\omega)$ is the Fourier transform of $S(t)$, $f_p(t)$ and $N(t)$, respectively.

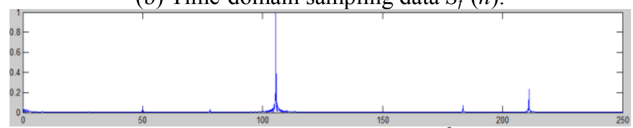
The sampling data of clinical case obtained by BRATUMASS is showed in Figure 3[3]. The positions of detecting points are illustrated in Figure 1(b). The actual measure distance is about 150~170mm, for patient posture is not perpendicular between mental slice and base circle.



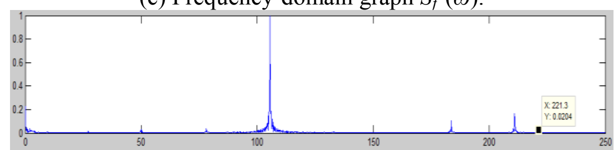
(a) Time-domain sampling data $S_f^0(n)$.



(b) Time-domain sampling data $S_f^1(n)$.



(c) Frequency-domain graph $S_f^0(\omega)$.



(d) Frequency-domain graph $S_f^1(\omega)$

Figure 3. Sampling data of clinical case obtained by BRATUMASS. (a) The time-domain sampling data $S_f^0(n)$ when mental slice wasn't placed. (b) The time-domain sampling data $S_f^1(n)$ when breast surface placed a mental slice with radius 1cm. The abscissa is the sampling ordinal, and the ordinate is the voltage level in (a) and (b). (c) The frequency-domain graph $S_f^0(\omega)$ corresponding to $S_f^0(n)$. (d) The frequency-domain graph $S_f^1(\omega)$ corresponding to $S_f^1(n)$. The abscissa is the frequency, and the ordinate is the normalized amplitude in (c) and (d).

According to the above graphs, we can see that frequency spectrum $S_f^0(\omega)$ is similar to $S_f^1(\omega)$. In Figure 3(d), the position of mental slice is marked (x:221.3/y:0.0204), 221.3Hz corresponding to the distance 169.5 mm. The efficient information of the mental slice couldn't directly extract from the spectrum.

Theoretically, there are no any targets of back wave information in the state of S_0. However, spectral lines about 105.7Hz and 210.6Hz always exist in the actual measurement. Their amplitude is higher than target of back wave signal and they also have the corresponding change with the change of objective and environment. Thus, elimination interference by filter is not properly. Consider the frequency characteristic of $S_f^0(\omega)$ in (6), quarter iteration of FRFT algorithm is presented to enlarge amplitude of object spectrum distribution in this paper.

3 Quarter of iteration of FRFT algorithm and the signal processing

3.1 Quarter of iteration of FRFT algorithm

$g(t)$ and $G(\omega)$ as a Fourier transform pair, the relationship can be written by

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(t) e^{-j\omega t} dt \quad (7)$$

$$g(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}G(\omega)e^{j\omega t}d\omega \qquad (8)$$

Following, brief write down for $G = F(g(t))$. $F(F(g(t))=F^2(g(t))=g(-t)$;

$F^4(g(t))=g(t)$. F^n indicates that operator is used N times.

$F(\omega)$ is the Fourier transform of $f(t)$, It has the following properties: $F(\omega)=\Im(f(t))$;

$\Im(F(\omega))=\Im(\Im(f(t))=f(-t)$. So repeat four times is periodic repeat.

From the perspective of nonlinear dynamics, the iteration of Fourier transom is an iterated function with period 4. Use the modular $\Re$ as iterative evolution function, for accumulating the same frequency during iterative evolution and discard the phase of signal. Then,

$$|F(\omega)|=\Re(\Im(f(t))) \qquad (9)$$

$$F^{(2)}=\Re(\Im(\Re(\Im(f(t))))) \qquad (10)$$

$$F^{(3)}=\Re(\Im(\Re(\Im(\Re(\Im(f(t))))))) \qquad (11)$$

$$\cdots\cdots$$

$$F^{(n)}=\Re(\Im(..\Re(\Im(\Re(\Im(\Re(\Im(f(t))))))))..)) \qquad (12)$$

After iterate N times, sorting frequency spectrum is obtained. Difference spectrum can be given as:

$$F_{\mathrm{di}}=F^{(1)}-F^{(n)} \qquad (13)$$

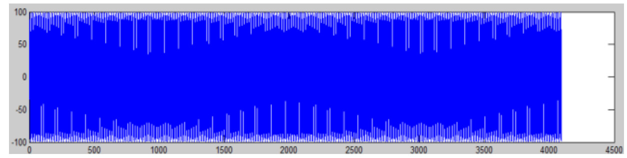
3.2 The influences to sinusoidal signal structure by quarter iteration of FRFT

Consider a signal $f(t) = A\sin(\omega t + \theta)$, where ω = 1.575GHz, θ takes random value between $-\pi$ and π. A might as well be valued 100， sampling frequency is 10 × ω and iteration number n = 4.

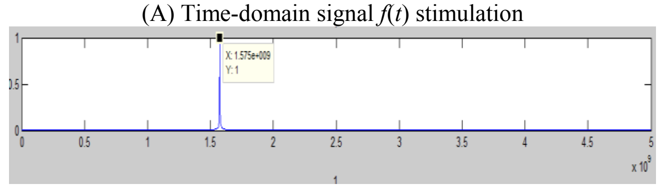
Fig.4 shows the processing of quarter iteration of FRFT. The iterative result in N=4 is the high order spectrum of signal. Consequently, the location of high order frequency in the spectrum is given.

3.3 Processing results comparison between Sf0(n) and Sf1(n)

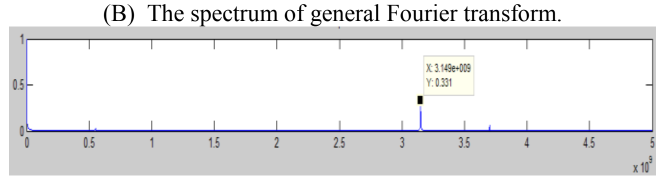
Compare fig.3 $S_f^0(\omega)$ and $S_f^1(\omega)$, Figure 5 demonstrates the processing result using the quarter iteration of FRFT algorithm. For the sake of convenience, the abscissa is transformed into distance which is corresponding to frequency, with unit mm. The ordinate is normalized amplitude. Figure 5(a) illustrates spectrum of the algorithm processing on signal S^0. Form (b), the back wave is saw clearly at 150~170 mm, which is corresponded to frequency 221.3Hz.



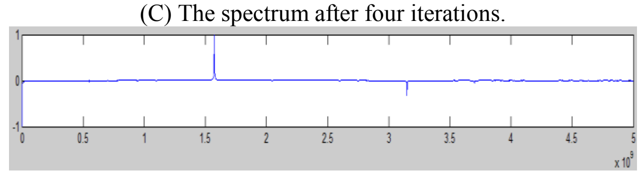
(A) Time-domain signal $f(t)$ stimulation



(B) The spectrum of general Fourier transform.

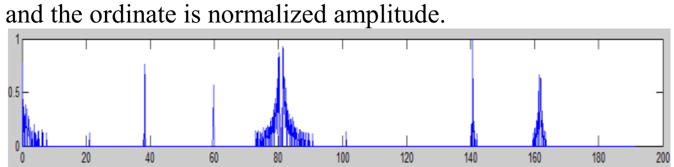


(C) The spectrum after four iterations.

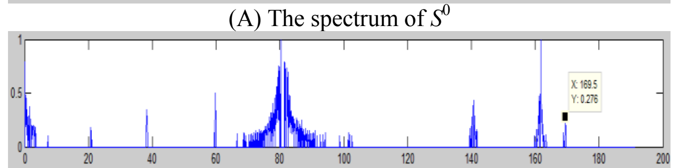


(D) the spectrum of $F_{di}=F^{(1)}-F^{(4)}$.

Figure4. Sinusoidal signal structure by quarter iteration of FRFT (a) The simulation signal $f(t)$. Its sampling depth is 4096. (b) The spectrum of normal Fourier transform, with N=1. (c) The spectrum after four iterations, with N=4. (d) The spectrum of $F_{di}=F^{(1)}-F^{(4)}$, the abscissa is frequency and the ordinate is normalized amplitude.



(A) The spectrum of S^0



(B) The spectrum of S^1

Figure5. The result using quarter iteration of FRFT algorithm (a) and (b) are the spectrums of S^0 and S^1 after separating by quarter iteration of FRFT algorithm, respectively.

4 Conclusions

The back wave spectrum amplitude is very small (see Figure 3(d)) in BRATUMASS. The amplitude of the back wave of the mental slice is only 0.0204, which is couldn't extract from the back wave. Nevertheless, invalid signal amplitude is high. (The system noise $N(t)$ and side lobe signal $f_p(t)$ are related to the frequency 105Hz and 210Hz, respectively. The amplitude of this two spectral lines is relatively large, and the amplitude of 105Hz spectral line even have verge on 1). According to signal characteristics, we proposed a method of weak signal separation in this paper. The amplitude of weak signal could be enhanced under the condition of reducing the loss of frequency as much as possible. In Figure 5(b), the amplitude of metal slice back wave is change from 0.0204 to 0.207(10dB). This algorithm is only one of the methods of the weak signal extraction in the preprocessing on BRATUMASS. For determine breast tissue properties completely, energy information extraction from back wave have to be further researched. Furthermore, target tissue properties are acquired. The problem on how to get character distribution in detection space by combining information, namely space image inversion [4] will be proposed in the following articles.

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6 Reference

1.Zhi-fu Tao, Xia-chen Dong, Meng Yao and Yi-zhou Yao. Biopsy Back Wave Preprocessing Research of BRATUMASS System based on Applications of Fractional Fourier Transform, Proceedings of The 2010 Inernational Conference on Bioinformatics and Computational biology Vol.II pp.258--261

2.Meng Yao, Zhi-fu Tao and Qi-feng Pan. Application of Quantum Genetic Algorithm on Breast Tumor Detection with Microwave Imaging，GECCO 2009, 2685–2688

3.Zheng, S. 2006. Breast Tumor Imaging Method Investigation in UWB near-field microwave environment. Master Thesis, East China Normal University，34-35

4. Zhifu Tao, Qifeng Pan, Meng Yao, and Ming Li. Reconstructing Microwave Near-Field Image Based on the Discrepancy of Radial Distribution of Dielectric Constant. ICCSA 2009, 717–728